PSYCHOLOGY: A NEW SYSTEM

By the Same Author

HUMAN DOCUMENTS

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APPROACHES: THE POOR SCHOLAR'S QUEST OF A MECCA

UNE QUESTION DE REPRÉSENTATION GÉOMÉTRIQUE

ETC., ETC.

PSYCHOLOGY
A NEW SYSTEM

BASED ON THE STUDY OF THE FUNDAMENTAL PROCESSES OF THE HUMAN MIND

BY ARTHUR LYNCH
M.A., C.E., L.R.C.P., M.R.C.S.E., M.P.

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PREFACE

To unravel complexities by the discovery of simple relations: that is, broadly speaking, the aim of all science. The expression of such relations as a law of Nature is one aspect of the matter; a law supplies a compendious reference to the whole field. Regarding then Psychology as a science which must be made as exact as conditions permit, we will be led inevitably in this region also to search for something comparable to a law from which all developments of the subject may be in due order evolved.

The problem itself should be set forth in its complete generality, and the solution stated ultimately in the plainest terms. Standing, therefore, before the unlimited domain of knowledge, the scope entire of thought or of mental experience in any form, we may ask: Is it possible to conduct the analysis through this vast and complex world so as finally to arrive at certain elements, the unanalysable processes; and to show that the modes of synthesis of these form the schema of all that can be known?

To offer an answer to this question is the object of the present work. The treatment departs from traditional lines, though that is not due to any desire

for novelty in itself; but rather, seeking to behold the diverse aspects of the question in the simplest forms, I have been led to considerations which are new not only in regard to subsidiary positions, but in the enunciation and solution of the basic problem of them all.

Book I. contains the formulation of the Fundamental Processes, and the proof of their necessity and sufficiency. Book II. gives illustrations of the application of the principles; Part I. dealing with various fields in which the operations of Reason are studied; Part II. being devoted to the elucidation of certain "positions" of prominence. Book III. discusses the development of Psychology in its historical aspects and in its future possibilities.

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In what way is a mind likely to be led to the consideration of the problems here indicated? Possibly by pursuing the question determinedly, What is after all the veritable sanction of reason? Or, What are the criteria on which finally we depend for the certitude of belief? Or, again, by virtue of suggestions from other sciences, it might be inquired: What can we find in Psychology comparable to that which the atomic hypothesis is to chemistry, or the cell theory to physiology, or the Cartesian method to mathematics? Or, if the Fundamental Processes be supposed to be similar for all minds, can we exhibit a basic general schema of the mind, and show that the differences of individual knowledge are due to the differences of the experiences that serve to fill in the schema? Can

we, in short, find a sort of organic alphabet on which the whole product of mental activity can be built?

It was only after many years of meditation that I arrived even at the formulation of such questions; the reason being that, when I approached them, they seemed to present no prospect of solution, nor even a point of attack. Moreover, supposing that one had by some fortunate insight lighted upon the veritable discovery, how would it be possible to prove that such an organic alphabet was necessary (that is to say, as forming the irreducible minimum required in the explanation of mental activities) and sufficient to account for the whole scope of things in knowledge?

In mathematics, it is true, we often show that certain conditions are necessary and sufficient to provide a solution of a problem; but there the field is definitely circumscribed and the character of the propositions allows of rigorous demonstration. But in regard to the range of psychical activity in the widest scope, how is it possible to make sure that the limit of analysis has been reached, and how can it be ascertained that no region of possible application is left uncovered?

The more natural way of presenting arguments would appear to be that of retracing the original steps; for if any mind, led on incessantly by the desire to know the secrets of mental acts, had arrived at conclusions in the objects of its quest, it would seem feasible to exhibit the route in all its successive turnings. But then the question of time enters, and also that of the bulk of exposition.

From early days these studies have held me in thrall, and I have been led from point to point in the pursuit of their truths; so that in my mind a certain way of thought, a complexion of meditation, has been formed; and examples of ordinary occurrence day by day have, through the years, helped to keep the course. This is the case, no doubt, with all who think for long and absorbedly on any subject. But how is it possible to transcribe into the form of demonstrations the evidence that depends really on habits of thought? The most that one can do is to formulate the definite basal propositions, and to indicate their applications, and in all these matters exhibit proofs as rigorous as the subject permits. This is done even in mathematics, for we find that the mode of exposition is hardly ever that by which the solution of a problem has been attained, although on the other hand it is interesting and illuminative to know the course of the original investigator, even of his mistakes, and of his tentative efforts.

. . . . .

The desire that has possessed me to pierce down to the ultimate basis of belief in all things has affected also my study of mathematics, for, apart from the formal demonstrations, there is the interest of discovering the origins of problems, and of searching for something that underlies the assumed basis itself. In this way I have been brought to meditations on the axioms of geometry, and also on that which is the real foundation of all systems of calculus, the process of counting.) Studies such as these may appear

at first sight not merely recondite, but trivial or futile. I am convinced, however, that no principle laid bare in the investigation of Nature remains long barren. And it has happened in the history of science that the speculations the most refined of a Descartes or a Gauss, or observations the most remote of an Oersted or a Branly, have subsequently resounded even in the realms of practical life. This is especially the case where the verities sought are those that lie at the base of a long series of analyses.

To understand the mechanism of thought, we must become acquainted with the finest wheels of the system, for in that way, and that way only, can we trace out in the closest succession the results produced in regular sequence. I would hardly care to say how much time I have occupied in posing, and in endeavouring to answer, questions relating to the axioms or to counting. Yet even now I do not see how I could have avoided the delicate but trying toils that this involved.[1]

The examples of other sciences serve to teach us also in what way by persistent analysis we may be led to the discovery of what are called laws of Nature, and indeed the history of the sciences might be indi-

[1] I am far indeed from holding that analysis consists in fine-spun disquisitions. On the contrary, I think that the characteristic sign of the true analyst, the thinker of veritable illumination, is to be found in the deep trenchant strokes by which he cuts to the basis of things, and the manner in which by bold generalisations he shows resemblances or correspondences in phenomena that had been thought to be far remote. Such is the style, each in his science of predilection, of Darwin, of Schwann, of Sophus Lie.

The study of the axioms was a lifelong preoccupation of Gauss, and the thoughts that he suggested have been developed by thinkers of the calibre of Bolyai, Lobatchewski, Helmholtz, Riemann, and Clifford. The question is, however, here approached from a different standpoint.

cated as that of the successive discovery of laws, more and more deeply based and possessing an ever-widening scope of application.[1]

Considerations of this kind induced me to turn again to the study of past masters in the province of thought—Aristotle, the German philosophers from Kant onwards, and the line of English thinkers from Locke to Bain, passing through Berkeley, Hume, the Mills, and Herbert Spencer. None of these, however, gave me precisely what I wanted. Only two—Aristotle and Kant—seemed to have approached the main problem I have indicated at all, and in neither case was the treatment at all satisfactory.

The great intellect of Aristotle was occupied with too many subjects of positive science to permit him to focus his attention on Psychology, and moreover he was working in the dawn of our civilisation. Kant's categories were not found as the result of an analysis expressly undertaken towards that end; the Koenigsberg thinker simply had recourse to the current expositions of logic. Moreover, his modes of thought throughout his system will be seen, by those who have patience to thresh out its complexities, to belong to the intuitive rather than to the analytical faculty, and his extensive expositions are in reality a vast *petitio principii.*

From the school of Experimental Psychology, and notably from a respected teacher, Ebbinghaus of Berlin, I have adopted some hints which I have utilised in the

[1] Leibnitz indeed conceived the genial idea that by generalisation on generalisation we might reach a fundamental law from which all others might be deduced.

study of *Memory.* *My* contact with them, moreover, and particularly the reading of Weber's "Tastsinn und Gemeingefühl," reinforced the desire that had been gathering strength to see adopted in Psychology a method as severely scientific as possible. Subsequently I completed a course of study covering the range of the objective sciences, undertaken partly with the hope of obtaining light in the observation of methods, partly for the reputed bearing of certain sciences—biology, anatomy, and physiology—on Psychology; and partly also with a view of extending the field of application for principles which were becoming more clear in definition.

Some of the observations derived from these studies I have touched on in the course of this exposition; they did not, however, greatly advance me in answer to the questions that still haunted me. I began to see that I should finally arrive at a mode of presentation quite different in form to anything that I had known; and, moreover, that in order to find a basis there was needful something more than persistent analysis along selected lines—something like a new cast of mind bearing with it that flash of illumination by which a picture is recognised from disparate parts.

Finally, I beheld the system which it is the object of this work to set forth. The task of exposition, however, presented difficulties owing to reasons already referred to, and I determined that, without much preliminary, I would simply posit the series of the Fundamental Processes, and subsequently prove them to be correctly established. This method involves no loss of rigour.

The care for rigour has indeed induced me to develop a great bulk of the argumentation in reference to mathematical processes. No special knowledge of mathematics is required to follow this exposition, although to those who have been previously unacquainted with mathematical studies the demand on the attention will be greater. I know no way of rendering facile the study of a subject which is itself both vast and intricate. It would be absurd also to attempt to make the discussion more interesting by any departure from the form of consecutive exposition. The interest of Psychology should be in Psychology itself.

What I have done, however, in the hope of lightening the reading is something that perhaps even the austere may forgive. I have sought my illustrations in current science, and that throughout a considerable range. One often observes that the study of mental phenomena appears dry because the examples given are merely schematic, and the reader has the impression of learning Psychology with the same air of bewilderment and futility that Molière's Monsieur Jourdain learned prose. There are times when for precision of statement, and for concentration of attention, it is advisable to use schematic forms; but wherever the exposition could otherwise be well carried on, I have rather chosen examples from what is being discussed around us in the realm of positive science.

Incidentally also I have referred to the works of previous writers, but it has been impossible to do this systematically, for on the one hand the gist of the

matter has been here reached, by a discipline of original thinking in which the old landmarks have been purposely abandoned, and on the other hand I have drawn my references, where necessary, not only from the technical works of Psychology, but freely from other studies.

The chapter on Localisation of Functions of the Brain will show the impossibility of making progress by the attempt to straighten out the tangled skein of the discrepancies of authorities. In this part of the subject the reference to the writings of others is more frequent than elsewhere, and as some of the questions are amongst those most eagerly debated nowadays, I have indulged in the exercise of a little polemics, if only to indicate how misleading may become the arguments derived from the battling of authorities.

Here as elsewhere the solution of discrepancies has been sought, not by tracing out comparisons in detail, but by seeking a deeper base from which the whole subject may become illuminated. The essential of the matter is, I repeat, the establishment of the Fundamental Processes of the Mind. Once that position is clearly apprehended in its true meaning and spirit, the rest follows inevitably as a consequence.

For that very reason a new treatment will be found in many of those points of detail which have become famous positions in Psychology. Amongst these may be mentioned the question of Externality. I seek, in this a ground of reconciliation between the Idealists of Berkeley's school and the Common-sense philosophers of Reid's; that is to say, not in all their conclusions,

but in what is valid in their arguments. This is to be achieved, not by frittering away the main positions of either, but by pressing the analysis of both more persistently and consistently than heretofore. The whole scope of application of Association will be found greatly widened. There is embodied also a new study of Memory leading to definite issues. The problem of Will is taken up anew, and the departure, carried out by various thinkers from James Mill to Muensterberg, from the mediæval position of an entity, Will, is reconsidered with circumspection but with more determined analysis; and Will emerges finally in clear light rescued from the destructiveness of these attacks. Various matters of peculiar interest are examined from new standpoints, as, for example, Fechner's Law, the axioms of Euclid, the operation of counting, the principles of Mathematical development, the conception of Infinity, Dreams, the Feeling of Effort.

All these matters have a bearing on the study of Reason, and that accounts for the elaboration with which that question is treated here. The study of Reason may indeed be looked upon as the origin of the whole exposition, or again, quite consistently, as the culmination of the investigations of which the basis is formed by the Fundamental Processes.

Various matters that have filled a large bulk of previous writings will be seen here to arise in the nature of corollaries from the positions definitely established. The system of the Transcendentalists will be brought to the touchstone of the analysis of Reason and of Will, and a clear light will be thrown on this

whole subjective architecture of imaginings. The principle of Hedonism which Epicurus and Herbert Spencer considered as vital to their ethical systems will be found presented at a deeper base than that at which they sought it. The position of Belief is formed naturally from the conclusions with regard to Reason. A note is appended with regard to formal Logic, and the lines are indicated, on which a new and useful classification of errors might be elaborated. The application of the principles of Psychology to the question of Localisation is illustrated by a discussion of aphasia, and the old notions of Localisation are shown to be too narrowly conceived. Finally there are suggestions with tentative indications of methods of that alluring subject which may be called—The Logic of Research.

In recent developments of Psychology there is a tendency to give the science an aspect as objective, or impersonal, as possible. But for the needs of the actual work I have been compelled to appeal largely to introspection; hence a certain personal note becomes manifest, but I would like to see this deprived of any merely self-assertive aspect. Thus when I say, I find so-and-so, I mean simply to appeal to the reader to test the matter for himself. Should he not at first, however, be inclined to agree with me, I do not become discouraged, for introspection itself is not always a steadfast, certain guide.

In the course of dealing with subtle, elusive thoughts there is met with a process of mind suggestive of the setting up of an instrument, or of attaining a point of vantage from which to illuminate the object. For this

a certain hermit-like quietude is necessary, and yet, such is the thinker's paradox, there must not be the hermit's remoteness that deprives thought of its zest and reality. It may be, therefore, only after a long period of study, and after patient, persistent efforts, that the thinker obtains a clear vision of problems that have puzzled him. But as with the microscope we may search long in vain for a clear definition of our object until in some fortunate circumstances it swims into our ken with all the unmistakable marks of reality, of shape, and function, and is henceforth held to be recognised; so, with introspection, something not altogether unlike may happen.

It is necessary to approach this subject with sensitiveness and with innocency of mind devoid of prejudice; but these merely candid virtues are not enough; there are also required a liveliness of zeal in the pursuit, patience grown to a habit, and yet withal a certain unresting, wrestling quality; above all, illumination. It is a toil in which the intellectual tools that are necessary are the pick and the lamp.[1]

[1] If it be at all pardonable to intervene with regard to the reading of the book, I would suggest that not only should the main divisions already indicated be kept in mind, but that the detailed *Table of Contents* should be continually referred to; and that the notes be disregarded in a first reading. If this course be adopted the consecutive character of the exposition will be observed, even if not on every point ostensibly indicated. But in pursuing the main course we obtain glimpses of alluring side vistas. The notes serve to point the paths to these, and hence on a second consideration they may be found interesting and useful. In no case, however, are they used to reinforce the arguments of the book by the mere weight of authority. That indeed would be contrary to its vital spirit.

TABLE OF CONTENTS

TO VOLUME I

BOOK ONE

PREFACE, V–XVI

CHAPTER I

INTRODUCTION, I–22

CHAPTER II

THE FUNDAMENTAL PROCESSES, 23–85

CHAPTER VII

NEW VIEWS OF GEOMETRY, 173–193

CHAPTER VIII

PROBLEMS OF INFINITY AND IMAGINARIES, 194–214

CHAPTER IX

EXAMINATION OF FECHNER'S LAW, 215–233

BOOK TWO

REASON

PART I

CHAPTER I

(For continuance of Table of Contents see Volume II.)

PSYCHOLOGY A NEW SYSTEM

CHAPTER I

INTRODUCTION

BEFORE entering definitely on the exposition of the system set forth in this book it may be well to touch, however cursorily, on certain of the more notable researches of the past.[1] We shall be thus enabled the better to ascertain what bearings in the vast sea of philosophical speculation any new line of thought, any original notion, may possess.

The question is difficult to answer: What is the province of Psychology? Possibly it is not well at first to bind ourselves by rigid definitions; for we find that the scope of the subject has included at various times not merely discussions ranging between formal logic and theories of morality, but also, amongst the Pythagoreans, the Leibnitzians, and the Lotzians, a sort of high regard or worship of numbers and "laws" in themselves; and amongst the disciples of Kant—Fichte, Hegel, and others—a worship of ideas, or rather abstractions derived from ideas—Transcendental Ideas. At the present day, however, the field of Psychology will be

[1] The present chapter is intended rather for those newly attracted to the subject. In the chapter "The Development of Psychology" references are given to modern works which will enable the student to find his way eventually in the whole field.

fairly enough appreciated if we say that it deals with what is *subjective* as opposed to *objective*; or, since our intention is to arrive at a fair understanding of the matter rather than to discuss on the threshold all manner of subtleties, we might express it by saying that Psychology is concerned with the inward processes and affections of the mind as distinguished from external things and their external interactions.

In the investigation of the problems that arise it is not now considered necessary to place any restrictions on methods that may seem helpful; and in our time particularly the sciences of physics, biology, physiology, pathology; curious special studies of heredity, lunacy, hypnotism, hysteria; direct experiments on various faculties, such as Memory; observations and experiments on the lower animals—all have contributed to aid our grasp of the subject.

In this subject, as in so many others, the first light seems to come to us from the Greeks.[1] Aristotle particularly dominated the minds of men for centuries, and his influence still prevails in the teachings of the schoolmen's metaphysics and logic. But Artistotle suffered from two disadvantages of the pioneers of thought: firstly, the positive knowledge accessible to him was much more meagre and more obscured than that which centuries of science have provided for ourselves; secondly, he endeavoured to build up a complete system, explaining the world on this too slender and faulty foundation. Even the prodigious mind of Aristotle fell far short of its self-appointed tasks; but, in enunciating his

[1] F. A. Trendelenburg and others have discussed remoter origins, and in Greece, long before Plato, these problems had been discussed; but Aristotle may serve here as a good landmark. Amongst the authorities may be cited F. Ueberweg ("History of Philosophy"), J. E. Erdmann ("History of Philosophy"), J. Burnet ("Early Greek Philosophers"), and Theodor Gomperz ("Greek Thinkers").

"categories," or forms of judgment, he laid one of the foundation-stones of the study of Psychology.

Kant, whose mind was greatly influenced by the works of Aristotle, nevertheless found the categories of the Stagyrite unsystematic; and then, searching in the orthodox text-books of logic for the enumeration of different kinds of judgment, and believing the classification to be satisfactory, he simply adapted it to his own point of view, *e.g.*:

Judgments	*Categories*
Universal	Unity
Particular	Plurality
Singular	Totality

etc., etc.

Kant's main inquiry was concerned less with the sources of our ideas than with the problem—what are the processes of the mind in receiving and dealing with ideas?

All this, however, is involved in Kant's peculiar Transcendental ethical system, and couched in the verbiage not always easily intelligible by which he represented that system. Kant had in fact posited, rather than exposed by analysis, a transcendental world where "categorical imperatives" held sway, where Pure Reason became resolved into Pure Will, and where the Ego, untroubled by the stimuli of the senses, existed in harmony with things-in-themselves.

The question, therefore, that arose in Kant's mind was how to communicate between the sensual Ego and the supersensual Ego. And it was in his efforts towards a solution of this problem that he finally reached the classification mentioned of the *forms* in which the mind can have access to sensation and thought. Kant elsewhere speaks of the Ego having three powers or faculties (Vermögen), of feeling, of

knowing, and of willing. All this is required in order to assist the single and unchangeable Ego to lay hold of the external world. Kant, however, disclaims the notion of his "Kritik" being regarded as a psychological work. It is rather the exposition of an *Erkenntniss-theorie*, a theory of the manner in which we arrive at knowledge.

This reference to Kant is not remote from the task of the present work, and it may be well to pursue the matter a little further. Suppose we draw a boundary between the outside world and the Ego. Locke, for instance, finds in the intellectual world of the Ego a great many terms not simple, which are in fact representative of syntheses; and his main business is to free the mind from various kinds of confusion respecting these, to analyse them, and thence to indicate their source: they have come into the mind through experience. Kant is not immediately interested in these ideas as they exist and combine in the mind, or directly in the external world from which they seem to arise; but he is bent on watching what happens at the moment of their traversing the boundary. He asks, By what manner of process, or under what conditions, is it possible that the outside world should impinge on the Ego?

If, then, Kant's analysis with regard to the elementary faculties of the mind had been complete and satisfactory, we might remove from his exposition his peculiar verbiage, derived in part from Aristotelian and Leibnitzian conceptions and imagery, and by substituting the physiologist's model, adapt his positions to modern views.

But, regarded in this manner, Kant's analysis is imperfect and confused, while the whole tenor of his thoughts is inconsistent with any evolution of his system in accordance with the march of science.

Much more helpful in the development of Psychology has been the line of the English thinkers from Locke to John Stuart Mill, passing through Berkeley, Hume, and James Mill; while Herbert Spencer and Alexander Bain have imported into the study new views and new illumination.

A very rapid reference to the history of Psychology as exemplified by these names may be useful in order to indicate the situation of this present study.

Locke himself was not a learned philosopher, and a great deal of his success was due to the manner in which he was able to disembarrass his mind of much that had passed for wisdom in the science to which he applied himself.

Locke writes in popular language, and his style is often bald, and the untechnical character of his phraseology sometimes allows opportunity for misconception, but his candour, as well as the excellent balance of his judgment, appear on every page. On the whole, it may be said that where he fails it is not by faulty analysis of the problem, as he conceived it, but rather that in some cases his analysis is not carried far enough, while in other cases he lacked certain positive knowledge that would have been of great service to his work. For instance, when dealing with the question of innate ideas, he seems to have formed too crude an appreciation of the position which he attacked, and his description of the child's mind before the access of experience as a *tabula rasa* is not altogether happy; for he ignores the doctrine of heredity, and gives no hint, as in such a phrase, of the great differences of aptitude of various minds to receive certain experiences, nor of that *spontaneity*, insisted on in a later day by Bain—the out-throwing of the mind, even under deficient external stimulation, towards that

development which it is destined to reach.[1] Locke's main position is that we gain all our ideas through experience, and that experience arrives through the medium of the senses. His dealing with the question of mind and matter is cautious and safe. He comes to the conclusion that there is a difference between the two; that nevertheless there is interaction between them, and that he does not know how this is effected. Locke, though remarkably astute, is often too limited in outlook. He places his whole reliance on introspection, and he does not use even this instrument to the full extent of its powers.

Berkeley, who followed Locke, pushed on the positions of the great path-opener, and widened the general scope of Psychology. The impression he gives is that of a man of genius with brilliant flashes of insight, and, it must be acknowledged, an inevitable tendency to wild vagaries. His literary style is captivating, too much so; for whereas Locke's baldness of expression bore with it a certain security against mystification, Berkeley is often able to conceal very irresponsible reasoning by rhetorical and plausible expression. Finding all our knowledge to be composed of ideas, he sweeps away all other substances except mind and ideas. What is called substance is to him merely the ideas of the qualities of that substance. What is called the outer world is merely an assemblage of ideas. Such is the main feature of Berkeley's formal psychological position; but there is a deeper notion beneath all this, for he refers to a supreme mind, that of God, of which the whole universe forms the ideas. Hence the inter-

[1] Karl Groos in his book "Die Spiele der Thiere" ("The Play of Animals") enters into this question in a manner somewhat adverse to the theory of Bain. Groos sees in the play of animals mainly a preparation for their life's work. But this would not be possible except for that spontaneity which Bain has so well thrown into relief.

action of mind and matter falls at once; he has the satisfaction of contemplating nothing grosser in existence than pure spirit.

Berkeley's emotional nature and his theological principles were intimately bound up with his psychological exposition. In dealing with successive points of issue he attained valuable results, for in endeavouring to explain the notion of externality as but an assemblage of ideas he was veritably making an extensive analysis of all that is contained in that position. Some of Berkeley's deductions, however, appear in the light of modern sciences absurd, for he altogether disparaged their methods of research as a means of acquiring knowledge. He displayed quite an intellectual fury, for example, against mathematics, a science of which Darwin later said that it seemed to give an extra sense to the thinker.

Hume, who followed Berkeley, treated the importation of a divine mind as having no warrant, and reinforced his position by denying evidence of any mind at all. He reduces, then, all knowledge simply to successions of ideas without the nexus of a controlling mind. A new position was also embraced in the province of Psychology, viz. that of cause and effect, which Hume analyses down to mere sequence. Further Hume seems ready to deny that any knowledge can be certain except what is contained in *analytical* propositions, that is, propositions in which what is asserted is really implied in the very meaning of the terms.

With respect to Hume's arguments in Psychology, it may be said that no one, not even Hume himself, has rested satisfied with the conclusion that there is nothing more in the universe than discrete ideas. The difficulties of such a position have been well expressed

by many, especially by John Stuart Mill, and it would appear that Hume's best service in this respect has been to demonstrate the unwarrantable nature of many of Berkeley's divergences from Locke.

We habitually act on the assumption that there is more in substance than the qualities by which it makes itself immediately known to us. The question will be considered in the analysis of Externality in this book. Again, with respect to Hume's analysis of cause and effect, there are sequences, even invariable sequences, which are not spoken of as cause and effect; accordingly the matter requires circumspection. But if the analysis shows clearly what manner of sequences and what restriction of conditions are implied in a proper use of these terms, then Hume's position is brought into agreement with that which is accepted, at least in scientific circles.[1]

Hume appears nowadays to have been remarkably shrewd and clever in exposing untenable positions rather than profoundly philosophical. He advanced the study of Psychology by his impatience of loose speculation, by cutting off the tendency towards various paths of error, by his stimulative style, and by his insistence that only by definite analysis must even his paradoxes be unlocked.

The problem of the Association of ideas was dealt with in a remarkable exposition by Hartley, who is in point of time really between Berkeley and Hume. James Mill made another definite advance by attacking the position of Will and doing much to free it from confusion. It is not meant, of course, that these questions had not been dealt with before; for indeed the whole problem, or again the whole series of the problems, had

[1] A book recently published by Dr. Karl Pearson, "The Grammar of Science," represents this matter very clearly.

been discussed with more or less insight or confusion from the beginning, and the arguments of time-honoured questions have by no means yet come to a conclusion—for example, in respect to the distinction between sensation and feeling, as instanced in Kant's Vermögen; or with respect to Will, and Kant's Vermögen of doing, this latter again being identified with the power of *conation*, conation again implying a sense of energy put forth, and this issue again provoking endless discussions anew, though withal involved in Berkeley's theory of vision.

Besides these names, there are scores of others in English philosophy marking either some advance or retrogression, or difference of handling, of Psychology. Sir William Hamilton and Mansel are the two most prominent English disciples of Kant, and both are very acute thinkers at their best; but at their worst sophisticated and mystifying, writing to defend pre-established positions rather than pursuing their thoughts with the sole desire for the elucidation of truth. Reid was the great champion of "common sense," and he and Brown did good work in making more secure the analysis of cause and effect. Dugald Stewart, Abercrombie, and others succeeded in diffusing a taste for psychological studies by a style of exposition more popular and interesting than scientifically valuable.

Turning to France, we find in the early days of modern science, Descartes, a man of great enlightenment, particularly in mathematical subjects, to which he gave enormous impulse. The ingenuity of Descartes, even in the domains of speculation where he most signally failed, was extraordinary, and certainly he was not at a loss for suggestions when he tackled the great problem in which all his other work was contained. He seemed to regard the soul as a sort of entity, which sat on the

pineal gland as a throne, and from that coign of vantage surveyed the movements of the thoughts, which were very subtle substances, and as they flew past directed them to their places. No thinker has ever followed Descartes seriously in these views, and indeed it may be said that Psychology was the branch of study in which his thoughts have been most barren of results. The luminous mind of Pascal, hardly inferior to Descartes in mathematical genius, contributed nothing definite to technical Psychology.

In a later generation of French thinkers we find in most of them a philosophy which is rather a general discourse on life, or, as in Voltaire's "Candide," a witty indirect comment on the inconsistencies of ethical systems in vogue, rather than a thrashing out of technical questions. Condillac and his school have been reproached by John Stuart Mill with missing the spirit of Locke's teaching and fastening with peculiar eagerness on his weaknesses. Condillac's style is popular, brilliant; his intention is excellent, but the analysis is not sufficiently profound. Preceding the French Revolution, and exercising a considerable influence on the course of events leading to that mighty upheaval, we find a number of men—Diderot, La Mettrie, d'Holbach, Helvetius, and others—representative of French Materialism. They have done service, even by their exaggerated positions, in preparing men's minds for scientific treatment of the whole problem. In this tendency their work has been supplemented by the great physiologists and biologists of various countries, from Johannes Müller to Claude Bernard, from Lamarck and Geoffroy St. Hilaire to Darwin.

So far then, fixing our attention solely on the works that have added essential links to the great chain of consecutive exposition in Psychology, and regarding

only the most salient positions, we might thus roughly tabulate results:

Aristotle.—Defines the subject. Posits the categories.

Locke.—Clears the ground. Method of introspection and analysis. Source of Ideas: Experience.

Berkeley.—Analysis of externality. Attempts at analysis of time.

Hartley.—Doctrine of Association of Ideas.

Hume.—Analysis of cause and effect. Ideas: Reproduction of images but less lively.

Kant.—Space and Time, fundamental positions. Imperfect tabulation of mental processes. Review of faculties: Feeling, Cognition, Conation.

James Mill.—Examines will.

John S. Mill.—Sets up mind, against Hume. Establishes logical groundwork.

We meet with the modern treatment of Psychology in Alexander Bain. He took the nervous system as the basis of his explanations, and his lucid expositions throw an entirely new light on the matter. The value of Bain's method would be great even if the nervous system were to be regarded only as a symbol, a graphic representation to enable us to indicate the "forms" of intercommunication, for example, of Kant's inner and outer worlds; or again, if it merely served to give a sort of classification, a continuous *course* to Psychology, instead of, as we have seen, the successive and somewhat disconnected attacks on various positions. But the real value is far greater than all this, for there is a close and invariable connection between physiological processes and psychical experiences.[1] No man had previously approached the work of Psychology with a mind both so well adapted and so well trained in the requisite subsidiary sciences as Alexander Bain. He reviewed the

[1] Höffding, whose psychological studies are weighty with erudition, has drawn up a long list of the analogies between the nervous system and the mind. Bastian, "The Brain as an Organ of Mind," may also well be consulted. An article, "Brain and Mind," by Dr. Charles Mercier in the *Lancet*, November 1909, though short, contains many observations of value on a subject in regard to which great misapprehensions exist.

work of his predecessors, and being inferior to none of them in acuteness of analysis, and possibly superior to all except Locke in balance and candour, he was able to conserve that which was valid in their teaching, while cutting off those extravagances which have in general resulted from the straining after some emotional satisfaction or theological need.

Bain's style is flat and hardly adequate to the best expression of his thoughts. His meaning is generally plain enough, though even on this ground he has been severely criticised. Thus, in explaining the law of Relativity, he says: "We do not know anything of itself, but only the difference between it and another thing." He has been accused of confusing here the two meanings of the word "know," corresponding to *savoir* and *connaître* of the French, but the intention of his words is clear.

Bain has been attacked also for what is one of his characteristic positions, the recognition that belief must be considered as bound up with an *activity* of the mind, being a manner of regard in passing from one set of ideas to another.[1] Bain's principal contributions to the advance of Psychology in the way of technical study have been luminous expositions of the law of relativity, the law of spontaneity of ideas; while at the same time he made clearer such positions as those of Association, Belief, and Will.

Herbert Spencer analysed to a deeper base even than Bain; and by his work we are able to join on, and exhibit the bearings of, the researches of Darwin and all the great biologists. Bain's work may be regarded as supplementary to Spencer's, dealing with the more technical points of issue, whereas Spencer laid down the great framework and formulated the main principles.

[1] The whole tendency of the present exposition will be to assert the *dynamic factor*, having a physiological basis, in every state of belief.

Spencer traced up the development of ideas from rudimentary facts of consciousness, there being a continuous progress involving in its chain sensations, feelings, instincts, passions, emotions, intellectual ideas and complex states, reasonings, beliefs.

To understand Herbert Spencer aright, one must, throughout his works, hold the germinating idea whose development produced them all. Spencer, seeing in the world on all sides evidences of progress, even though towards a distant, ill-defined goal, seized for contrast upon the organic beings at the two ends known to us of the scale of creation—amœba and man. Examining attentively, and endeavouring to understand corporal peculiarities and functions on the basis of broad general ideas, Spencer formulated the differences as showing, in proceeding from the lower to the higher, a change from homogeneity to heterogeneity, from simple unco-ordinated functions to complex co-ordinated functions, from indefiniteness in time and space to precision in time and space.

The doctrine of evolution does not itself always afford a standard in respect to higher or lower development, for all manner of questions may arise with respect to proportionate importance of the factors it involves. Even the conception of the amœba adopted in this comparison may be held to correspond less to the creature's real constitution than to the defects and limitations of our microscopical observations.[1] Moreover, the doctrine of evolution is not a law of nature in a sense comparable to that of universal gravitation, for side by side with evolution in Spencer's sense there is a continual contrary set of reactions; the factors are not

[1] Recent work by H. Pénau and by C. C. Dobell shows that bacteria, described generally as enucleate cells, are far more complex than hitherto supposed. As the powers of the microscopist increase, "structureless" living things seem to grow in development.

quantitatively expressed, and it is not clear how, as is always the case after the formulation of a natural law, new discoveries result from deductions and applications of its principles.[1]

The doctrine of evolution may be regarded rather as a sort of principle of classification, enabling a complex subject to be dealt with in an intelligible manner. The whole of Spencer's synthetic philosophy consists of an exposition of various subjects, extending from those of the individual mind to the relations of man in society, according to the scheme of this doctrine of evolution.

Much of the vast bulk of Spencer's work is occupied with matters appertaining to special sciences, and of which the laborious handling is not necessarily a part of his synthetic philosophy. For instance, a detailed description of the nervous system, such as occurs in his "Principles of Psychology," belongs rather to the anatomist than to the psychologist. The psychologist should of course be familiar with the results that bear on his own science, and, apropos, should cite them. But if the philosopher delay to expound all the subjects that enter into his domain, he will find himself at length simply editing an encyclopædia, and in the course exhibiting some position in Psychology. His physiological psychology is apt to become, as has been said of Wundt's, a Physiology and a Psychology.

Moreover, in the course of the exposition of the synthetic philosophy, and in the works which are incidental to it, there is a good deal which is not necessarily dependent on ratiocination derived from the doctrine of evolution. Therefore, when we strip off from Herbert Spencer's works in Psychology all that is

[1] A reference may here be made to two recent "Herbert Spencer" Lectures: "Herbert Spencer and Animal Evolution," by G. C. Bourne, 1910; and "Evolution, Darwinian and Spencerian," by R. Meldola, 1910.

not consecutive, but adventitious; and if we refine it all to the continuous tracing out of definite lines of argumentation, we find that when we have thoroughly seized the meaning of the doctrine of evolution, its formula becomes a luminous lamp such as formerly Berkeley's doctrine of ideas.

Spencer's work has not said the last word in Psychology. On the contrary, as with Descartes in mathematics and Darwin in biology, we find that Spencer's expositions, opening up profounder views of the subject, are full of suggestion with regard to new researches.

Already, however, Weber and Fechner in Germany had laid the foundation of a new branch of psychological study. It may be observed that the German thinkers, from Leibnitz downwards, and including Kant, Fichte, Schelling, Hegel, Schopenhauer, formed in their succession a sort of spiritual lineage, while Herbart and Lotze refer more directly to Leibnitz himself.

Fichte proceeded from the moral standpoint even more emphatically than Kant, and enthusiastically seizing upon Kant's notion of a world removed from the coarser attributes of sense, he attempted to ascertain what then *is* the thing in itself. The world of Fichte is a conception of an absolute consciousness—"The Divine Idea of the World." Its germinative principle is the realising of morality; and the infinite spirit, or the absolute mind, develops itself or spins itself out spontaneously into the finite spirits; hence our world. Fichte is the high-water mark of Idealism. He is notable as having exerted a great influence upon Carlyle in his younger days, as exemplified in "Sartor Resartus," which derives much of its spiritual atmosphere, and much also of its incomparable, all-pervading humour from Carlyle's traduction of Fichtean ideas into the concrete and localised applications of the philosophy of clothes.

Schelling begins with the outer world, and gradually brings it all back to a manifestation of mind. However, in order to make the nexus, he supposes a certain Absolute to hold sway over both provinces.

Hegel takes up the speculation, and applies his dialectical powers to the discussion of what this Absolute may be. Schelling himself suspected that in the final analysis everything would be found to be something like electricity. Hegel, however, declares in favour of a Spirit, of which the essential being was to think in terms of Pure Reason.

Schopenhauer's theory, "The World as Will and Representation," is not unlike Schelling's, but he believes that that which remains when everything phenomenal is removed is Will. This is, again, not unlike Hegel, for though Hegel places Reason where Schopenhauer places Will, yet Kant, the original source of their inspirations, conceived that in this transcendental world Will and Reason coalesced in identity.

Lotze proceeds less from a psychological than from a theological standpoint. He does away with Schelling and Hegel *in toto.* His own system may be described as teleological Idealism. Like Leibnitz, he attaches great weight to principles or *laws* in themselves, for though in one point of view the tracing out of laws is our severe task, yet again they appear almost like mediators between ourselves and a personal God. When one gets a vision of Lotze's system, it has an alluring aspect, more fascinating indeed, even though not more substantial, than the dreams of Kubla Khan.

Herbart brushes away the categories of Kant, which Hegel had adopted and improved, and evolves the theory, with some hints from Leibnitz, that the world is composed of feelings and ideas, which, clashing in the mind, evolve a product—psychic experiences; and

that is what the world really means. He applied mathematics to these speculations in a most extraordinary way, and pointed out how a gradual increment of elements of feeling would result at last in a definite psychic experience. Herbart's style is not sufficiently lucid, but he has no lack of fruitful suggestions, especially if they be illuminated on the one hand by Fechner's psycho-physics, and on the other by the teachings of physiology.

Weber broke away from the old traditions of the German school with their grandiose but illusory system-building, and applied himself to tangible problems within the scope of Psychology. Fechner endeavoured to obtain accurate measurements of the amount of additional external stimulus corresponding to what we may appreciate with sufficient definitiveness as, say, twice or three times the amount of sensation or feeling.

Physiologists, again, working from more objective positions have shown that gradual increments of stimulation of a nerve will at length result in the conveyance of a subjectively appreciable impulse. Herein we have suggestions of two great developments, that may be treated separately, of the subject of Psychology. Psycho-physics, or Experimental Psychology, finds its field in the development of the ideas of Fechner and of Weber. Psychology, approached through the medium of physiology and biology, has been exhibited in England most notably in the works of Bain and Spencer, as we have seen; and in Germany in those of Wundt, who, moreover, has occupied himself with the research of Experimental Psychology.[1]

After this rapid review of the ground of Psychology

[1] It is difficult to make a selection from the works of later writers. Incidentally, in the course of the exposition, references will be found to various thinkers in England, France, Germany, Italy, and America.

we find that much requires to be done. For a series of generations the study of this subject was retarded by the influence of literary and historical forms of exposition. Rarely, except in restricted problems, has an attempt been made to deal with Psychology in the same manner as with other sciences, to present what is known in a clear, sequential style, and to indicate the lines of investigation that would lead to further knowledge.

Contrast with this the methods adopted in mathematics, which may be selected as the science which affords the best example of a connected exposition. The scope of Mathematics, including its manifold applications, is that of the relations of all possible groups of numbers and quantities, and the measurements of the external world and all its parts; so that the problem is of a maximum of complexity. But the long progress of patient analysis has brought the science to such terms, starting from simple postulates, that an instrument has been developed which, though imperfect, is yet admirable within its scope.

The whole science of mathematics could be expounded without any reference to the originators of theorems or methods. Certainly it would not be well always so to expound it, especially in its higher reaches; for if a theorem of a great thinker be presented isolated from the main body of thought which led up to it, we lose greatly in being deprived of the study of the development of the mind of the master, and of the multitudinous associations pointing the way to new investigations towards which suggestions arise. We recognise, however, that the science has a structure and development of its own, independent of the names that adorn it, or of the considerations that led to its theorems.

In the study of Psychology, philosophy, or metaphysics, the usual process has been to describe the systems of ideas of the great thinkers in succession, and often without indicating amid the heterogeneous assemblage of notions—some good, some false, some merely baseless fancies—the gradual progress of the science. As well might we teach mathematics by detailed examinations of the philosophical systems of Pythagoras, Apollonius, Archimedes, and Albertus Magnus, and that without regard to distinctions between works of permanent value and theories which have failed.

Unfortunately there is not yet sufficient agreement even regarding the fundamentals of Psychology, and the science itself is not sufficiently exact, to enable us to hope at this epoch for a formal and rigorous exposition after the style of mathematical reasoning. But at least we should always bear in mind the respect due in this science to accuracy, to candour, to closely consecutive, enchained, progressive thought. For, rightly understood, Psychology is the most general of the sciences, the matrix that contains them all.

The study of rudimentary mental operations is the foundation of this present work; but the starting point has been arrived at by processes of analysis proceeding from problems of more striking practical interest. The sources and the lines which have led to the base have been thousand-fold.[1]

For example, remarking the value of gymnastic

[1] I find in a syllabus of a communication to the British Psychological Society, 1910, "The Nature and Development of Attention," by a well-known author, Dr. G. Dawes Hicks, the following words: "A new way of interpreting these facts evinces itself when the matter is treated from the genetic point of view, and the attempt is made to form some conception of the circumstances under which the earliest manifestations of attention are rendered possible."

This is excellent. I do not, however, follow Dr. Dawes Hicks to his conclusions; my own views of this matter will appear in the course of the exposition.

exercises in developing the body, I asked, "Is it possible to formulate a series of mental exercises tending to strengthen the powers of the mind, and arranged, moreover, in such a manner that in elaborating the science of education we may be also preparing an education in science?"

Now, observing how many of the operations in various sciences resembled each other, except for differences produced by the actual objects of study and their associated details, the further question arose in my mind: "Is it possible to exhibit a schematic form of mental operations, so that the process of education in any particular science may be in the nature of filling up the formulæ with the concrete experiences obtained in that science?"

Or again, observing the increased efficiency due to good style in rowing, I asked: "Is it possible to cultivate a superior faculty of reasoning in order to apply this as an instrument in any particular problem that obstructs the progress of thought?" Here we are led to the question: "What is really the nature of reasoning, apart from the incidents that may produce confusion of mind in considering any particular examples of reasoning?" "What are the factors of reasoning that may possibly be exhibited as faculties of the mind capable of cultivation?"[1]

[1] Or again, to take an example which will be appreciated by students of mathematics: In the study of differential equations we reach a limit of the application of our calculus, and it has been suggested that the insoluble forms are the very expression of the terms of insolubility, and that our algorithms (or modes of mathematical symbols) are not otherwise capable of expressing—as in the manner usually recognised as a solution—the particular conditions, involved in these problems, of the variability of a number of variables.

The consideration of such a difficulty leads us to the study of the algorithms themselves; their limitations; their development; and hence of their origin. The study of their origin leads us directly to the study of the conditions under which the mind conceives of calculations of areas; hence of the principles of multiplication and of addition.

Or again, observing how in the course of mathematical progress the system of

In the study, and in the subsequent exposition, of this subject I have thought it well not to refuse light from any source; but considerable use is made of the old methods of mental analysis and introspection. It has been my endeavour to establish each position in succession in as clear a manner as possible, and then without elaboration to proceed to the next so that a consecutive development may be seen to arise. Absolute clarity of view is the main desideratum in Psychology; and this is generally best served by a plain, even arid, style of presentation. In incidental comments and illustration, however, there is no necessity to preserve this severity of manner.

Technical language has been avoided as far as possible, though with full recognition that it is often useful in giving precision to one's expressions. What is needed in a subject so abstruse as Psychology is that language should be employed in such a manner that the reader is led to form the clearest view of a problem, so that if he do not reach the solution to which one wishes to lead

co-ordinates of Descartes, the Differential Calculus, Determinants, Quaternions, and other modes have arisen as new instruments of calculus, the question might be posed: What other systems of calculus are possible? What is the great virtue of the differential co-efficient? It is the ratio of the increment of the function to the corresponding increment of the variable, when this increment is infinitesimally small. Now ratio, implying division, depends on multiplication, and multiplication is but a systematic way of representing addition; and increment also implies addition. Pursuing our analysis we shall find that addition is the representation of the process of counting, and the study of counting leads us to the consideration of the Fundamental Processes of the mind.

The study of the problem of estimating areas again leads us to the examination of the importance of the rectangle, and thence of the right angle itself. For example, we might pose the question: Can we sum up areas not by the juxtaposition of infinitesimally small rectangles, but by areas formed of curved lines? Our meditations on the importance of straight lines and right angles we shall find at length to conduct to the study of our physical condition and its limitations, and to the combination of our visual and locomotive faculties. And thus we may see how, step by step, our analysis may lead us from the heights of mathematical complexity, or from abstruse problems having an ethical bearing, to the study of rudimentary forms of thought. The works of Ernst Mach, truly profound, may be well referred to in this regard.

him, he may at any rate see without confusion the point of issue that has been raised. Moreover there should be no prejudice in the thinker's mind, no desire to attain this or that foreordained goal; for it is considerations of that sort that have rendered the works of so many generations of acute thinkers useless to the advance of truth.

CHAPTER II

THE FUNDAMENTAL PROCESSES

§ I. The Establishment of the Fundamental Processes

Let us pose the question thus: In what way can we obtain a principle of division in order to establish a classification of mental processes, and indicate the Fundamental, that is to say non-analysable, faculties, so as to make certain that our system is in this respect both complete and exclusive?

We might proceed by analysing the steps by which any of the great truths of science have been discovered. Following upon this exercise, the problem of reasoning, posed in more abstract fashion, might be attacked. Or some of the philosophic positions might be examined, such as that of the problem of externality, which while dealing with concrete and definite things yet involves continuous analysis to a fine degree. Then again we might examine current processes of ordinary life in order to seek the ultimate basis on which rests any belief that proceeds to action.

These methods are tentative. We are so far in the position of one examining a passage in literature in order to discern the secret of the alphabet. The examination by persistent analysis of complex positions can give no guarantee that our system is complete and exclusive.

From these tentative operations, extremely useful in themselves, suggestions arise of the true methods, which consist in examining cases in which the mental processes seem to us to be the simplest.

We may find these in the axioms; in the simple operations of counting, which we shall see to be the base of all mathematics of number; in the simplest examples of the mathematics of spatial relations; in the processes of mind which are at the base of the acts which preserve life.

From these we are led to consider the operations of the senses, individually at first as far as possible, and then in various combinations. In these operations involving single senses we find a certain community of processes. Such of these as are unanalysable we are justified in adopting in our classification of Fundamental Processes. We must then ask whether the senses, such as are usually spoken of, are the sole means of bringing to us knowledge of the external world; and we find that we must consider all sources for our mental activities, whether we apply to such sources the name of senses or not.[1] At this point we may state a fundamental division:

(A) External or objective factors.
(B) Internal or subjective factors.

Under the heading of External factors we consider the processes directly set in activity by the senses which bring us into contact with the external world. Under the heading of Internal factors we consider movements of the mind which do not depend directly on this stimulation.

[1] A considerable amount of work has been done of late years in regard to this question. H. E. Beaunis in "Les Sensations internes" has given us an admirable exposition of the subject, and his book has been productive of new research. The names of Meumann, Becker, and E. Murray may be cited in this regard.

We must then examine modes of combination which will build up our knowledge, and also we must make sure that in our review we fully consider the whole extent of the field of these combinations. Then further we must examine particularly the mental operations involved in such subjective phenomena as memory, fancy, imagination, dreaming, tentative combinations, invention, and the like, in order to obtain a clear view of the factors involved.

If then, after adequate consideration of the senses, and all the sources of mental impressions besides the senses usually considered, we can show that the whole of our knowledge of the external world, and the whole of the new combinations of the various elements within our minds, may be demonstrated by the use and combination of the mental processes we have called fundamental, we may be entitled to say that our classification of those Fundamental Processes is correct and sufficient.

We find here a case in which elementary physiological facts aid us in research in matters where it is difficult to see that introspection would alone avail. Thus if we consider the mechanism of the different senses we find that in each the essential condition of sensation is the bringing of external impulses into contact with some specialised form of nerve-ending of which the supplying nerve is in uninterrupted continuity with the central nervous system, which again must be efficient for its work. We are then led to consider other examples of nerve-endings. We find that the nerves terminate in muscle fibres, both striped and unstriped, found in various situations and including the gross motor muscles of the body, the heart, the muscles of the arteries, those of the outer parts of the walls and also of the mucous membrane

of the intestines; there are moreover nerves supplying the various mechanisms of secretion; there is the system of the sympathetic nerves with their inter-connection with the motor and sensory nerves; and there is the whole complex, with its extraordinarily intricate relations, of the central nervous system, including the spinal cord and the brain. Through the nerve supply to the muscles we obtain in part that sense of energy[1] which we shall find to be one of the factors of our appreciation of externality.[2] It would be fastidious at this moment to trace out the sensations, impressions, informations, or demands brought by the nerves supplying internal organs, and becoming revealed in consciousness; or, while working subconsciously, modifying the forms of consciousness.[3]

[1] This, which is now set forth on assumption, is fully considered in the subsequent exposition, and more especially in the chapter on the Feeling of Effort.

[2] See chapter on Externality.

[3] We are now far removed from what is called the "Sensationalism" of Locke. The mind of an infant is not a *tabula rasa*, but a magazine of stored-up forces, and though experience is necessary to its development, yet that experience conceived of as the chance impressions on a *tabula rasa* is absurdly inadequate to explain a child's mental development. Even the very experience itself is determined by multiple processes of instinct, choice, and more fundamentally by the actual scope of our senses and their limitations. Experience must be thought of not as an external something impinging on even a living apparatus; but as being a product formed by the mind from some external Unknown. Locke's clear candid style won Psychology from the realm of barren religio-metaphysical disquisition, and his work was thus invaluable. But it is precisely in dealing with Locke that we find the advantage of a knowledge of physiology in psychological study. It is generally difficult to specify the form of this influence with exactitude; but we may observe how great it has been in modifying the whole trend of our thoughts, in rescuing our notions of physiology as well as Psychology from the gross materialism of popular systems. Consider the case of that lady, Miss Helen Keller, who was born blind, deaf, and dumb, though endowed with an organisation and mind, the heritage of a long line of ancestors, accustomed not only to the full exercise of the senses but also to mental operations of a high degree of complexity. Here it is evident that experience must be limited to the senses which Miss Keller possessed in activity; while the most vivid, the most stimulating of the senses have remained blank. Yet the intellectual development of Helen Keller has been extraordinary, and has evidently been subserved by factors which have come to her by heredity, and of which the possession was possible only through a physical form wrought out in previous generations in harmony with these special r——

If now we examine the special senses in turn, we find, besides the peculiarities of each, certain Fundamental Processes of the mind necessary to this exercise. *These might* be drawn up and passed in review after such an examination, but it is perhaps better for the purpose of exposition to declare them beforehand, so that this examination may take place in a critical spirit with regard to their necessity and their adequacy. These Fundamental Processes will afford the basis of a lengthened discussion; though in a manner which is not intended to be dogmatic, but simply didactic, I will first formulate them in their nudity thus:[1]

I. IMMEDIATE PRESENTATION.	VII. FEELING OF EFFORT.
II. CONCEPTION OF UNIT.	VIII. IMPULSE.
III. MEMORY.	IX. HEDONIC SENSE.
IV. ASSOCIATION.	X. SENSE OF NEGATION.
V. AGREEMENT.	XI. CONCEPTION OF TIME.
VI. GENERALISATION.	XII. CONCEPTION OF SPACE.

A few explanations and slight amplifications may be given at this stage, though merely in order to prepare the way for subsequent fuller discussion:

I. IMMEDIATE PERCEPTION OR PRESENTATION.—This may be taken as inseparable from Sensation, but it must be understood that it should not be restricted to the organs of sensation usually considered.

II. CONCEPTION OF THE OBJECT AS A UNIT.—The questions here involved are considered in the chapter on the Conception of the Unit and in the chapter on Counting.

III. MEMORY.—Most persons will agree that Memory

[1] Certain groups of these will be afterwards considered in a manner which will enable us to form a comparison with previous analyses and to examine in what respect the present view of the subject may be more or less closely accorded with current expressions. It may also be mentioned here that the conceptions of Time and Space stand in a class apart from the other Fundamental Processes, and that the distinction will gradually become evident in the course of the exposition.

is a Fundamental Process. For the purpose of exposition merely let us assume something more than we have already considered, and let us speak of Idea as the image which the mind displays of a past experience, such as a particular Sensation. Then Memory stands to Idea in a relation similar to that in which the Fundamental Process of Immediate Presentation stands to Sensation. It may be that some will find a difficulty in distinguishing Immediate Presentation from Sensation, and the difficulty would be insuperable if there were but one sensation in experience; but, since there are more, the very fact that we can think of them in some aspect common to all enables us to speak of Immediate Presentation without identifying that Fundamental Process with any one particular sensation. Similarly Memory and Idea are not identical. Memory is the Fundamental Process by which the Idea is made known as existing.

The question of Memory will be subsequently considered in detail.

IV. Association.—To this a wide range of application must be given. It is not merely a matter of Association, say, of one object seen with another object occupying the same field of vision, or again between two objects presented by different senses. Association is formed by myriad cirumstances of the whole mental make-up at the moment—the subconscious forces in operation, the temperament and previous experiences of the individual—as well as by the accident of external association in the actual experience.[1]

[1] Such a consideration of this position will at length lead us to the explanation of the subtlest effects of poetry. The poet's Ideas come surrounded by an elusive, invisible cohort of suggestions, just as the melody of a piano is attended by its accompaniment of variations, or as the timbre of the note is modified by the presence of the overtones, as we are assured by Helmholtz's analysis. But as the experiences, and consequently the associations, of one mind differ from another, so it happens that poetry that may rouse one person to enthusiasm may

V. Agreement.—This process involves the recognition of objects successively presented as being like, or in Agreement, with regard to calling up of a similar set of associations in similar relations.

We have here a reference to Memory and Association, which are themselves Fundamental Processes, but the point herein discovered will be considered later (cf. p. 35).

We may find it indeed convenient here also to anticipate by referring to Discrimination, which is brought to light by the negation of Agreement, and which will frequently be referred to in place of Agreement, because the conditions to which attention is sought may be thereby rendered more evident.

When we say that Discrimination is brought to light by the negation of Agreement, we may state the process thus: The mind proceeds along a course marked by Agreement. For example, a child has been playing with a piece of red glass or ruby, and has noticed various associations about it. Then arises an interruption or negation of certain of these associations. The child has mistaken a live coal for the ruby. That negation calls attention to the differences. That is a process of Discrimination. The process is that of an Immediate Presentation of the difference, and this is joined in Association with those Presentations found in Agreement.

VI. Generalisation.—The Process which becomes manifest at a crisis where Discrimination intervenes is

disappoint or displease another. The poetry of Keats has a magic force at its best which could only be derived from profound and luminous thought, in accordance with his own expression, "I am a philosopher first, a poet afterwards." The verse of Milton, rich in so many varied and marvellous powers, has besides a peculiar quality of Association from the extent of his allusions to various forms of knowledge. In "Paradise Lost," particularly, this is revealed in such a manner of felicitous reference as to absolve erudition of pedantry, and : a difficult acquisition consonant with rarest poetry.

Generalisation, with which will be considered Classification and Symbolisation.

This arises inevitably in connection with Agreement, where a series, however long or short, of Presentations in Agreement is interrupted; for Generalisation is the recognition of common qualities, A, between two objects (A + a), and (A + a′); while Discrimination is the recognition of the differences, a and a′. Generalisation as a fundamental movement really always proceeds from Impulse and Association, succeeded by Association in Negative sense. Thus, whereas on perception of (A + a) and (A + a′) before the Discrimination of a and a′, the mind's Impulse proceeds to form for both the same associations, yet when Discrimination takes place, thus separating A from A + a and A + a′, the recognition of A as distinct, and with distinct associations, constitutes Generalisation. It must be noticed that this is Generalisation in its simplest form. Generalisation in its scientific use is not formed inevitably, but by exercise of a trained faculty; but this form of generalisation bears the same relation to the Fundamental Process of Generalisation as the discrimination necessary to appreciate, say, a subtle legal point does to the elementary use of Discrimination in visual experiences.

I have associated with Generalisation the processes of classification and symbolisation. For long I hesitated as to whether these, classification and symbolisation, should not be included among the Fundamental Processes. The fact that they arise from Generalisation in conjunction with other Processes is not decisive, because, as we shall soon observe, all the Fundamental Processes are interrelated (cf. p. 35). The question that must be asked is: Can these Processes be composed synthetically from Fundamental Processes? If the discernment of others is clearer than mine on this point I will not cavil at their

judgment; for long I have been wanting entire conviction in the matter. To me now classification and symbolisation appear as synthetic.

Classification.—This is the conscious recognition of combination of Discrimination and Generalisation. Thus:

$$\overline{A + a \mid A + a'}$$

Symbolisation.—This is a principle of mental operations involving all the previous factors. It may thus be ranked as a secondary movement of the mind, though considerations based on the study of Impulse may tend to make us regard the faculty of Symbolisation, not necessarily expressed in words, as absolutely fundamental. Thus in A which involves the system

$$\frac{A}{A + a \mid A + a'},$$

A, if conceived as a Unit, is the symbol either of A + a, or A + a′, in as far as the possession of A is alone considered.

The system may be possibly involved:

$$\frac{\overline{A + a \quad \mid \quad A + a'}}{A + a_{\prime} \mid A + a_{\prime\prime} \mid A + a'_{\prime} \mid A + a'_{\prime\prime}}$$

and so indefinitely, and if A be elemental it becomes a symbol for any one of the system in as far as A only is concerned.

Let us give an example, simply to make the meaning clearer, though it involves many complex factors, far remote from the elementary cases we are considering. A soldier is instructed to fire on all enemies; the enemies are marked by a white cockade. The first enemy he encounters, may be tall, the second short, another may

have the uniform of an officer, another of a foot soldier. Now with each of these variations there will be distinct associations, but these associations become disregarded in view of the main directive. Finally, we arrive at the recognition that the whole impulse of his attack is stimulated by the sight of a white cockade on a man; and this cockade is represented by A of the system given.

Symbolisation takes place inevitably at the moment Discrimination takes place. For if A be considered the whole object, the Impulse associated with A is summed up on the Presentation of A. When A is found to be associated with other elements, and these are separated by Discrimination, A still remains as evoking the Impulse due to A; and the recognition of A in this relation is an act of Symbolisation.

VII. The Feeling of Effort.—This cannot be included among the senses, as usually understood, although it is always associated with them. It is not identical with what is known as the muscular sense, although its manifestations are more conspicuous in conjunction with that sense than with the others. It may become evident in the sense of hearing, as when one expects to hear a loud noise, and instead of that hears a faint noise. It is not identical with the actual sensation, for that may differ in quality and yet leave the Feeling of Effort unchanged within the limits of Discrimination. Or again, suppose that one lifts a heavy bar of lead; there will be found an appreciable, even though not well defined, muscular sense. Suppose now that, unknown to the operator, there be substituted for the bar of lead a bar of aluminium coated over with lead. Now as aluminium is much lighter, the effect will be surprising, for the operator will find his arm jerked into the air. There is something in the Feeling of Effort put forth

which is distinct from the sensations appreciable in lifting either the lead or the aluminium.[1]

VIII. Impulse.—That is to say, some movement of the mind, proceeding from and linked with the Unit, or thing held in attention. This will be understood by the objective consideration of the nature of the mind; that the mind, or at least its physical substratum as a complex living thing, is a magazine of forces. The question of Impulse is of special interest.

(*a*) Impulse may be derived directly from the Unit in conjunction with something forming strong immediate associations. Impulse, it must be remarked, always involves Association.

(*b*) Again, Impulse may arise with something not at all immediately associated with the Unit held in attention at that instant. In this instance the object is the starting-point of associations stored up from past experiences, and these associations may be far stronger than such as are immediately and externally associated with the object. A rough analogy, which is simply used for the purpose of exposition, is that the object may have to the associations involved a relation suggesting that of the button of an electric system to a mass of dynamite which is exploded when the button is pressed.

It is the Fundamental Process of Impulse which supplies the tentative efforts which we find continually necessary to mental progress in the course of reasoning. (See chapter on Reasoning.)

When we consider (VIII. *b*) that the Impulse is not necessarily derived from immediate conjunctions with the object, we find the path leading us from too restricted

[1] It may be well to indicate here that many eminent psychologists and physiologists deny that means are thus afforded of appreciating Effort. The question is considered in divers places in the course of the exposition, and particularly in the chapters on Brain Localisation and the special chapter on the Feeling of Effort.

limits of that "chain of Association" which is often popularly given as the representative of the course of mental processes.

Impulse must always be considered not only in relation to the object and its ordinary associations, but also in connection with that magazine of stored-up forces which finds in the object a point of instantaneous impact, while the new thought obtained and associated is the instantaneous resultant in the ever-changing form of the magazine of previously registered forces of impulses, ideas, and all that makes mind.

The meaning of Impulse, as well as that of Feeling of Effort, will become clearer in the course of the exposition. Some indication may be given by saying that the Feeling of Effort may be found to precede in some measure the full development of an actual sensation-experience, but Impulse precedes this, though precedence must here be understood in the sense in which the crest of a wave precedes the main mass of which it forms part.

Sensation and Impulse are two psychical phases of the magazine of stored energy which underlies, but may not except at moments become noticeable in, consciousness.

IX. Hedonic Sense.—(*a*) This involves not only special nerves of pain,[1] or some physical disposition giving such results in which in a negative sense it is most apparent, but also pleasurable or displeasurable Associations invariably discoverable in all perceptions.

(*b*) In addition to an immediate Hedonic sense there may be found closely associated, if not fundamentally so, in some elementary experiences at least, a more

[1] The question of special nerves of pain cannot be regarded as entirely decided. The literature of the subject is now so abundant that for this reason I refrain from citations, and the opinions of great authorities are diverse.

general emotional feeling. This has the same relation to the Hedonic sense as the Impulse of remote Associations connected with the mind in full working order (VIII. *b*) has to the immediate Impulse of close invariable Association (VIII. *a*).[1]

X. Sense of Negation.—It is necessary to include this amongst the Fundamental Processes, for it does not consist merely in the failure of these, but it introduces something of a distinct quality. It appertains to all, just as Presentation appertains to all sensations, or Memory appertains to all ideas. Thus the Negation of Association is Dis-Association; that of Agreement is Discrimination.

XI. Time.—This is essential to the perception of an object as a Unit (II.), and to all Processes when there is a change of attention from one object to another. Objectively it is evident that it is necessary to Immediate Presentation (I.); and subjectively, as (I.) cannot be represented to our consciousness except by Association with some other object. Time forms a factor even of elementary sensation.

XII. Space.—This is an invariable condition in regard to some at least of the objects of the senses.

. . . . .

It will have been observed that it is impossible to refer to any one of the Fundamental Processes without reference to others. Nevertheless they are distinct. An Immediate Presentation is not a process of Memory, but

[1] An acute modern thinker, M. Guyau, finds a certain "æsthetic" quality in sensation. Bain attributes a pleasurable quality, or the reverse, to sensation. Wundt speaks of Gefühlston (which is really what is here called the Hedonic sense) as a third characteristic of sensation, the others being quality and intensity. This was in the 4th Edition of his "Physiologische Psychologie." Later in 1896 he speaks of "Lust und Unlustgefühle," but he departs from the clearness of his original thought in confusing "Lust" and "Unlust" with affections. The matter is discussed by Mr. E. B. Titchener in the *American Journal of Psychology*, 1908: "The Tridimensional Theory of Feeling" (cf. p. 54).

in all cases of Memory a former Immediate Presentation is implied, and also with it the associations that have been formed by the Process of Association.

There is no especial difficulty in comprehending these relations. For example, we cannot think of a mother without implying a child; but the child is not the mother. We cannot think of one side of a penny without implying the other side. We cannot speak of a pentagon without implying five sides, all of which are self-subsisting. Or, again, we have an analogy in the working of a steam engine, where every process implies other processes, which, however, are distinct.[1]

Let us revert to the question of Discrimination in order to make clearer the explanations already given. Thus a previous experience A tends to call up B. That is to say A, as an Immediate Presentation, has had Association with B, so that when we find A either in Memory, or in reality again as an Immediate Presentation, then the image of B appears in consciousness; that is to say, Memory renews that Association. But as Memory is weaker in impression than the Immediate Presentation, only the chief features of B appear. But if in the reality A is now associated with B′, which has elements of Agreement with B, then the impressions of B′ override those of B; and where there is difference the consciousness of this is called Discrimination. Discrimination has implied Agreement, Memory, Association.

If it be disputed that Discrimination implies Agreement when we discriminate between a musical note and a billiard table, the answer is that we can discriminate in no closer sense than that in which Agreement exists.

[1] Or, in a still more complex way, in a class of equations called Abelian equations each root of the equation is a function of all the others. But the roots are not identical.

Here we have the succession of two different objects; but the fact that we consider them together as two objects already implies Agreement in some deep basis. That basis is discussed under the heading of Unit (cf. p. 86 *et seq.*).

§ II. The Series is not Redundant

A criticism that may be suggested against the present exposition of the Fundamental Processes is that the tabulation seems too detailed. There is no force, to be sure, in an objection of that kind, unless it can be shown in a particular instance that the series includes factors which should have no place there, or that it is in some respect redundant. I think it well, however, not to be content with refuting arguments, but rather to seek for some valid thought beneath the objection, and to show its bearing upon the question.

The process of thought is often very rapid, and in thinking of ordinary affairs there appears to be a freedom, and even a simplicity, inconsistent with the carrying on of a long series of processes. Something of the sort may seem implied also in the position that the actual present thought is conceived as a Unit. Moreover, the new disturbing stimulus—sensation it may be—is appreciated as a Unit.

The expression of such objections will help us to set forth, perhaps more clearly than otherwise, the significance of these Fundamental Processes, and give us the opportunity of demonstrating their necessity.

First, however, let us briefly consider Kant's analysis. The categories of Unity, Plurality, Totality may be interpreted in the present scheme by observing that that of Unity corresponds to the Fundamental Process of the conception of the Unit; that of Plurality involves, with

this conception, also that of Time, and that of Totality involves the conception of a complex as a Unit either immediately or by the aid of Symbolisation.

In Kant's analysis there appears to me to be something vague and tentative, as of a man dealing with verbal, or formal, presentments of a problem without working down to the essential realities. A similar remark applies to his Judgments: Universal, Particular, Singular. If any of these be definitely expressed it will be found to be explicable in terms of the Fundamental Processes of which the most prominent are those of Agreement (or its negative, Discrimination), Generalisation, with its corollary, Classification.

Kant's *Vermögen* also seem explicable in the light of this analysis. The *Vermögen* of feeling may be expressed according to the manner in which we understand feeling, either as Immediate Presentation of sensation (if the meaning of sensation be extended to its widest scope), or as the combination of such by means of Memory and Association, with the aid of the conception of the Unit.

The *Vermögen* of knowing is expressible in similar terms, with this difference, that the attention is directed more particularly to the Fundamental Process of Association in conjunction with Generalisation and Classification.

The *Vermögen* of doing and willing may be reduced to its components: the Fundamental Processes of the Feeling of Effort, Impulse, the Hedonic Sense, in conjunction with all that is involved in the other Fundamental Processes that form determinants to these.

Here again the difficulty is not to make the analysis when the position is stated with precision, but rather to obtain that precise statement, or to select some one of possible meanings from a vague statement.

In the exposition we speak of one of the Fundamental

Processes being conjoined with others. It should be insisted upon that they are really inseparable, and indeed inconceivable as actually separated; they form a constant complexus, and when we speak of any one, or any group, that is to be understood only as directing attention to such aspects of this complexus.

This has been already mentioned, but as it is better to repeat explanations than to leave in doubt another illustration may be offered. Any object, such as an apple, has form, colour, weight. We may change the form, but there will be some form. The colour may vary, but if the object be visible there will be some colour. Weight depends on universal attraction, but even without that hypothesis we could still think of the mass of the object, and we might indicate, as weight, the resistance of mass to force. Now the apple is not form, nor colour, nor weight; and form is not colour, nor weight form. Yet all these attributes are indissolubly connected, and they, with others, go to make up what we mean by the complexus: apple.

So none of the Fundamental Processes exists apart from the others, yet intelligibly enough for exposition we may speak of them separately. But because they are indissoluble, and because at the same time attention may be fixed on any one, or on various groupings, we find possible not merely such analyses as those of Kant, but also various popular expressions, which have necessarily some solid base, referring to reasoning.

Popularly the term Sensation is enlarged in application, and rendered less accurate. Suppose that, in order to meet the underlying reserve we spoke of, we adopt for the moment that term with its popular meaning. Sensation embraces then Immediate Presentation, Unit, with something of Association, and Agreement. If now we give the term Memory a widened application, so as to

include recollection with Association, Agreement, and also, though perhaps more vaguely, Generalisation with Classification and Symbolisation; and then if we form the popular synthesis of Will, including Feeling of Effort, Impulse, Hedonic Sense, with an underlying determinant from the other Fundamental Processes; while we speak of Time and Space, and all that conditions our experiences, as the environment; then finally we arrive at a means of speaking of the Processes of the mind, or of reasoning in general, as being a composite derived from Sensation, Memory, and Will, of a mind operating in a given Environment.

Now whether we accept this position or not, there is at least nothing in the notion that shocks us with a feeling of undue complexity. The objection to such a position is similar to the objection to Kant's formulation —the analysis is confused and incomplete, and the classification overlaps. When these objections become removed we are led again to the Fundamental Processes already set forth, and with that step we find that the original reserve in regard to these vanishes, while at the same time we have seen how it was possible for it to have arisen.

But with a hint derived from these considerations we might form new modes for presenting. Avoiding the term Sensation, we might define a term to be used as lately as we had used Sensation, and call it Perception. Then we might form a new group of Memory, with its stored-up ideas, by virtue of Association and Classification, and the conjunction of all this with what has been termed Perception we might call Combination. Then the activity of the mind under this stimulus and interaction, in conjunction with Feeling of Effort, Impulse, Hedonic Sense, we might call Adjustment. Then Reasoning might be defined in terms of Perception,

Combination, and Adjustment within certain universal conditions and special environment. There is nothing ostensibly absurd in that; simply the analysis is not clear.

Similar examples might be found by taking up almost at hazard a work of Psychology, and finding therein a treatment of reasoning.[1]

Suppose now that we take an illustration strikingly objective. Let the mind, with all its immense complexity of stored-up and inter-related ideas, be represented by a complex mechanical system. Let a new external stimulus be represented by a projectile impinging on and entering into the system. Then let all the interactions of the mind, including such as we think of as associated with will, be represented by the interactions of the system. All this is considered as within a certain environment defined for the moment. The resultant of the forces of the system corresponds to the new activity of the mind. So that what we spoke of, though with insufficient accuracy, as Sensation, Memory, Will; or Reception, Combination, Adjustment; we can now describe in terms of Force, System, Resultant.

[1] Prof. Th. Ziehen in "Die Prinzipien und Methoden der Intelligenzprüfung," speaks of faculties of Isolation, Complexion, and Generalisation.

Dr. Paul Kronthal, in "Nerven und Seele," refers to the soul as the "sum of reflexes."

Prof. Störring, in a work which has been translated into English, "Mental Pathology in relation to Normal Psychology," speaks of "activity-feeling," which is a "Condition of our regarding the spiritual self as within wide limits independent as against the material world."

G. Dwelshauvers forms a theory of synthesis at the base of mental life.

A. Binet, in his work "La Psychologie du Raisonnement," points out that Aristotle's formulation of the syllogism has injuriously affected the view of all subsequent thinkers—not merely Kant and J. S. Mill but even Herbert Spencer—in regard to the veritable processes of reasoning. Binet himself puts forward a "theory of substitution," but this is not at all convincing.

I refer to these particularly, for by comparison of any of them with the others we observe that there is in all a deficiency of the requisite analysis, and because each of these tentative theories will find its veritable explanation in the course of this exposition.

And once we clearly behold this graphic simile we see that, though not precise, it is in broad lines a correct representation. It might have been taken as a starting-point; and then the whole task of examination, and exposition, would have become the complete and precise definition of what corresponded to the mechanical processes.

Thus we are led to our Fundamental Processes with a clearer understanding why they seem complex, and why they are indissolubly united.

It is possible still further to simplify in appearance the processes by which the mind proceeds in its course. Let me give first a curious objective example. I remember at one time seeing in Melbourne the following experiment: A small object was hidden in a warehouse in the city. A celebrated "thought-reader"[1] took as his starting-point the steps of the Town Hall. By means of a small light metallic chain one of his wrists was attached to the wrist of a man who knew where the object was concealed. The thought-reader, who was blindfolded, I believe, though that detail is really not essential, rapidly made his way, even though surrounded by a dense crowd, to the place of concealment, and finally he discovered the object.

The explanation is that he had cultivated to a high degree the sense of touch, and the faculty of observation in such matters.[2] Whenever he showed a tendency to

[1] Mr. Stuart Cumberland.

[2] I have since seen in various psychological publications excellent studies of questions of this sort. The conclusions correspond to what is here given. The names occur to me of J. E. Downer on "Muscle-reading," and B. Bourdon on the "Sensibilité tactile." A great number of physiologists and psychologists have written on a subject which has some connection with such experiments as that referred to above—tactile localisations. Names of authorities are Weber, Hering, Kottenkampf, Ulrich, Barth, Lewy, V. Henri, F. B. Dresslar, Pillsbury. Legerdemain, an allied subject, has been studied by Dessoir, Külpe, Jastrow, Muensterberg, Pillsbury, and Triplett amongst others.

take the wrong direction, a slight indication, such as would be imperceptible to others, came by way of the chain, unconsciously tightened by the passive subject "concentrating" his attention. Now, just as in familiar games of children where indications are given by calling "hot" or "cold," it is sufficient to have at each point of progress affirmative or negative directions in order to arrive at the appointed goal.

Reflecting on the whole circumstance it seemed to me that Reason might be represented as an operator who at each turn receives the direction, "yes" or "no," and so eventually reaches a desired conclusion.

Here then, in place of the apparent complexity of our Fundamental Processes, we have a very simple image. But let us examine this image to ascertain all that it implies. What is meant by saying "at each turn"? That implies a crisis of experience. It implies Time and Space. It implies the experience. The simplest form of experience is that of Immediate Presentation of a sensation. "At each turn" implies also that the Fundamental Process of the conception of a Unit is involved, and moreover since there is movement, this momentary, or instantaneous, Unit is linked or in some way related to a previous Unit. Here we are introduced to the Fundamental Processes of Memory and Association.

Then the directive is "yes," or "no," at each turn; it is possible that the directive may under other circumstances have been reversed. At this stage therefore we meet with the Fundamental Process of Negation.

But these directions are not given at hazard. The directive is given after comparison with some actual, or implied chart of direction, or of something of which that forms the graphic picture. This comparison implies

Agreement, and as the comparison implies also factors reproduced by Memory from the storehouse of the mind, and represented by the help of symbolisation, we find here implied Generalisation.

In the fact of the operator's demand, either explicit or implicit, for the directive, and in the state of expectancy and reception, we have the Feeling of Effort, and in the resolute setting out again, on receipt of the directive, we find the implication of Impulse. In the feeling, whether of satisfaction or disappointment, accompanying each of these acts we have the manifestation of the Hedonic Sense.

Accordingly from the consideration of this reduction to simplicity we are led again to the Fundamental Processes as already set forth.

It is easy to adapt this form of exposition to the image of a "chain of Association." I once asked a famous German professor if he could give me an analysis of Reason. He replied, it is simply a "chain of Association."[1] This was not an analysis. But we have just observed in what way it may be made the basis of an analysis.

At this stage reference will be made to certain important positions usually discussed in works of Psychology, so that their relations to the series of the Fundamental Processes may be exhibited.

§ III. Certain "Positions" in Psychology

ABSTRACTION

Abstraction may here be considered. Let us fix our notions, for example, by means of a sphere. The sphere has not only shape, but colour, mass, and a number of other properties associated with the material of which

[1] This substantially represents the conclusions of John Stuart Mill's school.

it is formed. But we may have another sphere of different material, and the associations are different. By Discrimination as before we arrived at a generalisation; and in this way only is it possible, in that we still have before us a definite concrete something (A), from which a and a' have been separated by Discrimination, but which possesses its own associations.

But abstraction implies more than this. Abstraction implies dealing with some properties of an object, say a sphere, which is not conceived of as concretely existing. Generalisation in itself in its fundamental form is an inevitable, and partly unconscious, movement; but abstraction implies a conscious recognition of the process of Generalisation and also, not an inevitable, but a tentative exercise of separating certain associations and of conserving others.

Thus if in A + a and A + a' we disregard a and a', we still have A as a concrete thing, say a sphere. Similarly in B + b and B + b', separating the incidental associations b + b', we arrive at B, another sphere. Now we may consciously select and combine certain associations of A, and transfer them to B, and this with an active effort of disregarding associations arising from the difference between A and B.

Abstraction cannot get beyond the conditions formed by Association, and with it Dis-Association, Discrimination, and Generalisation, but it is not the same thing as the last step of inevitable Generalisation produced by Discrimination.

But since abstraction is so contained within the conditions of these operations we must seek an explanation of how it emerges into something distinct. We shall find, as in the further discussion on Discrimination (cf. pp. 53, 54 and 93, 94), that our mental operations are often determined, and, according to the

conception we are compelled to form, even aided by the limitations of our faculties (cf. p. 207). Thus if Discrimination were possible unto the absolutely distinct perception of the last intrinsic differences between things, and if the mind kept continually in activity all the associations formed by each of the elements, there would be no Generalisation.

Now consider a line. A line is always a concrete thing—a piece of chalk drawn out in extreme tenuity, or a piece of lead pencil, or the like. But apart from consideration of the material there is in drawing a line a certain form of movement given to the fingers, hand, and arm. It is also a fact that when the mind becomes accustomed to certain associations, these, being gradually formed with less and less effort, leave it free in its exercise to form new associations. So that the visual appearances of a line, associated moreover with a multitude of impressions involved in moving the ocular apparatus, become associated with the movements required in drawing it. Now these movements themselves are not very well defined in character, so that the movement in drawing one line does not leave in the mind an impression that enables it to be clearly discriminated from the movement involved in drawing a similar line. It is in the indefiniteness of such impressions, allowing the possibility of what is really a generalisation on a broad but vague basis, that we find the elements that in their combination give us abstraction.

Thus the notion of a straight line is gained by such acts as running the finger along the line, looking at it from one end to the other, or if it be represented as the edge of, say, a ruler, by turning the ruler round its edge as axis and observing that the edge maintains its position. There are formed then in the mind associations with one straight object, and these are transferred to another

straight object by virtue of disregarding differences, that is to say by Dis-Association in regard to these, and by Association with similarities of processes not easily discriminated.

In the case of a sphere we have the combination of a series of associations, relatively complex, of resemblances. Thus, say, we have a hollow sphere; there is the recognition of a centre, the drawing a line, perhaps represented by a rod, from the centre to any point in the sphere, and the observation that the rod remains of the same length while one extremity remains at the centre and the other is moved about on the periphery.

In all this, abstraction is greatly aided by the use of symbols, which becomes a strong bond for associations; but it must be remembered that symbols can only arise from the use, as has been indicated, of Association, Discrimination, Generalisation.

Thus, if we speak of a letter of the alphabet we must refer to some letter of the twenty-six. But our mental processes arrive at such a point of development that we can, by a conscious exercise of generalisation, form a phrase such as: A letter helps to form a word; and in this we do not think of the particular form of any one letter. We have here an abstraction. But that employment of language depended necessarily on the generalisation, letter; then on the observation that, in a certain definite instance, a letter helped to form a word; then on the further observation that there was nothing particular in the circumstances of this letter, that is to say that Dis-Association might here operate in certain regards while the main associations necessary for the purpose in view were conserved; and it is mainly by virtue of symbolisation that we can make the transference of the associations we require while neglecting

those we do not require. Hence we arrive at an abstraction.[1]

An abstraction therefore consists in the associations of certain circumstances with a symbol, which symbol may not necessarily be expressed by language, and which circumstances must be applied to certain concrete things, or the mental images corresponding, by virtue of conditions implied by the symbol, in disregard of other conditions not so implied.

The question has often been mooted as to whether the mental processes in the lower animals are similar to those of human beings. Many psychologists, especially psychologists of the more modern school, and including Herbert Spencer, believe that the processes are similar. The whole trend of the analysis of fundamental operations exhibited in this book would seem to point to like conclusions, but with this proviso, that the development of special senses varies enormously in diverse animals, and such development affects the whole mental life.[2]

It may be questioned, therefore, whether an intelligent animal, such as a dog, could reproduce the process

[1] Similar remarks would apply to the abstractions of mathematics, as for instance the formula of a curve involving the ordinates x and y. The formula has no meaning except in regard to definite particular instances of its application; that is to say, when definite values, in accordance with the formula, are ascribed to x and y. But the form of the symbols is the same, no matter what the particular definite values may be. In the simplest forms abstraction merges into what we have called Generalisation.' The consideration of an ordinate x or y, or of any straight line, does not give us the simplest case. The meaning straight line involves a complex relation of elements capable of Immediate Presentation and combined by virtue of Fundamental Processes. This will be seen more clearly in the discussion of Externality. A generalisation in regard to ordinates may be considered to refer to any ordinate, conceived of concretely, and in actual application to some definite ordinate. But any abstraction, as for instance of a straight line—length without breadth—not being representable concretely, really means applying a symbol for the processes of the mind themselves that operate on certain concrete things.

[2] In this regard the studies at the Institute of Zoological Psychology in Paris are interesting.

of abstraction. That would, no doubt, be impossible in regard to any high abstractions, for even with human beings a specialised form of language is required for such mental operations. But granting that a dog may exercise the Processes of Presentation, Discrimination, Association, and symbolisation, implying Generalisation, together with all the other Processes herein involved,[1] then it seems possible to imagine a case where a rudimentary form of abstraction must necessarily follow. For example: if a dog be fed always at the same hour, and the hour be announced by ringing a bell, there then will be formed in the dog's mind strong associations connecting his mental impressions of the bell and the dinner. Now, suppose further that there are several bells, and that while their sounds, though resembling each other, are nevertheless distinct, then the dog would form the association of any one of these bells with his dinner. Suppose further that the dog did not see the bells, but that the movement of a latch, which he could see, always preceded the sound of any of the bells, there would thus be produced in the dog's mind the associations—latch, bell, dinner.

When these associations had become confirmed, we might arrange an instance where the dog was prevented from hearing, as, for instance, by having his ears plugged; then the movement of the latch would produce the associations of latch—bell—dinner, not quite so vividly as if he heard the bell really sounding, but still in the nature of most of our associations. The bell

[1] Professor Lépinay, in a lecture delivered at the new Dogs' and Cats' Home near Reuil, Paris, expressed the belief that a simple kind of reasoning exists with such animals. The question has also been studied by Romanes, Lord Avebury, de Fouveau de Courmelles, Fr. Houssay, E. Alix, Karl Groos, W. Wagner, C. J. Cornish, and in an elaborate series of studies on monkeys conducted under the direction of Dr. Franz at the Government Hospital for the Insane, Washington. Most of the observers recognise in animals a manner of reason analogous to our own. This will be made clearer in the course of the present exposition of reason.

in this case would not be any special one of the bells. There would have already arisen in the dog's mind something similar to the generalisation: bell. The movement of the latch would have acquired the sense of a symbol of the whole associations. And then we should arrive at a fulfilment of the condition expressed in our consideration of abstraction in the case of human beings: An abstraction consists in the associations of certain circumstances with a symbol, which symbol may not necessarily be expressed by language, and which must be applied to certain concrete things, or the mental images corresponding, by virtue of conditions implied in the symbol and in disregard of other conditions not so implied.

In fact, proceeding from the associations of a symbol, the dog entertains the mental impressions of a thing, which is not any one of the things in his immediate experience (the real bells), and which produces associations that are to be found in the case of each one of these things in his experience.

But the human mind in regard to the abstraction: sphere, moves similarly. It proceeds from a symbol: the word, sphere; forms mental impressions accordingly which are not necessarily identical with those arising from any particular sphere; then forms associations, that is to say, those of the properties of the sphere; recognising that these associations are to be found in the case of any one of the particular spheres considered.

RELATIVITY

The notion of Relativity may be here referred to. Certainly it is true that all our knowledge is relative; that we cannot have an idea of the situation of a point in space absolutely; that we must consider it relatively

to some other point. Our appreciations of sensations are relative to those that immediately precede them.

Further, in the more complex life it is certain that no two persons represent the external world alike, for there is much variation in the relative acuteness of senses, in association with other senses and with the complex impressions of effort. Thus a person may hear the same objective sounds at one time faintly, and then, after some slight operation, say, on the ear, may hear them more loudly, and more distinctly, and more endowed with importance in regard to the impressions produced by other sensations and their combinations. He is then able to form a relative estimate of the previous impressions of the same objective sounds. But suppose that he had never experienced this change of hearing, then he would talk of these sounds as usual, apply the same names, associate them in the same way; but their standard in his mental life would be different.

But when we consider two persons, we should expect that the impressions brought by any of the senses would differ, even though relations persist similar in kind, but different in the whole scale of importance in regard to all the senses. That two persons are able to converse with close appreciation about colours, pictures, sounds, music, does not imply that their sensations are not different, but simply that these sensations are sufficiently similar to allow of the same symbols, as words, being applied, and that they possess similar relations to the complex of other sensations with their associations in the mind. Thus, to give an example, simply to make the meaning clearer. Two persons may each be looking at a map of a country, but the two maps may differ in their scale, in the relative size of the letters they apply to various towns, and also in the shade and vividness of

the colours employed, though the colour schemes be similar. These two persons might converse about their maps without discovering that they differed in any particulars. Thus the principle of relativity runs through all our mental life.

Subjectively, however, the notion in its fundamental aspects is contained in that of Immediate Presentation. Relativity, apart from this, is an objective condition of our mental appreciations, but not anything subjectively recognised. We find such conditions, for example, in the sense of sight. If the undulatory theory be true, a necessary condition for the perception of a distinct object is that the vibrations of ether which it sets up should reach the eye, and that the undulations should be within the limits of a certain scale; if the emission theory, nowadays seriously reconsidered, be true, then minute particles from the body, within a certain scale of dimensions, reach the eye; but the sense has no direct information as to these conditions. Relativity is a condition of ideas, even the simplest, but there does not seem anything in this position which, in regard to Fundamental Processes of the mind, would render inadequate the tabulation adopted.

§ IV. The Fundamental Processes Considered with Reference to the Diverse Senses

Coming now to a consideration of the various senses in turn, we shall find the whole series of Fundamental Processes reproduced in each; but the special characteristics introduce us to new fundamental data in special forms of experiences.

In visual sensations it seems certain that particular, unanalysable, Immediate Presentations are those of

colour,[1] intensity of illumination, form. There are also included certain of the elements which form our appreciation of motion and of externality, which will be considered later.[2]

We are fortunately enabled by physical science to assign to some of our fundamental perceptions certain objective correlatives, as, for example, that our sensations of colour depend on the frequency of the waves of light, red corresponding to 483,000,000,000,000 undulations per second, violet corresponding to 708,000,000,000,000 undulations per second. These correlatives in the physical world permit us to make analyses and discriminations in an objective way far beyond the limit of direct

[1] A curious and interesting study is this: By what process do we attain a sense of colour in general, so that although red things and green things are separate objects, we classify them under the idea of colour. Even if we cannot solve such a problem the fact that it exists may convince us how profound, how fundamental beneath even the processes of our conscious life, are the principles of Agreement and Generalisation. These are in fact not only Fundamental Processes of our minds, but have been Fundamental Processes in the development of our minds to their actual degree of complexity.

With regard to all that is involved in sensations due to coloured objects, a well-known student of problems of colour, Dr. F. W. Edridge-Green, calls attention to distinctions between light perception and colour (cf. Dr. Edridge-Green's "Colour Blindness and Colour Perception" in the International Scientific Series, and various articles in the *Proceedings of the Royal Society*). Sir Ray Lankester, in one of his studies in "Science from an Easy Chair," indicates in what way vision has become developed from its rudimentary form, the perception of light.

[2] With regard to the general question, duration must be considered a necessary condition, and therefore an inevitable presentation, of sensation. It has not been mentioned above so as to avoid raising a question as to the use of the word Immediate, but the difficulty has only to be mentioned in order to cause it to disappear. Immediate here does not imply a point of time without duration.

Bain gives massiveness as a factor, or attribute, of sensation. I am inclined to think that in the case of sight this might be compounded of intensity and extensity (form). The matter is made clearer by considering the objective conditions. The undulations, or vibrations (adopting for the moment the wave theory), have frequency, corresponding to colour, and amplitude, corresponding to intensity. Any other psychic effect must be due to addition of vibrations, and hence introduces objectively, at least, extension. This objective manner of regarding will enable us to criticise the force of a remark, in an article by Edmund Hollands in the *American Journal of Psychology*, where, speaking of clearness, duration, and extension, he says Wundt refuses to class these with quality and intensity as attributes of sensation.

Discrimination by our senses. Discrimination, though fundamental, is thus seen to be limited in application, and also as containing, beneath the limit of our Discrimination, a source of fallacy. Thus a and a′ are two separate things, but if the difference be beneath the limit of our Discrimination we consider them as the same.

This question will be referred to in the consideration of axioms, and in the discussion on reasoning. Our limitations enable us to build up a practical and intellectual life, served by the process of reasoning; but it must also be borne in mind that it is because of our limitations that such a form of arriving at results is necessary.

The question of intensity of illumination as perceived by the visual sense leads us to that of the quantity of light as measured objectively, and here we find also that the means of measurement in this objective way are far more precise than those of direct visual appreciation.[1]

The Hedonic sense is involved in the sensation of sight, though possibly less markedly than in other senses. That it is so involved, however, is made evident by the fact that it is capable of being greatly developed, as by artists, and by the fact that most persons are pleased by certain combinations of colours. We may be satisfied also that this Hedonic sense exists when we consider great quantities of colours. For example, even in imagination we find the difference in this respect between a sky of soft azure hue and a sky of scarlet red.[2]

[1] Various questions that here become suggested are, however, reserved for discussion in the chapter entitled "Examination of Fechner's Law."

[2] Bain and Wundt are amongst those psychologists who find in sensation that attribute here definitely thrown into relief as the Hedonic sense (cf. p. 35).

Mr. E. Bullough has given an interesting study of this question in a paper: "Perception Problem in the Æsthetic Appreciation of Single Colours" (*British Journal of Psychology*). He has enlarged the scope of the study in a communication to the British Psychological Society (1910): "The Æsthetic Appreciation of Simple Colour Combinations."

Experiments have been made in cases of insane persons showing that the influence on the mind of the colour of dwelling-rooms is appreciable.

The taste of artists, their appreciation of harmonies of colours, their delight in blending certain effects, cannot be cited as pure examples illustrating the Hedonic sense. Other considerations here intervene, such as the various associations involved in the word *métier* or profession, the sense of virtuosity in the handling of colours, the desire of experimenting for new effects, and the factitious associations therefrom arising, and also the cant of different schools of painting.

The sense of sight perhaps gives us more pleasure than any other, but that again depends on a multitude of factors apart from the direct Hedonic sense.

The sense of sight is generally assumed to be the keenest in Discrimination within its range, but that range only extends a little beyond 483 millions of millions of vibrations per second for red to 708 millions of millions of vibrations per second for violet; whereas the range of vibrations for some sounds perceptible to human senses is from 30 vibrations per second to 20,000 vibrations per second.[1] The medium of transmission differs, but what we are concerned with in sensation is not the medium itself, but the effect upon the end organs of the nervous supply concerned in the sense. Thus the vibrations of sound may be transmitted through solids, as when a vibrating tuning-fork is held against the temporal bone.

HEARING

It will hardly be necessary to discuss Hearing in terms of the fundamental operations in turn.

[1] More precise determinations give, for vibrations of the extreme visible rays 392 millions of millions of vibrations, and 757 millions of millions of vibrations, per second. For hearing the numbers given above are those of Chladni, but other savants show a more restricted scale. Biot takes the extremes as 32 and 8,192. Young gives 18,000 as the upper limit. This great range accounts for the fact that it is not possible to represent, for instance, a musical score by a spectrum of colours, although a certain relation between sounds and colours exists. This matter was recently discussed by Prof. Sylvanus Thompson in a series of lectures on Sound at the Royal Institution (1911).

The faculty of perception has a very great range in this sense, as we have seen. The faculty of Discrimination within that range is not usually considered relatively so acute as in the case of sight. Thus, for example, while the whole range of vibrations which give rise to hearing extend from 30 to 20,000 vibrations a second, even the highly trained ear cannot well detect differences of $\frac{1}{32}$ of a note; yet all the colours and their shades are contained within a scope of which the lower end represents more than one-half the vibrations of the upper end.[1]

If we follow objective indications it would appear that pitch in a note corresponds to colour, since both depend on the rapidity of vibrations. Loudness would correspond to intensity of light. Timbre would correspond to shade of colour.

Association between the senses of sight and hearing is not merely that of actual experience, for there appears to be a deeper sort of Association dependent on our physical constitution. Thus persons sensitive to impressions indicate certain associations between colours

[1] Dr. Edridge-Green, who has carefully investigated the matter, finds that most normal-sighted persons make about eighteen monochromatic divisions in a bright spectrum; that is to say, there are found effectively eighteen colours. This result will be surprising to many, and Lord Rayleigh believed that his power of discrimination of colours was much finer. Dr. Edridge-Green, however, attributed the discrepancy to various disturbing conditions, as, for instance the admixture of other light, in Lord Rayleigh's experiments. In his paper read before the Royal Society (1911) he refers to a former interesting examination by Helmholtz of Brewster's results in regard to three kinds of solar rays.

The discrimination of $\frac{1}{32}$ of a note will also seem excessive, for many normal persons can hardly discriminate two succeeding notes. On the other hand J. Kerr Love, who has studied this subject, says that an untrained ear, or slightly trained ear, can detect $\frac{1}{8}$ to $\frac{1}{10}$ of a semitone, and a trained ear $\frac{1}{64}$ to $\frac{1}{80}$ of a semitone. According to George Gore a practised ear can distinguish between 1209 and 1210 vibrations of the tuning-fork. These figures would indicate that discrimination in hearing is relatively sharper, as measured by objective tests, than in seeing; but the figures seem exaggerated for hearing. The explanation of discrepancies is partly to be found in the value of practice. It will be generally correct to say that for gross determinations the sense of sight is more constantly exercised; but that for the finer discrimination of experts the sense of hearing is more highly trained.

and the notes of various musical instruments, and there is a strong conformity in these indications when coming from different persons. A red colour is usually associated with the sound of a trumpet, a blue colour with that of a flute.

From objective considerations one might expect that a certain pitch of the note would rather correspond to red, and another to blue; but various other conditions enter into the matter. Thus the difference of timbre between two musical instruments is to some extent a question of pitch, for the timbre, as shown by the researches of Helmholtz, and more recently by the experiments of M. Devaux-Charbonnel, is determined by the value of the overtones, viz. in corresponding higher octaves, varying in various instruments. These overtones affect the shading of the note; and as the discrimination in respect to them is not clear in the sense of hearing, the differences corresponding to shading of notes become vaguely expressible in terms of differences of colour. Moreover, questions of the loudness of the notes are involved, and the fact that certain notes seem to set off the characteristics of an instrument better than others. Thus a loud, braying note on the trumpet would correspond to a red; a clear, soft, flute note of higher pitch to blue.[1]

That the intensity of light is associated with loudness of sound is recognised by most sensitive people, and particularly by those in certain conditions of nerve instability. Thus an idiot boy cited by Maudsley com-

[1] This corresponds to my own finding, and is the more usual, but there are great differences of appreciation with regard to these correspondences. The subject, under the title of colour audition, or synæsthesia, or pseudochromæsthesia, has been zealously studied of late, and considerable literature referring to it has already accumulated. Goethe referred to the matter. Hoffmann, a Swiss magistrate, studied it at the beginning of the last century. Since then Dr. Sachs, Wartmann, Fechner, J. Nuel, Von Kries and more recently Mary Whiton Ca
have contributed to our knowledge of the s

plained of the strong light of the full moon, describing it as "so loud."[1]

M. Hachet Souplet, of the Paris Institute of Zoological Psychology, has in a series of observations and experiments furnished proof that certain animals have senses far more highly developed than human beings.[2] Thus, for example, the wolf hears sounds that are imperceptible to human ears. Birds of prey have vision far more acute than men; while frogs have a certain sense so marvellously developed with regard to humidity that they are thus enabled to guide themselves towards distant places where water may be found. The recognition of such facts, together with those also of the greater importance to animals of the associations of their prodigiously developed senses, will help to explain many observations dismissed under the title of "Instinct"; and to show how a continuous path may be found between instinct and the highest reaches of our reason.[3]

The indications of Generalisation, classification, and symbolisation of sounds are not hard to discover. The notes of different instruments are not only distinct in themselves, but there are fine differences in the quality of the note sounded by musical instruments of the same kind but differing slightly in structure, even when the difference of structure is not perceptible to the eye. The notes produced by a Stradivarius in constant use have a quality, which a musician could detect, superior to that which the Stradivarius would possess if rarely played upon. Yet though this difference may be appre-

[1] The above remarks are put forward as suggestions of factors that are validly concerned in this matter, but the great secret probably resides rather in conditions of the development of the nervous system throughout the whole range of circumstances external and internal.

[2] Galton conducted a series of such experiments with an instrument devised by himself for increasing the pitch. At the London Zoological Gardens he found the animals of the cat-tribe to excel in hearing.

[3] This question will be referred to subsequently (see p. 569).

ciable, it is of another kind to that existing between instruments of different kinds. Hence we obtain classifications, following upon Generalisation and Discrimination.

If we take the case of a troop of cavalry accustomed to charge at the sound of some bar of warlike music, we should find that the first note would already produce its effect in causing the troop to prepare.

Impulse may be well recognised in the case of musical sounds. Indeed, a complete exposition of the æsthetic side of music would require not only to take such factors into account, but to relate them to questions of the mutual reactions of emotional and physical conditions[1] (cf. p. 60).

Memory, with regard to sounds, especially sounds in association, is in ordinary experience remarkably tenacious. A voice is remembered after an interval of years. But in this case the exercise of pure Memory of sounds is masked by the associations. This will be made apparent if we consider the difficulty one finds in repeating the sounds of words of a foreign language. With regard to passages of music not only are the factors of Association generally strengthened, but other factors come into play, such as the response to rhythm and certain expectancies created by familiarity with common forms of music.

[1] Great philosophers, and among them notably Schopenhauer and Spencer, have sought in vain for an explanation of the effects of music. Professor Sarron more recently has believed that the great virtue of musical harmony is to produce a certain rest from distracting influences, but this explanation is not satisfactory. In a work recently published in London, "Art's Enigma," Frederick Jameson discusses this matter interestingly. He finds the secret in "artistic imagination," but he does not tell us very clearly what is artistic imagination.

None of the observations hitherto have struck deeply enough into the veritable nature of the mind's constitution, its relation to the body, and the further relation to concordances of the external world. The indications of a true æsthetic in such matters will be found in the study of such suggestions as that offered above, and in the persistent meditation on the Hedonic principle in simple cases.

The Feeling of Effort will be observed if we take the instance of one expecting to hear a cannon fired, and if we compare this with the condition of expectancy when a note is about to be struck on a lute. The same Feeling of Effort may be also observed in cases of accumulated efforts or repeated efforts. If a person listens to a simple sound repeated at short intervals it will be found that the sound drops occasionally from consciousness.[1]

A distinguished alienist, Dr. Hyslop, formerly of Bethlem Hospital, considers the mental fatigue caused by the incessant noises of London as a factor contributing to insanity. The appreciation of music also depends, amongst its multitudinous factors, on well-graduated and harmonised Efforts of hearing (cf. p. 59).

The Hedonic Effect is evident in sounds, for not only are sounds immediately pleasant or unpleasant in themselves, but as music is formed by the combination of simple sounds there must necessarily be Hedonic Effect in these sounds. The total effect of music cannot be ascribed even to combinations of such pure elements. The truth of this may be perceived by reference to the effect of passages of poetry where the musical quality may be faulty, but where powerful feelings are stirred by the immediate images evoked, and by the deep associations they suggest. In music also there is an effect, which should be taken into account, of the immediate physical impacts of the undulations arriving in well-modulated harmonic vibrations (cf. note p. 59).

Time is an invariable incident in the exercise of

[1] Here, however, a great many factors enter into the matter. I have observed in many trials that the sound does not fade away as the result of a gradual diminution. It would seem as if the ear had some sort of faculty of accommodation. Fernet, in the *Semaine Médicale* (1911), discusses the subject interestingly, and points out the importance of the action of the muscles of the middle ear.

hearing. With regard to no other sense do we find, apart from the influence of other associations, such a clear appreciation of succession.

Space enters as a distinct element in every sensation of hearing. We are so accustomed to associate our notions of space with the visual sense that we do not always remark the importance of the space relations in sounds. It is true that we recognise at once whether a familiar sound is near or far, but we are inclined to attribute our knowledge in this respect to past experience. Such experience is very helpful in enabling us to form just estimates of distance in connection with sounds, but similar remarks hold true with regard to the visual faculty.

It will be seen, in the analysis of Externality, that a great many factors combine with the Immediate Presentations of vision to give us our notion of space. It is evident in the case of hearing that the single unaided sense is inadequate to give us just notions of distance, and this consideration may help us to appreciate the analysis of Externality as presented mainly through vision. But it is also conversely true that a certain element of the notion of space is involved in hearing.

But sounds are not merely near or far. They have in the ordinary language "volume," that is to say, they convey the impression of volume in the external source of stimulus. Poets speak of the "thin" piping of a flute, of the "long, sequacious note" of the violin, where "long" is not expressed wholly in time, of the roll of the organ, of the waving texture of the music of Wagner's "Valkyrie"; while Keats, so often acute in introspection, speaks of "music that comes swooning over hollow ground." Human beings in civilised life generally find the sense of vision adequate for most of what they require to know of spatial relations, but savages and

some of the lower animals have extensive fields of space relations also built up through the sense of hearing.

A detailed discussion of each sense in turn in its relation to the Fundamental Processes of the mind would take too much space for our present requirements. A suggestion, however, is hereby afforded for particular studies in this respect which could not fail to be interesting.

In the tactile sense the attention may be called specially to the Hedonic principle or quality. By our sense of touch we are informed as to whether objects are hard, or soft, or solid, or liquid.

Some sensitive persons are greatly affected by peculiarities of touch. I knew a young man, a poet, who experienced peculiar feelings when he touched velvet. There is generally a direct pleasure in touching furs, and particularly in stroking the glossy fur of a well-groomed animal.

The sense of touch is not a single sense. It is the generic name for the various senses of touch in the different parts of the body, and these senses vary greatly in the fineness of their perceptions and in their quality.

A considerable number of experiments have been made in regard to the acuteness of the sense of touch, and it has been ascertained that the most sensitive parts of the body in this respect are the tip of the tongue and the tips of the fingers.[1] Thus though touch gives us

[1] I refer the reader to text-books of Physiology, and particularly to the admirable work of Weber, "Tastsinn und Gemeingefühl" (149 in Ostwald's series), which laid the foundations definitely in this branch of science. But since Weber's time a great deal of work has been done in various countries, as by Hering, Funke, Volkmann, Vierordt, Kottenkampf, Ulrich, Barth, Lewy, Titchener, Dresslar, Pillsbury, and V. Henri (cf. p. 42). Goldscheider has devoted himself to this and kindred studies. J. Bernstein in the *Naturwissen. Rundschau* discusses a New Theory of Sensation of Touch.

information respecting space relations the information may be often defective, and in all cases it is seen to be not precise within the limit of the Discrimination of vision.

But in this respect touch differs only in degree from the other senses. From objective science we know that touch depends on the "end-organs" or tactile apparatus developed at the extremities of the terminal filaments into which the great nerve trunks ultimately branch. In some parts of the body these organs are more finely developed, and they are also closer together than in other parts. But similar conditions hold with respect to eye-sight and hearing.

In the visual organ the sensitive portion is confined to the retina, which forms a relatively very small part of the human body, and the retina consists, as far as its sensitive elements are concerned, in an assemblage of organs, closely packed together, representing the terminal development of nerve filaments, and consisting of a complex arrangement of "rods" and "cones" and nerve fibrillæ.

The manner of operation of these rods and cones is not understood, but it is evident that some agitation of the organ is the usual preliminary to a visual sensation, and it is equally evident that the stimulus to this agitation is an impulse of "light." What is the exact nature again of this stimulus we do not know; we are not even sure whether, as Newton supposed, and some modern philosophers have thought possible, the stimulus is derived from corpuscles emitted from the luminous body, or whether, as Huyghens and Thomas Young and Fresnel thought,[1] and most scientists since

[1] Grimaldi and Hooke gave expression to this hypothesis before Huyghens. Thomas Young revived it in opposition to Newton's corpuscular theory. Fresnel conceived it more clearly in the manner now generally adopted, and with marvellous analytical skill gave it the modern mathematical form.

Young's days have believed, the stimulus is due to the vibrations of a pervading ether which is perturbed by the source of light.

The retina is more closely packed with rods and cones and more highly developed in one region—the macula lutea—than nearer the periphery. Therefore when we affirm that there is not one sense of touch but innumerable senses distributed over the body, we do not in this way express an exceptional character. The sensation of vision depends on the co-operation of a number of separate organs each performing its own function. This is more strikingly shown when we consider both eyes. The impression made on the right eye is quite distinct from that made on the left eye by the same external object. The two impressions are not even formed together quite accurately, and it is indeed to this circumstance that we owe in part our notion of "solidity" or relief in the outside world. When one eye squints it may form its image so entirely distinct from that of the other eye that no fusion occurs at all. Squinting is generally due to the inefficient working of one of the muscles that direct the eyeball, such as the external rectus or internal rectus.

Accordingly when the whole scope of the visual faculty is considered, the muscular apparatus directing the eyeballs should be considered as part of the organ of sight. In the operation of sight with respect to determining our notion of Externality we shall find that such factors must be combined with many others before the ultimate impression is attained. The sense of hearing likewise depends on the co-operation of a great number of complex parts, and even the end organs—the organs of Corti—have varying degrees of development and concentration in that part of the apparatus which they occupy.

The fact that the sense of touch shows such independence of function of various parts, and such disparities of impression, is revealed to us by the sense of sight. We see that there are two needle-points resting on a part from which is conveyed the impressions as if from a single point. We have no sense which we can immediately direct upon the visual apparatus to give us similar information.

The sense of touch, though less acute in regard to certain aspects of the external world, has perhaps a stronger Hedonic Effect. The very words "comfort" and "luxury" are associated more or less closely with this sense.

It also appears that as the sense takes less heed of the fine Discrimination of its more sensitive apparatus, and becomes interested in the more massy general conditions of agreeable contacts, the Hedonic Effect is brought more into evidence.

SMELL

The sense of smell has no doubt been enormously diminished in human beings during the course of their civilisation, and it differs widely in different individuals. Its Space relations are weaker than those of vision or hearing, and its acuteness within its range of Discrimination is generally considered less.[1] On the other hand its Association incidence and its Memory are often more tenacious.

Its Hedonic Effect is also greatly marked. Wordsworth is said to have been very deficient in the sense of smell, but on one occasion he smelt fresh

[1] Jacques Passy, whose studies in this sense were admirable, found it to be "the most sensitive, but on the other hand the least precise" of all. Individual variations are, however, enormous in regard to odours.

violets, and remarked that it was like a glimpse of heaven.[1]

There is no doubt that the comparative deficiency of this sense is due in part to want of cultivation. It would be possible to obtain by study and experiment a graduated scale of odours, and it would also be possible to obtain a basis of classification of odours by their qualities. This would in turn afford the basis of an æsthetic development of the use of the olfactory sense after the model of the development of the sense of hearing in music. The effect of music is only in part due to the immediate appreciation of the sounds by the ear; and as the Hedonic Effect is originally even stronger in the sense of smell there seems to be no reason in nature why a distinct powerful art should not be built up on a gamut of odours.

The olfactory sense ought also to be investigated with a view to determining whether the appreciation of odours sums up its functions. The sense of pungency, as of various acids and of smelling salts, must be distinguished from that of odours. The sensations produced by the presence of noxious gases, the peculiar effects of chloroform or ether, even as affecting the olfactory sense or some sense referred to a wider region, differ from those either of pungencies or of odours.[2]

[1] Milton, in a fine passage of "Comus," uses an expression derived from the sense of odours to reinforce the power of his description of music (cf. p. 267):

> "At last a soft and solemn-breathing sound
> Rose like a steam of rich distilled perfumes,
> And stole upon the air, that even Silence
> Was took ere she was ware, and wish'd she might
> Deny nature, and be never more,
> Still to be so displaced. I was all ear,
> And took in strains that might create a soul
> Under the ribs of Death."

[2] Associated with Jacques Passy, as well as both before and after him, have been many savants who in the study of odours have given us a charming chapter of science. The principal names are Weber, Paulsen, Zwaardemaker, to whom

TASTE

The sense of taste is relatively not strongly developed with regard to Discrimination within its scope. On the other hand it is powerful in its effects of Memory and Association. Its Hedonic Effect is also relatively very strong, so much so in fact that in the perversion of the doctrines of Epicurus such as we find in decadent modern societies the importance of this sense has become exaggerated.

It is deserving of study, however, in a scientific way, and for this purpose it would be necessary to establish a scale of taste as accurately as possible. The quality of tastes differs widely, and here also accurate studies with nomenclature referring to good standards are called for.

The sense of taste will be found to be more complex than generally imagined. Alexander Bain calls attention to the distinction between tastes and relishes.[1]

we owe the invention of an olfactometer, Kayser, Funke, Aronsohn, Fischer, Petzoldt, Zuckerhandl, Patrizi, C. Henry, É. Morin. G. E. Müller has studied the Eindringlichkeit, or insistency (to adopt Passy's term) of odours, by testing the minimum quantities necessary to excite sensation. Bailey and Nichols have worked in the same direction. Lombroso and Ottolenghi have tested this sense in criminals. Lehmann and Dr. Wm. Elder have studied memory in regard to smell. Eleanor A. M. Gamble has experimented on smell in regard to Weber's law. Exner and Lustig, working by aid of histological methods, found all cells in the olfactory region to be sensitive. Classifications of odours have been given by Linnæus, Geissler, E. Rimmel, and also, on the basis of chemical composition, by Berthelot.

For the work of Jacques Passy, the "Reliquiæ" edited by his father may be consulted, as well as *L'Année Psychologique*, 1895-6. This may well form an introduction to the whole subject.

[1] Since Bain's time a considerable amount of research has been carried out with regard to the sense of taste by Ranvier, Engelman, Retzius, and Tanzi among the histologists; and recently by Ponzo by experimental methods. Experiments have been carried out in Smith College, U.S.A., by A. H. Pierce on the effect on hearing of the stimulation of the sense of taste. Dr. P. Michelson has studied the sense of taste in the larynx. Bailey and Nichols have experimented on the delicacy of the sense. Lombroso and Ottolenghi continued their researches in the case of criminals with respect to this sense also. Camerer studied Weber's law in regard to taste. Keppler, J. Corin, and Karl Rittmeyer have devised various experiments. N. Cybulski and H. Beck found in a clinical case of extirpation of the tongue that taste persisted for sweet, bitter, and sour substances, but not for salt.

NEED OF ACCURATE STUDY OF THE SENSES

Accurate study in regard to these special senses is still a need in the science of psychology. Most of the experiments in experimental psychology deal with the visual sense. One interesting fact, which lies at the threshold of the study, is that in visual sensations one-tenth of a second has been found to be the time required for the objective stimulus to make its full impression, and that one-tenth second is the period during which the impression persists.[1]

Similarly studies should be made in connection with all the other senses. Such data as would thereby be afforded would assist in the establishment of a definite art of odour impressions.

The time during which the external stimulus must act seems to be longer in the senses in which Discrimination is less acute and the time of persistence of the impression is correspondingly longer. The Time element is thus less precisely marked. Correspondingly too we find the Space impressions becoming vaguer. The elements which are most concerned in intellectual development, Discrimination, Classification, are less marked in these senses. On the other hand Memory and Association are strong with regard to them, and Impulse, Feeling of Effort, and the Hedonic Effect more remarkable than in the senses of finer discriminations.

This is well in accord with the necessities of nature. For the senses of the strongest Hedonic Effects are those most fundamentally related to the necessities of life; and even in regard to comparisons in the individual

[1] In Ribot's "Les Maladies de la Mémoire" some figures are given with regard to other senses, while those for light are taken higher. Wundt's results for reaction times have been generally accepted, but the celebrated Italian psychologist, Sergi, finds differently. What is here suggested, however, is that the work should be carried out systematically on the basis of good classifications.

senses themselves the strongest impressions have similar relations. Certainly the visual sense is of great importance in the preservation of life, although it is not vital; but a good deal of the development of its discriminative faculty is not so closely related, as in the indications of other senses, to the fundamental physiological conditions of sustenance.

The five senses do not include all our sources of communication with the outside world. The distinction, for example, between tastes and relishes may lead us to inquire more closely in how far the other senses may have their data subdivided, and also whether the different kinds of information they convey may not be subserved by different kinds of nerve-endings. Already in this respect a difference has been established between the functions of the rods and cones.[1]

In addition to such possible subdivisions, we have other distinct senses, certain of which should hold equal rank with those of the five. The sensations of heat and cold in no wise resemble any of the other sensations. The seat of their end-organs no doubt is in the skin, but these sensations are not to be confounded with those of any kind of tactile impressions, and in fact they are subserved by a distinct sensory apparatus. The nerves conveying the impressions of cold are considered by some physiologists to be distinct even from those of heat.[2]

The examination of this part of the nervous system

[1] In the macula lutea (yellow spot), which is the most sensitive part of the eye, the proportion of cones to rods is greater than elsewhere. In the central fovea of the macula lutea there are no rods. The magnificent histological work of Ramon y Cajal, Müller, Ranvier, Schultze, Retzius, and others has revealed to us the complexity of these organs of vision, but hitherto not much has been ascertained precisely of their functions.

[2] Goldscheider, however, came to the conclusion that the difference of the sensations of hot and cold was not due to difference of nervous paths.

has advanced but little, but the general remarks regarding the increase of the Hedonic quality with the diminution of the acuteness of Discrimination will here also apply.

It was for long assumed that the organs of the ear were concerned in hearing alone, but the anatomical observation of the ear, and particularly the examination of the semicircular canals, suggested other functions. The semicircular canals are arranged approximatively in three planes at right angles to each other. The analogy of the three planes to which Cartesian co-ordinates in geometry are referred would naturally suggest that the function of the semicircular canals has to do with the determination of position, and hence of the balance of the body, and of orientation, or estimations of direction.[1]

This supposition is strengthened by the fact that in homing pigeons the semicircular canals are enormously developed.[2] It is not assumed that the semicircular canals are alone concerned in such judgments.[3] In man especially the appreciation of balance is derived from various nerve sensations, coming from the feet,

[1] Consider, for example, in a room the floor and two walls which meet in right angles. The position of any point in the room may be determined if we know its height from the floor and its perpendicular distances from the walls. On this principle Descartes founded his system of co-ordinates, as the distances which determine the point are called.

[2] A great number of experiments by Flourens and a host of subsequent scientists have been made to study the results of destroying the canals. The indications all tend to confirm the view that the semicircular canals are concerned in our appreciation of balance and orientation. The recent experiments of Weill, Vincent, and Barré on the Voltaic Vertigo seem to confirm this theory of the functions of the semicircular canals. H. Held ("Untersuchungen über den feinern Bau des Ohrlabyrinthes der Wirbelthiere," Gesell. d. Wiss., Leipzig, 1907) has well demonstrated the finer structure of the labyrinth.

[3] Hitzig's work, "Der Schwindel" (Vertigo), of which a second edition has been edited by Ewald and Wollenberg, of Strasburg, contains arguments in favour of a vertigo centre in the brain connected with the centre of equilibration in the medulla, and distinguishes between cerebral vertigo, ocular vertigo, and labyrinthine vertigo.

for example, in walking, associated in experience with various positions of the body.

The muscular sense is distinct from the Feeling of Effort noted in connection with the other senses, though that Feeling of Effort appertains also to this sense. This may be strikingly brought to light under favourable conditions. If a person experiment in lifting a bar of lead so far as to obtain a fairly accurate notion of the effort required, then if, as has been previously indicated, a bar of aluminium, or other light material, be presented so as to look like the bar of lead, the effort is made in the same terms as before, and a little shock of surprise is found in the expenditure of so much nervous energy in vain.[1] This indicates the Feeling of Effort accompanying the exercise of the faculty, but the actual muscular sense in operation, and *per se*, is not coincident with the Feeling of Effort.

The muscular sense in regard to different muscles would present diversities analogous to those observed in regard to the sense of touch. Moreover, the fineness of Discrimination greatly depends on cultivation. Even in muscles which work automatically, and without giving any definite indication in consciousness, some derangement of function will often call attention to a definite muscular sense. Further, even when the exercise of the muscular faculty produces no definite effect in consciousness, yet the messages continually sent up to

[1] The experimenter would probably laugh when he recovered from his shock of surprise. This experiment is interesting as indicating the primordial base of that sentiment which in its developed form we recognise as "humour." Kant says that it arises from the collapse of a wrought-up expectation, but he should have added that the shock of surprise should not be accompanied by any feeling of deep pain, as might happen in some cases of collapse by expectancy. For example, the humour would not be marked if the result of the experiment should happen to be a serious injury to the eye.

I have made use of an analogous illustration in a study on humour in animals. In such a case we must make our analogies deep enough to exclude the accidentals of language, or of modes of expression peculiar to ourselves.

the central nervous system form part of those innumerable factors of the subconscious life which, as we shall afterwards see, modify our conscious impressions.

The functions of the principal organs are carried on automatically, but even in cases of perfectly normal working they are not without producing an effect in consciousness. The expansion and relaxation of the lungs, especially in deep breathing, are accompanied by well-recognised indications of the senses subserving the parts. Similarly the working of the normal stomach in full operation makes itself known.

All the organs give indications through the senses when difficulties arise through disarranged functions. The heart, which is a powerful hollow muscle acting as a pump, may in the case of highly sensitive people give indications even of normal working. The heart labouring against obstruction makes its efforts known through the special sense affected. The arteries under undue tension also give indications of that condition through their nerves of supply.

It would lead too far to consider in turn each one of the natural processes and the indications of its normal and abnormal working; but from the suggestions offered it becomes apparent that the five senses are far from representing the whole of the impressions which arrive at the sensorium which help to form the consciousness and to direct the current of the thoughts.[1]

The sense or feeling of hunger and that of thirst

[1] H. E. Beaunis, in his work "Les Sensations internes" (1889), has laid broad foundations of systematic study. Amongst others who have worked at the subject are Külpe, Höffding, Wundt, Ladd, Kroener, Hermann, Schiff, Richet, Ebbinghaus, Titchener, Meumann, Sherrington. Elsie Murray in a thoughtful article (*Am. J. of Psych.* 1907) finds fault with the classification of Beaunis.

There are two sorts of receptive tissue in internal organs—free sensory endings and Vater-Pacini corpuscles. Poivier and Charpy found in the mucous membrane of the ileum nerve cells similar to those of tactile corpuscles.

are quite distinct from any of those hitherto considered. That is also true of the senses connected with other deep processes of prime importance to the race. In these cases we find confirmed the inverse relation between the acuteness of Discrimination and the intensity of the Hedonic Effect.

To the cases of general hunger we may add those of local hunger and the sense of inanition or want of exercise of individual parts of the body.

Investigation is needed regarding other sources of information of the external world. For example, there appears to be a definite sense of humidity and dryness, analogous in human beings to that so enormously cultivated in the batrachians (cf. p. 58). It would be interesting to experiment so as to determine what parts of the body are most sensitive in this respect; what type of people, what ages, and what conditions of health best exemplify the function.

There is no doubt also a distinct sense of pressure, as for example atmospheric pressure. It would be well to inquire in how far any part of the auditory apparatus gives us indications in this regard. There would appear also to be a sense of vibrations as apart from those which come into consciousness as sounds.[1] And there may be

[1] A case may be here cited, in which, however, it is difficult to exclude mystification? A Norwegian, Emil Knudsen, aged 36 (1910), claimed to possess a sixth sense. At the suggestion of Professor Friedenreich, a leading neurologist of Denmark, an experiment was tried. Knudsen was blindfolded, and placed at the wheel of a steamer in Copenhagen Harbour. He is said to have piloted the vessel through the intricate channel with perfect confidence.

Another experiment of a much more rigorous character was that of Professor Watson of Chicago, who destroyed the sense organs of the ordinary five senses of a rat. The rat nevertheless found food which had been so placed as to make the test conclusive.

Of recent years there has been a good deal of writing on the subject of a "sixth sense," as by L. Truschel, "Das Probleme des sogenannten sechsten Sinnes der Blinden," and F. von Velden, "Der Sechstesinn" (*Fortschritte der Medizin*, 1906). V. Forti and B. Barrovechio in *Med. Klin.* (1905) and Herzog in *Deutsch Zeit. f. Nervenk.* (1906) have discussed the Vibrationsgefühl (Sense of Vibrations). Within

also a sense of rhythm in the change from one sound to another, for musical notes are not defined alone by sounds or by the combinations of sounds that correspond to their chords.

There would appear also to be a distinct sense making us acquainted with the change of electrical conditions in our neighbourhood and consequently in ourselves, when such changes reach a certain importance.[1]

And again there is probably a distinct sense of radiation, as may be observed in close dry atmospheres, and which is not to be confounded with any sense of light or heat.

A combination of indications such as those referred to, from the mention of humidity onwards, may account for the fairly clear premonitions some people have of the approach of storms.

§ V. Impressions Produced by Internal or Subjective Causes

But besides such sources of information the mind has to deal with subjective impressions. Let us consider for a moment roughly the sequence of physical phenomena which precede, say, a sensation of light or colour. We suppose first a luminous object. The luminous object communicates a stimulus, whose nature we do not know,

recent years the question has been carefully studied in America, notably by F. B. Dresslar, whose paper on "The Pressure of the Drum" (*Am. J. of Psych.*) may serve as an indication of the methods of experiment. Helen Keller, who, though blind and deaf, can enjoy and discriminate music, said: "Every atom of my body is a vibroscope."

[1] Hachet-Souplet in an article entitled "Les facultés mystérieuses des Bêtes," refers to the high development of the sense of electricity in birds. Dr. Ph. Maréchal has published a work with the title "Supériorité des Animaux sur l'Homme."

An observation by M. P. Dellile, an engineer on board the French war-vessel *Descartes*, in which he traces certain forms of nervous trouble to the use of wireless telegraphy, may be found to have a bearing on this question.

which reaches the cornea, which traverses the anterior chamber of the eye, the crystalline lens, the vitreous body, which finally acts upon the rods and cones of the retina in some manner likewise unknown to us.[1] But none of these passages or disturbances constitutes a sensation of light or colour. Nor can we say that the retina is the organ that causes sensation, for if the process be stopped at the point to which we have traced it, there will be no sensation. The retina is simply one of the essential parts of the optical apparatus. The crystalline lens, it will be noted, is not an absolutely essential part, for it is removed in the operation for cataract, and its function in causing the rays of light to converge at the retina is more or less well carried on by the glass convex lens which the surgeon selects according to certain indications varying with the individual.

The retina, however, may be intact and yet from various causes no sensation of light or colour may occur. The excitation of the retina communicates in turn some form of stimulus to the nerves leading from the retina to the brain and which, on each side, combine to form the ophthalmic nerve. Of the nature of this stimulus we really know nothing definite. We have only a choice of ingenious theories supported by celebrated names.

The course of the stimulus from the retina to the brain depends upon the part of the retina concerned. Most of the rays which fall on the inner side of the retina find their effects carried through the optic chiasma, or crossing-place of the two ophthalmic nerves, to the opposite side of the brain. Along the optic tract, with which a vast number of junctions are formed along the different portions of the brain which it traverses, the stimulus is carried until it reaches what is called the

[1] The main parts only are here noticed. The cornea is itself formed of five layers, and in the retina eight layers are distinguished by histologists.

centre of vision in the occipital lobe of the brain. This centre has been located by various clinical observations, biological examinations, and experiments. What is the precise relation of a centre in regard to the stimulus or to the sensation we do not know. Nor is it even determined that this centre is the ultimate destination of the stimulus.[1]

Now at any part of the route which the stimulus traverses there is no sensation. The stimulus itself changes its character, and we are acquainted only with vague general indications regarding the conditions under which this stimulus operates. It may be taken as established, for example, that the retina responds, in general, only to the excitation of which the source is a luminous object.

But that excitation of the retina is not a sensation, nor is it an "image" of the object, though this term may be useful in ordinary language. It is necessary to insist on these points, for much confusion has been caused by the loose use of language in this regard. A question has been frequently asked, for example, How is it that if the image on the retina be reversed we do not see things upside down? It is true that, as shown in elementary books on physics, the stimulus proceeding from the lower part of the object affects a part of the retina which is above that affected by the stimulus from the upper part of the object. But that is in accordance with the whole manner in which we obtain information of the external world. To understand how it is that no confusion arises

[1] Sir William Gowers has published in a pamphlet, "Subjective Visual Sensations," some arguments in support of a theory given out by Ferrier, that there is a higher visual centre in the region of the angular convolution. A. W. Campbell's work, "Histological Studies in the Localisation of the Cerebral Functions" (Camb. Univ. 1905), forms a good introduction to this subject. The whole field bristles with difficulties, and not less so, as we shall see, in cases where all recognised authorities had concluded that the question had been settled. The matter will be found discussed in detail in the chapter on Brain Localisation.

on this account is no more difficult than to understand how men can live and walk at the antipodes, and that in fact we ourselves are at the antipodes if their standpoint be taken as the basis of comparison.

Now when we recognise the fact that we may be considered as living at the antipodes, we find the answer more readily, viz. that we do not experience in any way the necessity of having to make corrections arising from this fact, that in fact the terms up and down are relative, and that we call down what as viewed from the antipodes would be up; and all that we know in this way being similarly circumstanced our view of the external world is consistent.

Even when by artificial means a correction is made, as for instance when an engineer directs operations while looking through the telescope of his theodolite, which reverses the "image," it is remarkable how soon one becomes accustomed to disregard any difficulty that might arise on that account.

But with regard to the "image," the term "image" really presupposes a seeing eye, apart from the object, be it retina or screen, on which the "image" is supposed to lie. The luminous body does not of itself, and absolutely, send forth images. It sends forth perturbations, corpuscular or undulatory, which if brought into contact with a Crookes radiometer suitably arranged produce motion; which if brought into contact with a mixture of hydrogen and chlorine produce a chemical combination; which if brought in contact with a sensitised plate under suitable conditions produce a photograph; and which if brought into contact with the retina produce another kind of agitation to which the retina is specially adapted. But the disturbance of the retina is not a photograph, nor is it an "image"; for just as the photograph requires the combination, speaking broadly, of the

luminous rays and the sensitised plate, so does the image require the combination of the luminous rays and the receiving systems which may be physically represented by the entire apparatus including the visual centre in the brain.[1]

It should be remarked that of all the rays which proceed from the luminous body only a small portion affect the retina in the manner which produces conscious impressions. The retina is a little instrument set up in space, and responding to only one class of the innumerable agencies that surround it, and only to a limited section in that class. The notion of the image formed on a retina is due to the fact that we imagine projected on it such impressions as are derived from the luminous rays reflected from a screen, for example, and which after their course through the retina to the visual centre result in an image.

If the previous exposition be clearly understood, then not only will all difficulties about seeing things upside down vanish, at least according to these terms; but also the usual explanations of physiologists and psychologists regarding them will be seen to be based on misapprehensions.

Further, many such crude notions regarding the physical basis of thought will vanish also. The excitation of the retina is not a thought; it is not even an image. Yet the discovery of as much as we know of the curious mechanism of the retina has proved extraordinarily interesting and has marked an advance in our knowledge of vision. But

[1] W. MacDougall (*Brain*, vol. xxxiii. Pt. IV.) has elaborately discussed the question of "The Relation between Corresponding Points of the Retinæ." Newton supposed that there was a common centre in the brain for corresponding points, and he was followed by many authorities in this assumption. MacDougall points out a number of circumstances (as, for instance, rivalry of the images from the corresponding points) which are not consistent with that view. He concludes in favour of separate centres. Histologically, the matter is not determined. Psychologically, it will appear in the following exposition in a new light.

if we still pursue our investigations and place on an equal level of certitude the existence and mechanism of the visual centres, and even the higher centres; or even if we could observe, directly or imaginatively, the final co-ordinated movements of all the very molecules themselves of this intricate nervous apparatus, we should still be as far off as ever from the understanding of thought.

If one were to speak of thought as a function of the eyelids, the absurdity of the statement would be immediately evident. Yet, usually, in receiving the impressions from external luminous bodies the eyelids must be held apart, that is to say, in general language, must have a certain determined physical condition. But at what stage should we stop in tracing the process of stimulation from the external eye to the visual centre?

The nerve cells of a portion of the brain must undergo certain determined physical conditions before a sensation of light or colour arises in the mind; but here, so far as we know, or conceive, we have to deal with nothing more than a physical antecedent, just as the opening of the eyelids was a physical antecedent.

That the analysis may never be carried further it is impossible to say, for who can impose limits on the progress of the mind? But the question becomes merged in those obscure depths where we seek to find the nexus between cause and effect, and where we have been unable to solve the far simpler problems in the tangible world about us.

But to speak of thought as a function of the brain, in any manner, inconsistent with the foregoing exposition, is absurd. All that we can affirm is that a certain course of physical excitation is the antecedent to thought. We can say it is the cause of thought, if the word "cause" be invested with no other meaning than that which is involved in such a conception.

MORBID SUBJECTIVE IMPRESSIONS

The foregoing discussion has been necessary in order to consider properly other sources of information to the mind, viz. those of subjective impressions.[1] The retina, we have noted, responds in general only to one kind of stimulus, viz. that of a limited section of the spectrum of light rays. It is true that various experimenters, such as Helmholtz with electricity, and Purkinje with different kinds of processes, have produced images probably due to the excitation of the retina, but such images are vague and not correspondent to the outside world.

The excitation of the nervous cells of the visual centre usually depends on that of the preceding excitation of the retina, but it appears that these cells may be excited independently, though whether in that case the stimulus is of a similar or different character we cannot tell.

The matter may be made more graphic thus: The excitation of the retina is representative of certain movements of the rods and cones, and the stimulus under various forms is carried at length to the visual centre, and in the course towards this the arrangements of mere nerve conduction have been replaced more and more by arrangements of intercommunication between all and all. Now this web-like tissue suffers a peculiar perturbation corresponding to each kind of stimulus. That form of perturbation or excitation we may, with the restrictions already noticed, speak of as the cause of sensation.

Now the nerve centre knows nothing directly of the retina. And moreover under various conditions of nerve instability it is possible that the excitations of the web-like structures of nerve ganglia, nerve cells, nerve filaments, may occur from conditions within the brain itself.

[1] In this chapter the word "subjective" is used in a narrower sense than usual. It refers to the impressions which are independent of any immediate stimulus from the external world. It is not found that confusion is thus likely to arise.

Accordingly sensations are presented to the mind similar to those that arise from stimulation in the usual way.[1] These sensations may be combined in various ways; and so at length we may obtain subjective impressions of a complex and vivid character which make an impression sometimes almost as strong as the realities of the external world. In many instances they are mistaken for such realities, and these are then a fruitful source of the stories of ghosts, apparitions, appearances of various kinds and of more or less distinctness.

It happens, however, often that these subjective impressions occur in the case of those whose minds are trained to observe and who have a good general knowledge of the causes of impressions objective and subjective. Accordingly a great deal of material for study has been collected, and many interesting facts about subjective impressions have been noticed.[2]

Subjective impressions are most commonly found in those of a highly sensitive temperament, either overwrought by excessive work, stress, or deprivation, or afflicted with disease.

Subjective impressions occur in well-marked forms in migraine and in epilepsy. In migraine the subjective impressions are often those of a kind of zigzag in colours, or "fortification spectrum," occupying a considerable part of the field of sight. The impression is definite, it lasts a considerable time, from a quarter to half an hour, and it is followed by a headache lasting for hours.

The subjective impressions in connection with epilepsy

[1] Bain gives reasons for believing that the same central fibres are stimulated in subjective impressions as in those due to external stimulus. Intense subjective impressions of colour may even give rise to the complementary colour, as in the ordinary case.

[2] Brewster, Féré, and others have shown that certain hallucinations are deviated by the prism. This indicates their dependence on objective stimuli. Parinaud found that consecutive images—for instance, those of complementary colours—were not deviated by the prism.

are much shorter in duration, lasting only a few seconds, but never elaborate, and they constitute part of the "aura" or premonitions of an epileptic fit.

Sir William Gowers mentions a case in which the sequence was the following: First the beating of the heart was felt; this impression seemed to ascend to the head, where it became audible as a sound. Then two lights appeared before the eyes and seemed to approach by jerks, synchronously with the pulsation. The lights then disappeared, and were replaced by the figure of an old woman in a red cloak, who offered something that had the smell of Tonquin beans. Then consciousness was lost.

This case is interesting for many reasons. It will be noted that the visual impression here is both elaborate and distinct. The first indication is the increased beating of the heart, and subsequently the fact that the pulse-beats marked the jerky progress of the approach of the lights indicates that nervous discharges which correspond to sensations and their combinations depend on the supply of blood. Moreover, as the disturbance becomes more severe a larger area of the nervous web is affected, as shown not only by the greater elaboration of the subjective images, but also by the fact that successively the areas corresponding to the centres for sound and the centres for smell are invaded and perturbed. Finally the disturbance becomes so great that the nervous discharges find outlet also in the violent movements of an epileptic fit.

In this particular instance the impressions of sight dominated; but in other cases of subjective impressions, particularly in cases of delusions, the dominance of aural impressions is common.

Each sense may yield subjective impressions of a characteristic kind. It is not for our purpose necessary

to work them out in detail, but attention may be called to the striking example of pains referred to a great toe in the case of a man who has had his leg amputated.

In all these cases we find subjective impressions arising spontaneously even in spite of determined efforts of the will of the subject, and finally dominating his consciousness. We find something analogous in the movements of Memory, which we will discuss later, and in all manifestations of mental construction, invention, fancy, or imagination.

It may happen that the imaginative faculty is much more highly developed in an individual than the faculties which tend to criticism and good judgment. Also in such a subject the conditions may be found of overwork and stress, perhaps following an artificial stimulation. Again, highly intellectual men, from Julius Cæsar to the poet Alfred de Musset, and, it has been affirmed by some, Napoleon Bonaparte,[1] have been subject to epileptic attacks.

Hence has arisen a certain general impression summed up by Dryden in the lines:

> Great wits to madness sure are near allied,
> And thin partitions do their bounds divide.

It would not be well, however, to push arguments of this kind too far or to draw hasty conclusions. We shall afterwards see that the very way of progress in thought is by a series of tentative combinations, afterwards tested by reference to the realities of the world. In men of high intellectual endowment and fine spirit the play of the

[1] By Talleyrand, whose evidence would be decisive if it were reliable. Many French medical men class Napoleon with the epileptics. Amongst other famous men subject to epilepsy may be mentioned Richelieu, Montesquieu, Swift, Chateaubriand, Dr. Johnson, Lenau, Crebillon, Carducci, Schiller, Petrarch, Molière, Alfieri, Flaubert, Buffon, Pascal, Newton, Mozart, Händel, Paganini, Mahomet, Peter the Great, Charles V. There are grounds also for including

constructive and inventive faculty is likely to be active, but there is no reason in the nature of things why the analytical faculties should be defective. If they were, the great wit would go only a small distance even in achieving a reputation for ability.[1]

Moreover, the efforts of fancy and combination are within a certain measure controlled. They do not entirely tyrannise over the mind, as in the case of lunacy or epilepsy. Such diseases spring from material causes which are as likely to affect those of vulgar minds as the "great wits," though when a mind of great energy is so affected its aberrations become more remarkable.

Moreover, in these matters of popular judgments the great wits are judged by a standard which is itself faulty, viz. the entire sanity and reasonableness of the average of mankind. All kinds of persistent errors in conduct or thought which do not stand the test of conformity with the great truths of the universe are in themselves elements of madness; and minds little cultivated are hardly more likely to form correct judgments than those of great reach and fine illumination.

Nevertheless, the popular saying indicates the dangers which lie near in cases of great effort in mental operations. The man who flies is liable to fall, and the more audacious his flight the more striking will be his fall. But flying is not falling. There must also be noted the aspect of fatigue and its consequences in faculties which become habitually over-exercised. The failure of genius is sometimes a case of "trade-maladies."

We have now taken a review of the senses, and we

[1] The critical faculty is singularly deficient in such books as Nisbet's "Insanity of Genius," and also in the work of Lombroso himself. The collection of facts has interest and value, but the deductions drawn from them are often absurd. A thesis recently read by M. Giraud at the University of Lyons, "Névroses à décharge," is no less interesting in regard to examples, and is couched in a more reasonable strain.

have considered them in connection with the series of fundamental operations which have been posited; we we have considered various sources of information, apart from the "five senses"; and we have noted in dealing briefly with subjective impressions that the physical organisation which subserves thought is to be considered otherwise than as something inert which comes into activity when touched by outside impressions: that, to quote a simile used by Sir William Gowers, it must be viewed not as a piano but as an organ full of pent-up energy waiting to be released.

This manner of regarding the mind and its physical substratum will be confirmed in all that follows.

We will now consider more precisely some of the positions laid down, and particularly such as demand for their full comprehension determined analysis and fine introspection.

CHAPTER III

CONCEPTION OF UNIT

BEFORE definitely entering on the consideration of the Unit I would like to refer to a theory of Monism put before me by a distinguished philosopher. He said that all the faculties of the human mind may be reduced to one, viz. Memory. He then entered into an explanation of the cosmos, which in the last resort he declared consisted of nothing but vibrations. All was vibration. The combination of vibrations produced the world and what we know of it.[1]

Whether this be true or not, it is difficult for our minds to seize it, and it is still more difficult from that basis to develop a system of knowledge by which we can proceed to new and hitherto unknown results. The most fruitful products of human intelligence have been derived from minds in the operations of which each step has been founded on a solid basis of understanding, even though in the long line of the analysis the retrospect may give the impression of something beyond ordinary reason.

Newton's thoughts are less "lofty" in the sense of flight than close to earth always in the plainness and

[1] I have no wish to suggest that this intangible expression of Monism corresponds to the conception of all Monists. There is no lack of men of eminence in science amongst them. I cite at hazard Haeckel, Romanes, Binet, Carus, Dessoir, Morselli, and Mach, and the Biochemicals generally. Formerly DuBois-Reymond, Virchow, and Wundt might have been added, but they abandoned the position, and Haeckel scolded them soundly for doing so.

solidity of his reasoning. "The thoughts that wander through eternity" must first climb step by step; but all true thinking opens up a portion of the universe, and it is the contemplation of that result that inspires the imagination with the lofty and the wonderful.

The idea of Monism, that All is One, may be tested by the series of the Fundamental Processes already set forth. It is evident that Immediate Presentation is not the same thing as Memory. Memory is called into play when the thing perceived awakens the image of another with which it becomes linked in Association. Similarly the recognition of a Unit cannot be an effort of Memory; nor Discrimination, though it depends on Memory; nor Generalisation, with its corollaries Classification, Symbolisation. The Impulse carrying the mind along with effort of Association is also clearly not Memory, nor is it the Feeling of Effort accompanying a conscious movement. The Hedonic principle is not Memory. Time and Space cannot be called Memory.

Therefore if these represent real Processes of the mind it cannot be denied, on the ground of being included in Memory, that they are Fundamental.

But close comparison brings this much to light, viz. that they are not one of them independent and distinct. One implies the others.[1] A reference may here be made to a previous discussion of this point (cf. p. 35 *et seq.*), but it is not superfluous to repeat the statement, for the full argument regarding the Fundamental Processes is really to be found in the whole course of the exposition.

It may be well to state the position categorically at the beginning. The mind attends to one thing at a time, and only one: the Unit. I have used the phrase "attends to" in preference to any other because it is necessary to

[1] Cf. Herbert Spencer ("Principles of Psychology"): "Manifestly, associability and revivability go together."

insist on the fact of attention. The mind may be influenced by a great number of objects at one time, and this manner of influencing may modify the character of the impressions and their consequences derived from the Unit in question; but in the mind's attention at that moment there is only the Unit.

The recognition of this position must be brought about mainly by an appeal to clear introspection. But various suggestions may aid in the matter, and even if the truth of the statement be not immediately conceded, it will be gained at length by persistent analysis and by removing sources of confusion and obscure ideas.

Consider, for example, a bundle consisting of a variety of flowers. That forms a multitudinous presentation to the visual sense. There is nothing to suggest a unit in this. But if the whole be lifted up by the hand then the effort of attention associates that act, which in itself implies unity, with the total appearance. A symbol, such as the word "bouquet," enables the mind to include this complexity under one label.

Now it may be argued that the mind from the first, and even after the application of the label, recognises these things as various, and that it separates the whole into its true units, the individual flowers.

The mind paying attention to each flower which it recognises in turn beholds in it a Unit. But remark that the impression of each flower in turn is capable of yielding by analysis an interminable number of separate Presentations, to each of which in turn we could apply the term Unit; while, if we were consistent in our objection, we would refuse it to the individual flower.

On the other hand, the fact that we can indicate the first still greater complexity, the bouquet, by one symbol is proof that we can regard it as a Unit.

Even if the object were much more complex and

diverse we could still think of it as a Unit. For example, we can designate by a term which implies the conception of Unit each of the following complex things: A public meeting, a nation, an *n*th differential coefficient, the analysis required to demonstrate it, a book, the world.

We may take then as established a part of the position set forth, viz. that anything whatever formed by whatever combination held in consciousness may be regarded as a Unit.

We must now proceed to the consideration that the act of attention on the Unit excludes, for the moment of that attention, the conception of another Unit.

The preceding discussion has cleared the way even for this position, for if the added object were simultaneously included in the same conception with the first, then the Unit would really be the complex of the two; and we should not recognise that complex as being composed of two until we had successively carried our attention from one Unit to the second.

Consider again one of a pair of dumb-bells. We call it a Unit. But if we consider two knobs separated by a little distance we call them two Units. That is because we have already made the separation by proceeding from one to the other. In the same way we may consider the two knobs of the dumb-bells as two Units. Or again we may consider the pair of dumb-bells as a Unit, when comparing with another pair of dumb-bells.

Presented in this way the position may be seen to resolve almost into a truism; but that is the characteristic of any truth seen clearly in all its relations. The main difficulty of accepting the position arises from the fact that the Unit present to the attention at a certain moment is usually a thing of great complexity and that the mind in regarding it is influenced by that complexity. But this is perfectly consistent.

I open my eyes, for instance, upon the room in which I sit. I see at once what may be resolved into a vast number of separate things—the walls, floor, carpet, chairs, a fire, pictures, a cat.

But we have already seen that no complexity prevents an object being regarded as a Unit. And though the impression immediately presented may be resolved into separate units, it becomes clear on introspection that that is only in consequence of successive acts of attention, each corresponding to a Unit.

It must be remarked that the first Unit formed of the complex things is really determined in its character by the presence of these complex things. For instance, if the cat be absent from the position it occupied the Unit presented would not be the same as the actual Unit.

We discover by subsequent analysis that the Unit is really of great complexity; but even at the moment of recognition of the Unit, its character, its Effort, its Impulse, the forms of its Association, are really determined by what we may subsequently recognise as its component parts.

The Unit is not present to a *tabula rasa*, as the old notion of the Locke school would have it, but to a mind behind whose immediate activity is a vast fund of experience, of associations, the dynamical basis of Impulse, all magazined in certain particular forms, all ready to co-operate in new forms of Associations, in new Impulses, and all influenced in that activity by the objects presented, even when these objects do not actually find expression in the consciousness.

If this consideration be well taken, and if the extreme activity of the passage of attention from one Unit to another be recognised, the difficulties will disappear of establishing the position set forth. The mind attends to one thing at a time—the Unit.

But the study of the Unit is not exhausted by this discussion. A great many questions become suggested, the analysis of which eventually leads towards the same conclusions, and which throw light not only on the Unit, but also on much of what follows.

We have seen that after accepting a flower as a Unit, we separate it, by directing the attention from part to part, into a great number of other units. Each of these in turn may be again separated into a number of units.

The question may therefore be asked: To what will the ultimate analysis lead us? All these objects of mental phenomena have a chemical and physical base.[1] In the objective world the whole may be resolved into the things themselves, the elements, and the mechanical relations subsisting between their parts, considered atomically.

But the most persistent analysis and introspection will never lead us to the discovery of the so-called chemical elements, nor to the solution of the question now being attacked from many sides, as to whether these elements represent the final, or original state, of their matter.

Yet our analysis should lead us to some boundary which marks the limits of our conceptions, and the points so reached should represent elements of Immediate Presentation—Units.

Certain other questions may be put forward, which will on the one hand throw light on the nature of the problem we propose, and the solutions of which, on the other hand, will be found all consistent.

For example: We lift the bouquet with the right hand. By that act of a single movement we are helped at least in our contemplation of the bouquet as a Unit. Suppose now that we had lifted it with the left hand.

[1] The works of Ernst Mach, original and profound, are interesting in regard to this question.

We would arrive at a similar conclusion. But what is there in the nature of our consciousness that associates the movement of the right hand with that of the left so that we recognise this act as being of the same kind?

Or again we see a green leaf; then we see a red leaf. What is there in our consciousness that enables us to think in such a way that, though we are compelled to hold these things quite apart as two Units impossible of coalescence (for red is 'not' green), yet induces us to contain them under one head of classification?

We reply that both are colours. But what is colour as distinct from the concrete realisations in red and green, and the other distinct colours?

Or again, if we proceed to sound: Why can one sound be linked in association with another distinct sound? And why can a number of distinct sounds, as for instance in a spoken word, be linked together to form a Unit?

The matter is more complex when we seek the ground of Association of very dissimilar things. For example, when I move my right hand this motion is habitually associated, let us say, with a certain sound. A certain expectancy of Association arises as I begin to move my right arm. Now if I were to move my left hand similarly, would the expectancy of this Association arise?

How is it that at length we can think of, as a Unit, "A song and dance"?

We may arrive at the answer to the problems thus: Consider the colour, red.

The actual sensation involves, as we have already noted, Feeling of Effort and Impulse.

If immediately afterwards be presented a slightly different shade of red, there will be again a Feeling of Effort and Impulse. Further, a Process of Discrimination arises; and now with the two objects (the two shades of

red) presented in consciousness, we obtain the whole scope of the Fundamental Processes established.

Now we have noted that as the discrimination becomes less, the importance, the more vital character of the movements, becomes greater.

In these two colours then, while the shades of red are discriminated, the Feeling of Effort and Impulse are so indistinguishably in common that a basis is formed for the Association of the two.

We find in the whole course of the exposition that it is the fact of a limit to our faculties of Discrimination which affords a basis to our whole life of ratiocination.

In this elementary instance it enables us to form Association, thence Generalisation, thence Classification, and Symbolisation in regard to two shades of colour.

It is not necessary for this that the mind should have any clear perception of the analysis, or of the veritable underlying conditions or nature of this Association. The process is as natural and unconscious as that by which at night a black cat might be taken for a grey cat.

It must be noted that with exercise not only does the faculty of Discrimination become sharpened, but also the faculty of finding essential resemblances in things apparently remote, the faculty which in one aspect is referred to as Generalisation, in another Classification.

There does not, for example, seem at first to be much in common between a fly-wheel and a reservoir, yet their functions are analogous, as may be seen by describing each in the most general terms with regard to function, viz. that of converting intermittent supplies into continuous supply.

This remark may call attention to the continued process of our education in Discrimination and Generalisation. Hence when we refer to a mind little developed we should expect first that colours of considerable difference

are associated together by virtue of the inaccuracy of Discrimination. And also that when the difference is great the generalisation may not extend so far as to include them in one class. It is only by experience then, I take it, that the relation of colour to colour is fully appreciated.[1]

By no amount of experience, or by the exercise of Generalisation, could a colour and a sound be referred to the same class in respect to their peculiar qualities. Yet they must have something in common in order to be counted in a series as units. The Unit therefore must refer to something deeper than the external aspect of things. We find that common base in the act of attention itself, and attention is really associated indissolubly with the Feeling of Effort in receiving an impression.

The process is : Effort or attention,[2] springing from a source fundamental enough to lie beneath Discrimination, and then the full impression received, the Immediate Presentation, having its own characteristic, whether it be of a colour or of a sound that determines that particular Unit.

When we speak of an "abstraction," as we have elsewhere noted (cf. p. 44 *et seq.*), in the sense of considering

[1] It is generally supposed that women possess a more highly developed colour sense than men. In a recent number of the "University of Colorado Studies," Mr. V. A. C. Henmon shows that the experimental evidence in support of the theory is unsatisfactory and inconclusive! Experiments undertaken by Nichols showed that men were decidedly more sensitive in the recognition of red, yellow, and green, while women were more sensitive to blue. Tests on the discrimination of differences in colour show that women are in general superior to men in the discriminative sensibility, and as Mr. Henmon shows, particularly in their discriminative sensibility to blues and greens ; for he too believes that men show a decidedly greater sensibility in discriminating reds. He finds that in schoolchildren between the ages of eleven and fifteen there is no significant difference in the perception of difference in red, and that the variability and range are slightly greater in girls.

[2] This is not here intended as a definition of Attention as a Position in Psychology. A note on Attention is given at a much later stage, but here the desire is simply to make the meaning plain with regard to the Unit.

numbers as apart from the circumstances of the things numbered, it is by virtue of the Fundamental Processes here described that we are enabled so to think.

In all these cases of our active mental lives we are greatly aided by the employment of symbols, which have become indeed so habitual to us that they mask our view of the process itself. The symbol is taken as the Unit, and we carry on our thoughts without reference to all that it symbolises, until at any moment we arrest our attention more definitely and proceed to develop what the thing symbolised presents. And so it happens that anything that can be represented by a symbol can be taken as a Unit.

There is one more point to be made clear in this discussion. Although it is the case that the mind entertains but one thing at a time, yet in order that the procession of thought may be established, it is necessary that the mind should note also the passage from one Unit to another.

Here a reference may be made to the physical structure whose stimulation underlies the processes of thought. Enough has been already said by way of guarding against confusing such stimulus with the image, or idea, or thought. But by the very nature of the structure of the brain it serves excellently for illustration in making abstruse matters clearer.

The nerve fibres themselves seem to act only in the way of conducting stimuli. But when the stimulus reaches a ganglion in the brain, then what is called an "explosion" takes place. The disturbance in the ganglion seems to be the immediate antecedent to a sensation or image. It is not meant that any explosion in any ganglion is followed by a phenomenon in consciousness, for there must be continually in the brain disturbances of the kind which, though modifying thought, do not reach the

intensity necessary, or are not so situated as to "cause" ideation.

When the explosion in a ganglion takes place impulses are given out in various directions along nerve filaments to surrounding ganglia, which are disturbed according to the intensity of the stimuli, and according to the freedom of the paths along which the stimuli have passed, and according to the dominance of the stimuli in certain directions beyond others. Hence new ideas succeed the old ones.

Now graphically representing the Unit by means of a certain ganglion, the succeeding Unit would be represented by the next ganglion, the advent in consciousness of the second Unit closing the attention of the mind on the first. The pathway, which need not be single, between the ganglia would represent the relations between these two Units.

This leads us to express more clearly what was suggested in a preceding paragraph. The mind has consciousness of a Unit, and also of the nature of the passage in thought from that Unit to the succeeding Unit, and the complete comprehension of that Unit closing the process with regard to the first and preparing the way for the passage to the next succeeding.[1]

The underlying mechanism must be thought of as a multitudinous, highly complex, and well-charged system of cells linked in an elaborate way, sensitive to impressions, and requiring strong inhibitory forces in the mechanism itself at every passage to prevent explosions. A stimulus not only causes explosion but produces

[1] It must be again insisted upon that the mind may be likened, not to a *tabula rasa* at any time, but always to a highly complex system of activities held in control. At any moment an Immediate Presentation is determined not only by the objective stimulus but by many conditions which may not be made to the consciousness. The Presentation immediately antecedent to a certain Presentation is one of the factors that determine the character of that Presentation.

something like a struggle for dominance of the currents along various pathways.[1]

Those that fail to produce conscious impressions yet are not lost. Such a condition will be immediately understood by those accustomed to dynamical problems, where forces are spoken of as acting to their full extent, even though the result be equilibrium or a motion contrary to the direction of any force considered. This must be always kept in view in considering any Unit at any time. The Unit must be considered as the resultant of a vast system of profound forces, representing potential energy always on the point of becoming kinetic energy, yet in the main held in equilibrium.

When these forces by added stimulus burst beyond control the conscious thought arises, rippling above the sea of the forces that have produced it. The Unit does not remain independent and detached, but is modified by all that has preceded it, while in turn it must be considered with the whole system of these factors in determining the next.

The movement of attention at the Presentation of a new mental impression implies the cessation or modification of other activities, a suspension of certain predetermined acts both of mind and body, so much so at times that a slight sensation may be the cause of considerable efforts of readjustment. Increased physical activity in any part implies in general a demand for an increased blood supply to that part, and consequently a diminution in other parts. Hence in the movement of attention to any one sense, other senses are indirectly involved; and these expenditures and modifications of energy are them-

[1] There is found here something like a struggle for existence of ideas, with a survival, if not of the "fittest," at least of the most congenial to the environment and most favoured by the actual circumstances. All this, moreover, is consistent with a phrase of Goethe: "Dicht bei einander wohnen die Gedanken" (Closely dwell the thoughts, one with another).

selves so obscurely presented in consciousness, and yet involve not only, more or less remotely, the five senses, but other less evident sources of sensory impressions; that herein we find, at the limit of Discrimination, the base of Generalisation of which the immediate object of attention is the Unit.

At any time the impression in consciousness of an active dynamic system in one part of the brain, and referring to one set of ideas, may be obliterated by explosions in another part of the brain arising from a powerful stimulus of a kind unconnected with that previously accounted for.

The foregoing arguments are the substance of what I am able here to offer in order to lead the mind of the reader to the conclusion stated.[1] But I do not know that I should be discouraged by a failure to attain this position. It is contrary to the pre-judgment of many, and these pre-judgments, even in the minds of those most ostensibly candid, are often insuperable.[2] The conclusion was not one to which I expected to be led when I first looked into the question, and it was only after many hesitations and deep obscurities that its determination seemed to be revealed as consistent with reason and as inevitable.

To throw light upon it from another source I will quote from the celebrated work of Weber, "Tastsinn und Gemeingefühl." I do not wish to cite Weber for his authority, however great; for the whole character of this book is to seek new truths through an original course of thought, and moreover there are passages in Weber's book where I consider his analysis both incomplete and faulty.

[1] Hamilton thought that the mind could not entertain more than *six* presentations simultaneously.

[2] The matter will be again referred to in the consideration of Counting in its diverse aspects.

Weber's style often suggests Locke's in its clearness, circumspection, and candour, but Weber had a great advantage over Locke in the discipline of his scientific training, his deep general knowledge, his bent towards quantitative expressions, and his acquaintance with the field of this research.

Weber,[1] moreover, seems to have hesitated about accepting the conception of the Unit, as here expressed, until moved to the position by the force of the reasons advanced.

On p. 13 (in vol. 149 of Ostwald's Klassiker) Weber says:

"I am able, while I count the ticking of a clock, also to see the form of a flame, and feel the form of an object which I hold in the hand, and it seems accordingly that one can at the same time entertain various sensations. Such an experiment, however, is not sufficient to *prove* that; for we may consider that in the intervals between the tickings the attention was directed to the flame, and then to the form of the tactile object, and that this changing of the attention may be so quick and so often repeated, that we receive the impression that, simultaneously and without interruption, the three sensations have been present in consciousness.

"How little time is necessary to receive impressions may be observed in the case of correctors to the Press, who read a sheet with considerable speed, and yet must look at each letter precisely in order to notice errors.

"In the case of vision I can show that the part of the retina with which we see clearly has the dimension of $\frac{1}{3}$ of a line only. We must therefore move the eye from one part to another, so that every part of a large object may make its impression on this small sensitive spot of the retina. Nevertheless we believe that we take in at one view objects which could not possibly make their impression simultaneously on that spot of the retina.

What we complete in successive efforts we believe we have accomplished at once; but the more precise observations of Bessel[1] seem to show, on the contrary, that it is impossible to entertain quite simultaneously a visual and an aural sensation.

"In observations with the transit instrument it falls to the astronomer's duty to estimate twice the distance of a star from the cross-thread which the star passes, and to determine how far the star was from the thread at the first pendulum beat of the clock before the star reached the thread, and how far on the other side of the thread at the second beat.

"It is brought to evidence that the observations of the most practised observers differ not inconsiderably, because, as Bessel maintained, one first hears the pendulum beat and then sees the distance; another, on the contrary, first notices the distance of the star from the thread and then hears the beat.

"Bessel's assumption seems to me to be confirmed by the following observation which I have made and which appears to prove that one is not able to represent in their time-relations two aural sensations of which one affects the right ear and the other the left ear. When I hold two watches, which do not tick quite at the same rate, near one ear so that the ticking is heard by this ear and not by the other, then I am able to distinguish the periods when both tickings agree from the periods when the tickings of one fall in the interval between the tickings of the other, and I can make out a rhythm which becomes repeated. But if I hold one watch near to one ear, and the other near the other, I can observe certainly that one is ticking faster than the other, but I am not able to make out the repeated rhythm, and the tickings of the watches produce an impression quite different from that of the first case. On the same grounds one is prevented from hearing the beat of the heart and feeling the pulse at the same moment."

[1] Bessel, "Astronomische Beobachtungen," VIII. Abteilung (Königsberg, 1823), Introduction.

This passage from Weber is extraordinarily interesting on many grounds. It exhibits, for one thing, how objective science may be called in aid to deal with a question of psychology.[1] It also makes evident Weber's scrupulous care, and reluctance to form a decided opinion except on incontrovertible evidence. As a matter of fact I do not think this evidence in itself quite conclusive, but it reinforces all that has been said.

In a subsequent chapter Weber gives some interesting experiments which he undertook to find how many letters he could see at one moment with fixed eye when looking at a page of ordinary print, directing his gaze precisely at one letter. He thought that he could recognise three or at most four letters. This, however, is not inconsistent with what he has affirmed about the impossibility of entertaining two sensations simultaneously. One may, for example, see the whole dome of the sky and all the stars as a Unit. Subsequently, and while not actually gazing at the stars, certain of them may be recalled and counted. So with the letters of a page. One may see, though not altogether clearly, the whole page as a Unit, but to ascertain how many letters one may recognise at a moment requires subsequent mental operations.

There is another chapter of Weber (cf. p. 10 of Ostwald's Klassiker, 149) which, though not ostensibly dealing with this question, yet throws light upon it:

"We can bring two portions of the skin in contact with one another and thereby effect that one produces upon the other an impression by virtue of pressure, heat or cold. This is not possible with the other organs of

[1] The observations referred to with the transit instrument also marked the first step in the history of Experimental Psychology. The Astronomer-Royal, Maskelyne, in the year 1795, noticed that the observations of his assistant differed from his own, and he dismissed the assistant. Subsequent examination revealed the cause of the discrepancy in what is now known as the "personal equation."

sense. Thus we cannot, for instance, see into one eye with the other.

"The question now arises whether both impressions which affect simultaneously the organs in contact coalesce in one sensation in consciousness; or whether in that case we ourselves can determine, by domination and intentional direction of our attention, which of the impressions shall appear in consciousness, or what other circumstances bring about the result, that one or the other impression comes into consciousness.

"The experiments which I made proved that the impressions did not coalesce into one sensation. For example, if we bring a cold limb into contact with a warm limb, we do not feel a middle temperature, but in many circumstances cold, in others heat, and sometimes, alternately, cold and heat. When the sensations of hot and cold change quickly, we arrive at the representation that hot and cold objects lie near each other or behind each other, but we are not able to image the sensations hot and cold to coalescing into one, somewhat as we represent a high note and a low note, while we consider their relation to a third."

Here again we have clearly shown the impossibility in Weber's mind of entertaining at the same instant two sensations even of a certain similarity. It may be remarked that the two notes of music may coalesce into one, but in that case the sensation is only one. This is more clearly observable in the case of vision. If the stimuli which correspond to the colours, red, orange, yellow, green, blue, indigo, violet, be combined in a certain way we have not a simultaneous presentation of the respective sensations. We have simply a sensation of white.

Another quotation from Weber, containing a passage wherein I do not find myself in accord with him, will make the whole position clearer. It occurs in the course of observations leading to the first of the quotations already given:

"In order that the representation (Vorstellung) of a sensation should become determined the attention must be directed upon the sensation to be represented, whereas the sensation alone also becomes determined when we direct our attention as earnestly as possible upon another object. Sensations which we have represented in the categories of Space, Time, and Number are more easily kept in Memory, while pure sensations, which have not been so represented, make no durable impression, and do not easily form associations. Every one knows by experience that many objects fall into his eye, while he sees only the few upon which he has directed his attention, and that, while he is absorbed by some occupation, many sounds come to his ear without his hearing them."

He goes on to say that he believes that even these impressions modify consciousness. All that has been discussed is in accord with that last opinion. It is in the opening remarks of Weber that I find a lingering false influence of Kant, from whom Weber, however, marks a decisive break. Weber speaks of the categories of Space, Time, and Number. But what has preceded in this work, and what will be shown in the chapter on Counting, will make clear that the conception of Number may be analysed, and that it may be shown to consist of a complexus, including Time, by the aid of the Fundamental Process of Association, and by those Processes which we have already discussed in dealing with Abstraction.

In regard to the representation of a sensation of which Weber speaks, it should be remarked that there is no word in English which precisely translates *Vorstellung*. It means not merely being aware of something, but perceiving it in its relations. It is a combination derived from Immediate Presentation and Association, where perhaps the scope of associations is wider than we have

understood in regard to Association as a Fundamental Process, although capable of being covered by the repeated application of our Processes.

Now if, for the purpose of exposition, we keep in view the image of the mind as a system of active forces held, for the most part, in equilibrium—though that equilibrium be of moving parts, as in machinery—we will understand that everything that impinges on the senses, or indeed on the constitution of the body in any respect, and which consequently affects the physical apparatus correlated to consciousness, must have influence on the outcome of the Fundamental Processes.

We say, in the physical world, that the throwing a stone alters the centre of gravity of the earth. That is true, though the effect be inappreciable. When a disturbance reaches a certain degree of intensity the effect may be observable. And in like manner we can understand that the forces that play on our mental lives, and which really modify the results in consequence, may be so slight as to be virtually negligible or so intense as to dominate the whole physical activity.

At any instant that which is in consciousness is a Unit. It is not here meant, however, that the Unit is coincident with a sensation or that it is necessarily composed of factors from one sense. It is really determined by a vast complex of factors. I open my eyes and my ears to an aspect of the landscape with its multitudinous intimations of land, and sea, and sky. That instantaneous aspect is a Unit.

I direct my attention more particularly to a feature of that landscape, a bright flower. But in that movement of attention there has been a change of Unit, a change involving a change, for example, of Association, of Impulse. The attention may now be "concentrated" on some one minute thing—as a coloured spot—in the

flower. That implies a new Unit, with appropriate change in Effort, Impulse, Association.

Now reverting to the passage of Weber where he speaks of *Vorstellung*, it will be understood that something which, regarded objectively, may be present in the field of vision, and which, moreover, may possibly be recovered in Memory as a Unit, may in the original Presentation have been but a component of the Unit. The effects of the Process of Association depend on the intensity of the Presentation, and there are degrees of the strength of Association. But now if the attention be diverted to a sensation, so as to obtain a *Vorstellung*, this means an intention of widening and strengthening the effects of Association, and of directing the new Impulse in particular Association with that sensation.[1]

We can make the sensation the object even of all the complicated operations of an extended argument; and we can thus give it greater durability in Memory, but in doing so we depart from a simple expression of Fundamental Processes, though the whole of the operation may at length, by analysis, be expressed in terms of these. And this is true, within a smaller scope, in regard to *Vorstellung*.

It may happen that the attention is being directed to a mere Presentation, and that the Feeling of Effort and Impulse arise in Association with a new field of sensations and the like, by reason of some involuntary, violent stimulus, as for instance by a sudden blaze of light, a thrilling screech, or by something of dominating importance appearing in the operations of our own bodies which had not hitherto been manifest in consciousness.

[1] It is not implied that this is done consciously, but no difficulty is thus presented. We desire to walk, and we walk; and we thereby direct, although unconsciously, a complex muscular system with the nature of which no one, except possibly an anatomist, is acquainted.

The word "attention" therefore must be here employed with a meaning freer than seems usual.[1]

COUNTING

The discussion of the Unit has prepared the way for understanding the nature of Counting.

We are so familiar with the process of counting, in the ordinary conduct of our lives, that our clear view is apt to be confused by familiar methods which may seem to be necessarily associated with it.

A few questions that we meet with in the analysis of the position may therefore be set forth in order to disturb the mind from too facile acceptance of familiar things:

1. Is there any meaning in numbers apart from a series of concrete things?

[1] If the above exposition be valid, then we shall find, as always happens upon the statement of a new and true principle, that the results reach far beyond the direct intention. For example, if the principles of the system of Copernicus were correct, he established not merely that the earth moved round the sun, but by the same breath he blew away the vast systems founded on false bases, and opened the way to new research.

If the analysis here exhibited be rigorous, then by applying a touchstone to Kant's conclusions of the coalescence of Pure Reason and Will, we shatter his whole system, and all of those that were derived as corollaries from it. If the account here given of *Vorstellung*, and that which is subsequently given of Will, consistent with our Fundamental Processes, be true, then the whole system of Schopenhauer, "Die Welt als Wille und Vorstellung," must go by the board, in as far as it professes to be founded on an ultimate analysis, and be not merely a series of opinions, suggestions, aphorisms, criticisms, which may or may not be justified, though exhibited in a glowing, striking, and often captivating style.

And further the inefficacy of the whole school of the Doctrine of Association, or of the Chain of Association, is shown.

I say, if the exposition here offered be valid. The answer must be not to take refuge in the illusory strongholds of great authorities—always the mark of inferiority of intellect—but to show definitely in what vital particular this analysis fails in what it professes to accomplish.

Let us strike all principles as with a Thor's hammer; let us probe into all positions as with Ithuriel's spear. If they be true, they gain in strength; if they be false, they are shattered; if they be in part faulty, then from that discovery arise new and luminous pathways.

2. In what way are we enabled to number unlike things?

3. If the answer to this question be the reference to the ordinary process of counting involving unlike things, might it not be remarked that these things are symbolised, that the symbols are like, that the first process has been the establishment of the series of the symbols?

4. Is there in things, as dissimilar as we please, a common element which enables us to remember them in the same series?

5. Is it possible to count without the use of symbols?

6. What would be the operation of counting if we had no language?

7. Could there be numbers if nothing occurred twice in experience; that is, if the march of phenomena was continuous but not repeating?

8. Do animals count?

9. If not, in what factors forming the operation are they deficient?

10. What are the faculties necessarily involved in counting?

We see an object, say a sheep. The operation of the mind in proceeding from one sheep to another is distinct; and if we consider the two sheep as similar so that we take no note of accidental circumstances, that is to say, if we proceed from a Unit to a like Unit, the processes involved are fundamental.

Counting of like things to the number two is possible without the use of symbols.

Also, it appears that in this simplest form of counting we count not symbols, but things such as in our more developed arithmetic the symbols represent.

If now, without counting, we saw a succession of sheep pass, say about 100, and then a second series of about 100. It will be agreed that without counting

by symbols it would be impossible to say that the second series was equivalent in point of numbers, or repetitions of the objects, to the first. At what point then could the unaided faculty determine an equivalence of such repetitions? Would it be ten, or five, or three?

If we have three simple objects such as three balls placed side by side, thus . . . , we say at once that they are three. But even in this case we are liable to be deceived by appearances. The use of symbols has become so much like second nature, and symbols may be formed by Association in so many ways, that we do not immediately recognise that in saying three, we may have performed no operation of counting at all, having simply taken the symbol . . . as familiarly and easily as if we had been told that the number of sheep was represented by the symbol 3, which obviously does not imply that in the instance before us we have counted the objects.

A bank clerk counting a great number of coins may count them by fives, but this again is by the use of symbols. He arranges them thus ⁚⁝, or thus ·.·.·, or even thus . . . . ., and each one of these symbols is almost as familiar to him as V, or 5. Hence this process of counting is analogous to that of counting little packets labelled 5.

Even when the clerk does not arrange the coins at all, it does not follow that he works without the aid of symbols. He might effect this in various ways. For example, if five sovereigns be weighed on good scales, the weight will be found so nearly equivalent to that of another series of five sovereigns that the possibility of mistake for four or for six sovereigns may be practically excluded.

Considerable experience in simple weighing by hand would give delicacy of perception sufficient to exclude

errors. The number of a pile of sovereigns might possibly be estimated in this way; but each five would really be determined by a symbol, which in this case would consist in the character of the muscular effort required.

Or again the length occupied by five sovereigns laid side by side is so constant as to exclude errors so gross as of four or six sovereigns occupying an equivalent length. With practice the eye could estimate this length with sufficient accuracy as to avoid errors. If counting were performed by such an aid, then the symbol of five would be the notation of the length corresponding.

Or again the estimation of the fives might be due to an addition of three and two so rapid and automatic as to escape the attention of the operator himself if he were unaccustomed to mental analysis. In this case again we would not have an instance of counting fives.

An ordinary person would find difficulty in estimating by fives in any case. An uneducated man finds a great difficulty in estimating by threes.

It seems to me that the pure faculty of recognising successions of units, apart from some form of symbol, does not carry beyond the recognition of two. In cases where we may be inclined to think otherwise, we must be careful to exclude such delicate estimations of effort as would imply a symbol analogous to that of weight, or of associations analogous to that of our ordinary use of symbols.

It is well to consider the use of symbols in an unfamiliar manner.

Suppose that we had a number of cards placed along a path, and that we took a corner out of each card, and kept these corners in a heap. Now suppose that a

[1] Cf. B. Bourdon, "Sur le temps nécessaire pour mesurer les nombres," *Rev. Phil.* 1908.

number of dogs walked along the path, and that each time a dog passed a card we took a corner from the heap and placed it aside. Now, at the end of the operation we may have forgotten individual cases of a dog having passed a card, but we know that there were as many such passages as corners of cards.

Further, if the operation were repeated we should know that at the end there were as many passages as on the first occasion. Now, further, if these corners were taken up always in the same order, and had each a characteristic shape, then the last corner taken up would symbolise the corresponding intermediate stage, and serve to indicate the exact repetition with regard to number of the process.

Or again, suppose we became acquainted with a number of posts in series, the first painted red, the second orange, the third yellow, and so on to a violet post. But suppose that we had not counted those. Suppose also we had a horse tied to each of these posts. Now, still without counting we could observe that the last horse corresponded to the violet post, and we could at any time reproduce the series, without mistake, simply by attaching a horse to each post in turn until we arrived at the violet post. The violet post would then symbolise the full complement of horses.

If the series of the posts became so familiar that we could easily present them in recollection, then if we saw a number of horses walking by, we could, by referring each to a post, determine whether there were more or less than the usual number as symbolised by the violet post.

And what we say of the violet post is true of any of the other posts. And so here we should have a form not of counting, as ordinarily understood, but yet of estimation of numbers.

Similarly by making use of the sense of touch alone, in connection with any series we wished to determine, we could ascertain similarity and differences with respect to any new series presented. It would be necessary that the series of touches should have distinct characteristics, and that they should be capable of reproduction always in the same order, and also of association with each Unit of the series in question.

Another form of estimation is when in listening to music, as for example of a marching tune, a step is taken at each beat of the music. Now all recollection of the peculiar character of each step may have passed away from the mind, and the recollection also of the characteristic of each beat of the tune; yet the whole series may be produced without fault from the beginning, each bar of the music forming associations awakening recollection of the following, and the last beat would symbolise the number of steps.

It is only necessary to observe that the sequence of the bars is correct, and that a step takes place at each beat; then at the end we have the certitude that the steps have been reproduced, although in the meantime all intuitive sense may been lost of the early elements of the series.

The reference to a musical form of numerical estimation gains interest also from this peculiarity that the passage from one Unit to the succeeding has a definitely marked and reproducible characteristic; whereas in turning the attention from one uncoloured post to another there may be nothing to seize the attention or facilitate recollection.

Other forms of estimation may be referred to, as for instance that of a parliamentary orator who might divide his speech into parts and look at one window, say, while dealing with one part, and a cornice while dealing with

another, and at the Speaker's chair for the peroration. These objects being always in the same place, no effort would be required in establishing the series.

A noted medical man interested in history thinks of the centuries as associated with the rungs of a ladder which he climbs, each rung having some peculiarity.

Counting on the fingers is a very good way for those who are not sure of the numbers otherwise. Most of us, however, have been taught to use numbers which can be recollected at will. Here we have an artificial series, and one offering no special features for associations, but by constant repetition the series has become familiar and facile, and so it is generally more convenient than those that at first might seem easier of application.

If our life could be supposed to consist of a series of like sensations, counting would be simply repetition. With two sets of sensations, one being used to count the other, counting becomes repetition with association of the successive members of the series.

When we count three eggs, for example, and three twinges of gout, and three poems, what is there in common so that we can apply the symbol three to each of these series?

That question has been answered in the main in discussing the Unit. The Feeling of Effort itself in the Immediate Presentation of a new simple impression, which may be a symbol of a complexity, is the something common to these disparate things which enables us to apply the symbol of counting. It must be noted that the complexity, poem, has already been symbolised by a Unit, the word poem.

A further illustration may make this matter still more comprehensible. If we fire at a target, the operation of firing is noted in itself. If then we fire at a tree, the similarity of the act of firing in the two instances is such

that the differences of the object do not bring in an element of confusion such as would prevent the same term being employed to denote the act.

The act of repetition of the symbols of counting becomes so facile that each symbol is associated without difficulty at the rise into consciousness of the object counted. The various characteristics that appertain to the object are not noted. We count the acts of attention, or those operations of which the base is the Feeling of Effort.

It may be remarked by way of objection that in counting coins we may have occasion to examine the coin closely to detect a possible counterfeit; but in that case the examination is something apart from the process of counting.

Thus, to revert to the previous illustration, a man may hesitate as to which target to choose, but when he has chosen his target the act of firing at one target is in itself similar to that of firing at another.

Counting essentially consists in the Presentation of a Unit, and the Association of another Unit with this, and a continuance in periods of Time, marked principally by the Feeling of Effort, of such Processes of Presentation and Association, according to some recognition of Agreement in the successive units.

Consequently the conditions under which counting is possible are those firstly under which the recognition of units is possible, and secondly the conditions under which a series of symbols may be established and linked by associations with any succession of units.

Accordingly we have necessarily implied as a basis for counting all the Fundamental Processes which we have seen already to be involved in the conception of the Unit; then we have the establishment of a series which implies a repetition of units, whether purposely or

accidently in experience, so that we have formed a strong Association between the successive numbers of the series; then we have the Association of each Unit of the counting series with the successive members of the counted series, the Association being with each only in reference to it as a Unit; the term series itself implying Agreement; the term succession implying Time; and finally all the Fundamental Processes.

To complete what is ordinarily implied in the term counting, the associations must be applicable indifferently to any series, so that comparison may be thus established between any two counted series.

Further conditions are implied, but they lie beyond the bounds of mental analysis. They are the permanency of the conditions of the external world in as far as the existence of the series is concerned; the permanency of our constitutions in regard to the response to the stimuli corresponding to the series counted; the permanency of our constitutions in regard to the reproduction of a series by virtue of stimuli arising, not now from the external world, but by the succession of the series itself.[1]

[1] I have not here entered into the discussion of the problem, Do animals count? further than to pose the question in order to indicate the necessity for recognising that merely conventional forms of counting do not sum up the whole matter. The preceding exposition, however, makes it clear that animals may be capable of a rudimentary form of counting. Hachet-Souplet remarks: "Certain hymenopteres paralyse the larvæ of other insects by stinging them, in order to place them in the nest where their young ones will be born, and the number so placed is always equivalent to the number of their young."

Some readers may object that this is not a case of counting but of "instinct," but since an effective count is made we may call it counting by instinct. In this wise our own Fundamental Processes are also instinctive, that is to say, according to our physical and mental constitution.

Levi Leonard Conant has studied the faculty of counting in animals.

Other references may be found in a symposium (*Journal of Psychology*), to which the contributors were Messrs. Wildon Carr, Lloyd Morgan, Stout, Myers, and McDougall (cf. p. 569).

CHAPTER IV

ADDITION AND SUBTRACTION

ADDITION

THE study of counting has prepared the way for the proper study of addition.

In the first place we shall find that we add external objects, units, together; and that if we can speak of adding numbers, as abstractions, this only means that we take the numbers themselves as units, and indicate the result by a symbol. Thus if we take, say, a series of two peas, and then a series of three peas (. .   . . .), we shall observe that consistently with what has already been discussed, we express each of the series as a Unit; and when we add the first series to the second series the process of addition means that we now express these as one series, and then proceed to count the units contained in that series. Thus $2 + 3 = 5$.

Here, however, by this experience we have done something more than merely count. The second series was associated with the symbols one, two, three. But now we have applied to it the symbols three, four, five; and we have assumed that the operation of counting in this case is equivalent to that of the previous case, that is to say, that we have added three units to two. Therefore we have really counted the symbols three, four, five, by means of the peas originally associated with the symbols one, two, three. If there is any difficulty in grasping this

argument the question may be put: How do we know that there are three in the series three, four, five? If we narrowly examine the manner by which we arrive at a correct answer the difficulty will disappear.

Thus by means of new associations we count the symbols themselves. It is more difficult to count the symbols directly.

It is true that in an ordinary way we do not count when we add, we simply say two and three are five; but it was necessary once to have counted in order to know what the sum would be. The formula of addition symbolises an operation and its results.

We may now examine whether anything further is implied in the formula: Three and two are five (. . . . .). In the first place we form the new series, that of three, and that of the two, each regarded as a Unit (though each containing units), since the problem is to add the first series to the second,; with this difference the first three units are counted as before. In the second series, which we have posited as consisting of two, we now count as two the symbols four and five.

There is a further step necessary in changing the order completely. Thus suppose that for convenience of reference we name the objects a, b, c, d, e. Hitherto though adding different series we have preserved the order; but suppose now that we add a, b, d to c, e. Here we call the series a, b, d three, implying that for each object there is a corresponding symbol up to three. But to effect the operation it must have been brought about that the associations giving the sequence of the symbols one, two, three have been made stronger in our minds than the associations of the original order of the objects.

The same observation is true in regard to the series c, e, which we recognise as of two. This series in its

weakened Association serves to define the new associations of four and five as being symbols of two objects.

We may convince ourselves by remarking that we are still adding three and two, and to know that the symbols four, five have been applied to a series of two, we must have counted c, e as of two.

Hitherto in adding these series we have supposed them placed before our eyes. This fact has supplied to the operation certain assistance which we have assumed as a basis. That is to say, in changing the order we have assumed that in placing d after a, b, we observe in the actual experience that we have not created a Unit, but have simply changed its place in such a manner that it is easy to omit it in the associations of sequence which it previously held when we proceed to consider c, e as the series formed of the objects that remain.

To bring the force of this remark clearly before the mind, we may take an extreme case and suppose that a thousand objects, which we had seen mingled pell-mell, were divided into two unequal series which we had to count and add from memory. It would soon be discovered that we could not be sure of having counted each object once and only once.

In order to accomplish that we would require that they should be displayed in a definite order before the eyes. The actual experience thus affords the associations necessary for carrying out the operation. And when we are able to proceed to a similar operation without such ostensible aid, it is because the associations have been repeated in our experience so often that we reproduce them in recollection.

Now when associations have been formed and frequently repeated so as to be very familiar, the associa-

tions cease to depend on sequence.[1] Thus A, B is an association always repeated in experience in that sequence, but after familiarity it will be found that in memory not only does A recall B, but that B recalls A, and that the association of A and B is stronger than that of the notion of their ordinary succession. And if we have a series established, A, B, C, D, E, quite familiarly, it becomes easy to associate any three objects with A, B, C; and then two other objects with A, B; and finally to associate D, E with A, B.

Now the symbols of counting have become so familiar that it is easy to recapitulate them without regard to the continuance of their original succession. Accordingly when in recollection we add by counting two series taken from five units by selection in any order, we shall find in general that we have in the first place counted the objects in each of the two series, and that in forming the two series, in any manner we please, we form associations with the symbols of counting; that then we count the symbols themselves; and by virtue of the associations mentioned, we know that we have thus counted the objects so symbolised. The association between symbol and object need be strong enough only to indicate that each symbol does represent an object to be counted.

In the ordinary exercise of addition we make use of associations based on the operations already considered. We say two and three are five, four and five are nine, almost as familiarly as we repeat the numbers in counting. If we have to add, say, fifteen and twenty-three, few people can give the answer in the same direct way.

In the manner of expressing fifteen, we meet with

[1] Thus in writing at great pressure, or in a condition of fatigue, it is not uncommon to displace a letter: terrible, for instance, may be written terrblie. The question will be considered later in the chapter on Memory.

a great advance in arithmetical development. We have the numbers grouped in series of tens and of numbers less than ten. This depends on a new operation of the Process Generalisation, and classification and symbolisation. A series of ten is treated as a Unit, and our enumeration tells us how many series of ten we have to deal with.

Thus in 15 we have one series of ten, and five units besides. Thus 15 is a compound way of expressing what otherwise we might express by $10 + 5$ or XV. Similarly 23 is a neat way of expressing two series of ten and three units in addition. $23 = 10 + 10 + 3 =$ XXIII.

It is not only neater but more explicative than either of the two last numerations, for it indicates at a glance, without the necessity of counting, how many series of tens are added.

To add fifteen to twenty-three we add together the numbers expressing the number of series of ten. Thus adding one and two we find three as the number of series of ten. Then we add the series of the other units; five and three are eight. So that we have three series of ten and eight units in addition; and, accordingly to the mode of expression adopted, this number is expressed as what we usually understand by 38. As expressed in familiar notion: $23 + 15 = 38$.

If we had required to add fifteen to twenty-nine we would have as before three series of ten and the sum besides of the numbers five and nine, that is to say, fourteen. But fourteen consists of one series of ten and four units besides. So that we have in all three and one (that is, to say, four) series of ten and four units besides. We express this as 44. . .

The great difficulties in the problems of counting, and accordingly in those of addition also, have now been overcome.

In our methods of enumeration we consider a series of ten series of tens as a Unit, which we call a hundred. And we express the number of hundreds taken by a symbol placed in the third position, beginning from the right and going to the left. 437 means four series of hundreds and three series of tens and seven units in addition. The clearness of this notation may be compared with that of the Romans, CCCCXXXVII.

These differences have proved, in the history of the world, to be of great practical importance.[1]

The Roman system of notation soon becomes unwieldy. In order, for example, that in this simple instance it should convey the same information as the notation 437 we must first count the number of C's, then we must count the number of X's, then to V we must add the result of counting the two extra units to obtain seven.

Now, in view of what has already been discussed regarding the Unit as the momentary fact of the mind, and also regarding the difficulty of remembering series which have not been associated in the mind by constant repetition, we shall readily conclude that the Roman method is not only more cumbrous in representation, but that it involves a far greater strain on the mind in each successive operation.

Moreover its development is not in accord in the best way with the principle that has suggested this notation. Thus in XV we have a single symbol representing five units, and we have a single symbol representing a series of ten. But in XXXV we have each of the series of ten expressed. That which was here lacking, in view of the establishment of a good system of notation, was the recognition that the series of ten might be considered

[1] C. F. Hill, who has written learnedly "On the Early Use of Arabic Numerals in Europe," finds the first examples in MSS. of the tenth century; Arabic numerals came into general use at the beginning of the thirteenth century.

a Unit, and that the numbers of such units might be directly expressed by a single symbol.

The Romans had, however, allowed an element of complexity to remain in that they did not use symbols sufficiently even for the numbers up to five; 3 is a better symbol than III, for it is taken as a whole.

Acting on this principle we might now express the number of tens thus: ^{3}XV. If now we compare this again with ^{2}XV and ^{4}XV, we see that by the operation of symbolification the numbers 2, 3, and 4 would express the numbers of the series of tens, even though the symbol X were faint or absent; thus: ^{2}V, ^{3}V, ^{4}V. So that even by this means we might have been conducted to that Arabic notation 25, 35, 45, which in its simple beauty seems like an inspiration of genius.

The development of arithmetic has been made possible by an appropriate choice of notation. The development of algebra has also depended on this principle, that wherever a complex has to be frequently considered in an operation, it should be expressed by a simple symbol.[1]

[1] The forms of the Differential Calculus are but extensions of algebra. Accordingly we should expect to find that the highest developments of mathematics would be found amongst the peoples who had employed the simplest and most scientific forms in their elementary expressions. History shows to us how profound and far-reaching the importance of such matters may be. The development of our mathematics made possible modern developments of the science of mechanics. Amongst the thousand products of this science we may cite the discovery of astronomy, and particularly that of the law of Universal Gravitation—discoveries whose consequences have determined the trend of our civilisation.

So true is it, that one might say that given two nations of equal material power at a certain moment, developing their civilisation apart from each other, the nation that had adopted the best form of mathematical notation would, *ceteris paribus*, inevitably dominate and lead the other.

The study of addition leads us also in another way to questions of great importance in the development of the higher mathematics. It is by a minute examination of such problems as have been here presented that we may be best led to the understanding of the fundamental principles of Vector Analysis, which is at the base of the system of Co-ordinates of Descartes, of Hamilton's "Quaternions," Grassmann's "Ausdehnungslehre," Plücker's generalisations of Descartes' principles, and thence through Klein and Sophus Lie to a magnificent modern development of the powers of mathematical analysis.

We have observed that in addition $2 + 3 = 5 = 3 + 2$. That is to say, the order of the number is indifferent. Now the operation would be similar with any two other numbers. Therefore by Generalisation we say, let a and b be symbols of any two numbers; thus $a + b = b + a$.

This property is expressed technically by saying that in addition the numbers are commutative.

If now we add a third number whose symbol is c, we have $a + b + c = c + b + a$. It is also true that $a + b + c = (a + b) + c$, when in the second case a and b are added together and c is added to the result, for this is the way in which the addition $a + b + c$ has been effected. It is also true that $a + b + c = a + (b + c)$, where b and c are added together and the result added to a. This may be demonstrated by reference to the exposition of addition as based upon counting.

Accordingly we establish that in addition we may group together any numbers of the series and add the result of their addition to the remaining numbers.

This property is expressed by saying that in addition the numbers are associative.

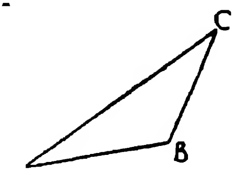
If then we had to deal with other objects, say geometrical qualities, in such a way that the processes were both commutative and associative, it would be suggested to employ the symbol of addition. It would be necessary to observe that nothing further was to be assumed in the use of this symbol, beyond the commutative and associative properties.

Now a vector is defined as a straight line having a certain length and a certain direction.

Direction must in this instance be distinguished from situation. Thus if the vector be moved parallel to itself into any other situation it is not considered to have changed in its vectorial properties.

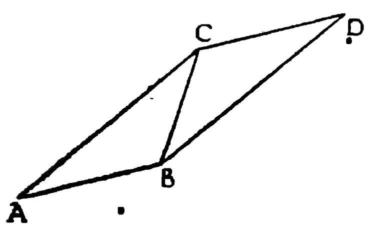
If to a vector A B we apply a vector B C, so that the

beginning of B C coincides with the end of A B, then in the mathematics of vectors we say A B + B C = A C.

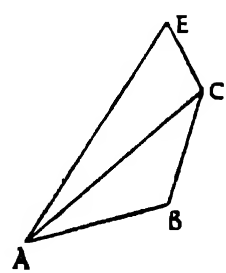


Here it is evident that the operation symbolised by + is not similar to that which we have already considered as addition, but we will be consistent in its use if we observe that it is commutative and associative, and that it is not used otherwise than in accordance with these properties.

It is commutative, as may be thus shown. Draw C D equal in length and parallel to A B. Join the points B D. Now we know from geometry that B D is equal in length to A C, and parallel to it. Therefore by our definition of the vector we have B D = A C; and also C D = A B. And by our assumption regarding the symbol + we have B C + C D = B D, therefore B C + A B = B D. Therefore B C + A B = A C. Therefore B C + A B = A B + B C.



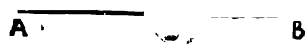
If now we add another vector C E, it is evident that A B + B C + C E = A E; and that (A B + B C) + C E = A E; and that A B + (B C + C E) = A E. It is easy to extend the demonstration to less simple cases.



We have here a certain form of calculus which, at the basis of the science of vector analysis, will be found greatly to simplify many geometrical problems, into which, however, we do not now enter.

We may now discuss a problem which arose before the development of vector analysis, and of which the terms appear to be simpler.

Consider a line A B,

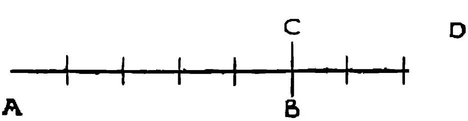


then another C D,

of which the direction is the same. For simplicity we may take the lines juxtaposed, so that the point C coincides with B.

Then we say A B + C D = A D. But although this process has become familiar to us, it really involves something beyond the forms of addition first discussed; for in that discussion we dealt only with the addition of series of units which resulted in a series of units. Here we have the addition of two spaces, or lengths, which results in a length.

Suppose, however, that A B were divided into a number of equal lengths, five, and that C D were divided into a number of lengths, three, each equal to one of the divisions of A B.



Then we could consider each of these lengths as a unit. A B would be taken as a series of five units, and C D as a series of three units; A D would be

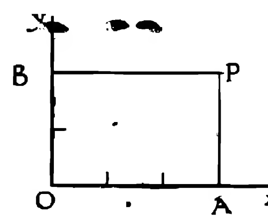
then a series of eight units. The principle could be extended to lines containing any number of units, if only the units were common measures of both lines. The principle would also apply to any number of lines in the same direction.

The suggestion would arise accordingly to apply the symbol + as in the simple case above.

Considerations of this kind led Descartes to establish the system which gave an impulse to geometry which was productive of enormous development.

Descartes established his system of co-ordinates to define the position of a point in space. Let us consider a simple case of two co-ordinates, O x and O y at right angles.

Measure off on O x three units to O A. Erect a perpendicular to O x at A; measure off along O y two units to O B.



At B erect a perpendicular to O y, cutting the perpendicular to O x at the point p. O A is called the abscissa of p; O B the ordinate of p. Thus when we have given that the abscissa is 3, and the ordinate 2, we define in this system the position of the point p.

The ancient Greeks, who were great geometers, within the scope of their own conventions, had not taken the step of dividing the straight line into units of length to serve as measures. This fact alone proves that, although the notion is familiar to us, it required persistent and clear-sighted analysis on the part of Descartes to arrive at the principle involved, and to apprehend its importance.

In the foregoing discussion, which has been offered to indicate the results that may flow from investigations of this sort, certain difficulties have been disregarded. For

example, how do we know that we can find a Unit that will exactly measure two straight lines, so that they may be defined by the numbers which symbolise the repetitions of the Unit? It may be said that we need only take the Unit small enough; but this is an assumption. In mathematics we meet with incommensurable quantities, and with lines that cannot be measured by parts, however small, of other lines. Certain questions of spatial relations associated with such problems will be dealt with later.

The operations involved in addition may be summarised as including firstly all those involved in counting; then the operation of uniting one series with another so that they may be regarded as one series for the purpose of ascertaining by counting the number of objects in the series so regarded; the expression of such a result by symbols; the Generalisation of this process; the Association in Memory of the symbols of various series, taken in turn, with the symbols expressing the results of such combinations of the series; the recognition, and all that this involves, that the operation is "commutative," that is to say, that the order of the series is indifferent; the recognition that the operation is "associative"; operations involved in the classification of series in a selected system of notation; operations involved in the choice of methods, that is to say, in adding series of units to series of units, series of tens to series of tens, and so on; adaptation to the system of notation, as, for example, in "carrying over"; repetition producing familiarity so that all of the operations themselves become so familiarised by practice that they are carried on step by step without immediate reference to their meaning.[1]

[1] American psychologists have of recent years studied addition, but from other standpoints, and mainly in regard to failures, or speed tests, in the usual forms. L. D. Arnett and C. E. Brown have written interestingly in this regard on counting and addition in the *American Journal of Psychology*.

SUBTRACTION

Subtraction is the "inverse" of addition; that is to say, if we have $2 + 3 = 5$, then if we consider first the series 5, and inquire what this series becomes if no account be taken of the series 2, we have the series 3. This is symbolised in the formula $5 - 2 = 3$.

Let us consider what is meant by taking no account of the series 2 in this process.

In the Immediate Presentation of what is externally an object there is found to be always associated a Feeling of Effort and an Impulse. The Feeling of Effort determines the object as a Unit, the Presentation corresponding determines its particular character, and the Impulse carries the mind to an Association with an object which succeeds it in attention.

Accordingly we remark that in every succession of the kind the disappearance of the object from consciousness is as vital as its Presentation, although it is not so striking a phenomenon. It might possibly have been included in the series of Fundamental Processes, but that seemed an unnecessary refinement, for it is included in the proper conception of the Unit, Association, and in Negation, as applied to these.

We note also that in our active mental life Association may produce a condition of expectancy in which the Feeling of Effort becomes manifest without the object expected. For example, if one believed that it was midnight, and listened to a clock solemnly striking at regular intervals, then after eleven had been counted the next stroke would be expected at the estimated time, and if it did not happen there would be felt a distinct shock of surprise.

Now in applying a symbol as in counting there is also a Feeling of Effort involved. Though not essentially the

case, this is often associated with noticeable physical movements. We may observe that when school-children count the whole body is sometimes moved in unison. There is a suggestion of positing or casting the symbol as if it were a physical object. If then a symbol were used in a strongly associated series, and the object were not produced in experience, then there would be a shock of surprise.

In such way the mind becomes familiar with the Disassociation of a symbol and its object.

In our active life what is called the Will comes into play. We perform continually acts of addition with a certain purpose, and we continually perform acts which are similar in character, though represented by different objects, with that of positing a symbol upon its Unit.

Thus the act of taking hold of an object is associated with that of dropping it. The act of seeking an object is associated with that of avoiding an object.

Accordingly we shall see that there is nothing in the nature of things why subtraction should not be performed independently of addition. We have a series five, for example. In counting the objects of the series five we take note of the series two, and we may here end our operation. But in so completing the count we do not necessarily note the remainder of the series as three. But now in order to know how many we have "subtracted" we make a count of the series left, and we find the number three.

Thus we need not have performed the process of addition, two and three are five. In this case the process of subtracting two from five is that of counting two, and then reversing the ordinary course of Association, and renewing the count on the remaining units, which we ascertain to be three.

But in all our experience the rise of an object, simple or complex, in consciousness is more striking than its passing. And in regard to these processes the fact that addition is looked on as the positive process, and subtraction as the inverse, proves that addition is in experience the more important operation.

Subtraction, as the inverse of addition, would be presented thus: We have become familiar with the operation of determining: two and three are five. Two is considered as a whole, a Unit covering two objects; similarly with three; and with five. Moreover, with familiarity of these processes we not only neglect the observation of all that they imply, but we also associate them with various operations of a more graphic character, but which apart from certain differences contain something essentially similar.

Accordingly there would be associations of such addition with acts, such as placing a little stick in continuation with a longer stick, and observing the combined length of the two sticks; or again adding a small weight to a greater weight, and estimating the combined weight. Hence addition has associations which make it comparable to positing, or placing, two things together.

Now we have already seen in the discussion of counting that when objects are familiarly associated together, that association is stronger than the association of sequence.

Accordingly after placing two things together to make a whole we are prepared to comprehend the process of taking one of them from that whole and leaving the other.

It seems therefore probable that subtraction has arisen from addition by conscious observation, and by the exercise of will in seeking the "inverse" of addition.

MULTIPLICATION

Multiplication is founded on addition, therefore eventually on counting. The process of multiplying three by two is that of taking a series of three; then another series of three, so that we have two series of three; and then adding these together. The process is symbolised thus: $3 \times 2 = 6$.

In this simple instance it is equivalent to $3 \times 2 = 3 + 3 = 6$.

We might, in order to save the trouble of counting in expressing the number of times the series is taken, symbolise the process thus: $3_{(1)} + 3_{(2)} = 6$.

Pursuing the line of discussion regarding notation, we observe the great importance of simplicity in symbolisation, simplicity not only in the manner, but also in the number of objects presented. In this way it is deemed inadvisable to distract the attention by anything beyond the essential denotation.

Thus instead of $3_{(1)} + 3_{(2)}$, we might be tempted to express the process thus, 32, where the first symbol represents the number in the series to be multipled and the second represents the number of repetitions of that series.

But the expression 32 already means something different. To avoid confusion we separate the 3 and the 2 by a symbol thus: 3×2.

In algebra, where numbers are symbolised by letters as a and b, we really do adopt the form, ab, for multiplication. No confusion here arises, for as the proposition concerning a and b is generally taken to apply to any numbers whatever, it is not necessary to divide them into a series of units, tens, and hundreds.

If now we multiply larger numbers, say 43 by 25, we introduce no new Fundamental Process, but we

combine in new forms some of the principles of counting, addition, and notation, which we already have discerned.

In multiplying 43 by 25, we multiply 43 first by 5; that is to say, we obtain the result of the addition of the series of 43 repeated five times. Then we multiply 43 by 20; that is to say, we obtain the result of the addition of 43 repeated 20 times. We have now the sum of 5 times 43, and 20 times 43. Now taking the series of 43 as a Unit, we have this 5 of these units and 20 of these units, and we know from previous investigations that the addition gives us 25 of these units; but (calling the units which form the series 43 the original units) if we consider all the units represented by taking the sum of 5 series of the original units 43, and add these to the units represented by taking the sum of 20 series of the original units 43; then it is not immediately evident that the total sum will be the same as if we added all the original units 43 together 25 times.

The determination of this matter, in a general way, brings into view the "distributive" character of multiplication. Let us begin with a simple instance. We require to multiply by 3 the sum of 5 and 2. Translating this problem in the most direct way into the language of symbols, we have $(5 + 2) \times 3$.

We may at once take the sum $5 + 2 = 7$; then the problem becomes 7×3; that is to say, the sum of 7 taken 3 times; or the addition of 3 series of 7 units; and this we know to be 21.

But we inquire, Do we arrive at the same result, if we take the sum of 3 series of 5, and the sum of 5 series of 2, and add these sums together?

The question may be discussed thus:

.... (1)
.... (2)
.... (3)

Arrange a series of 5 units in order; then add a series of 2 units; as in the first line above. Now consider this as a series of 7. We have introduced no new process. Mark this series by the symbol (1). Repeat the process, arranging the units in the manner shown. Mark the two following series of 7 by the symbols 2 and 3.

We have thus 3 series of 7; and it is evident that their sum is the sum of all the units shown above.

But from the manner in which the arrangement is made we see that we have also 3 series of 5, and 3 series of 2; and that if we first obtain the sum of the 3 series of 5, then the sum of the 3 series of 2, and then add the results together, the sum so obtained will be equal to that of all the units shown above. Therefore we shall have:

$$(5 + 2) \times 3 = 7 \times 3 = 21,$$

also

$$(5 + 2) \times 3 = (5 \times 3) + (2 \times 3) = 15 + 6 = 21.$$

The result here obtained, however, does not depend upon any peculiarity of the numbers 5, 2, or 3.

We therefore generalise the process; that is to say, as before discussed in connection with the application of symbols, we recognise the character of the operation in a manner independent of the differences of the symbols to which it is applied.

If therefore, instead of considering 5, we take the symbol a to represent any number; and similarly b to represent any number; and similarly c to represent any number, then:

$$(a + b) \times c = a \times c + b \times c;$$

or with greater simplicity of symbols in algebra

$$(a + b) c = a c + b c.$$

This formula expresses the "distributive" property in multiplication.

At this stage the "commutative" property may also be discussed; that is to say, taking a particular example, we ask, Is 3×7 the equivalent of 7×3?

This question may be solved by reference to the arrangements of Units (p. 95).

7×3 indicates taking 3 times the series of 7, and the result is the sum of all the units shown. Now we have already seen that the manner of arrangement of units is indifferent, except for purposes of convenience, in the process of counting, and therefore of addition.

We note accordingly that in each of the vertical rows we have a series of 3, and we see that by virtue of the first disposition of this arrangement we have 7 of these series.

The sum of the 7 series of 3 is the sum of all the units shown. Therefore finally $3 \times 7 = 7 \times 3$.

And we may generalise, as before, using algebraical symbols thus: $b\,a = a\,b$.

Another question in a similar order of ideas arises with regard to multiplication. Is it "associative"?

This question may be discussed also by means of a simple example. If we wish to multiply in order 2, 7, and 3, and if we express the problem symbolically thus: $2 \times 7 \times 3$; we may proceed in more ways than one.

If we multiply 7 by 3 we obtain 21; so that we might express the problem now as $2 \times 21 = 42$.

But we might have begun by multiplying 2 by 3, obtaining as the result 6. Then the problem would have been expressed $6 \times 7 = 42$.

But is this operation capable of generalisation? Consider again the arrangement of the objects (p. 131). Here we have a series representing the product 7×3, or 21. Consider now the first vertical row ⁝; if instead of each dot we now place two dots thus ⁝⁝, and similarly for all the other rows, we will have 21 series of two;

that is, we will have $2 \times 21 = 42$. But in the first vertical row, whereas we had at first 3 dots ⁝, we have now three series of 2, or 6, ⁝⁝ And in the whole arrangement this series is taken seven times. So that we have for the whole number of units $6 \times 7 = 42$. Therefore $6 \times 7 = 2 \times 21$. Therefore remembering the way in which we obtained these results $(2 \times 3) \times 7 = 2 \times (3 \times 7)$, or generalising, with the symbols of algebra we have $(a\,b)\,c = a\,(b\,c)$. This formula expresses the "associative" property of multiplication.

Do we multiply numbers or things? We propose to multiply 4 horses by 3 horses. We take a series of 4 horses. We consider this series as a Unit. We take another series of 3 horses; we associate the series of 4 horses with a Unit of the series of 3. We renew the operation but with different units; that is, we take another series of 4 horses and associate the series with another Unit of the series of 3. We renew this operation, associating another series of 4 horses with another Unit of the series of 3. The full operation is therefore: a process of taking, or positing, a certain Unit (in this case a series of 4 horses) in a manner of which the simplest example would be the positing of an object such as a stone; the repetition of this operation; the Association of each act of repetition with a Unit of a known series.

Now by the natural development according to which symbolisation arises, we find that the material characteristics of the series of 3 are indifferent. The result would be the same if the series consisted of 3 peas, or of 3 stones.

The essential is the repetition in order; and the positing of a particular series of 3 only serves to make the Association with each distinct, and to secure that the act of positing is not repeated beyond the number of that series. Now the repetition in order depends on the

element of Time, the recognition of which in this way is a Fundamental Process. The fact that a series of any 3 units will serve for the Association required proves that these units are only the symbols to mark the successive Efforts of the Time process. The symbols of counting are also nothing but such symbols, and they are really just as material, though defined by means of the sense of hearing, as the symbols previously used.

When we desire now to "commutate"—as, for example, in showing that $3 \times 4 = 4 \times 3$—we would meet with confusion if the units of the series of 4 were not of the same kind as the series of 3. Therefore the employment of the operation of commutation presupposes that, whatever be the series dealt with, the series of repetition is symbolised by the same kind of symbols; and as the symbols familiarly employed, those of the numbers in counting, are not such as arise in ordinary experience; and as the Process of Dis-association with the accidental characteristics of the first units demands repeated experiences; it appears that commutation can only be employed after the habitual exercise of multiplication.[1]

We may now resume the discussion of the problem

[1] The questions discussed in this chapter, and those dealing with addition and counting, have proved in the history of science to be of great importance. It is true, for instance, that persons habitually multiply and arrive at correct results who have never occupied themselves with the analysis here exposed; but that merely proves that they follow the indications taught by others. A person may use a telephone by following a few clear instructions, but operations of that kind would give him a very imperfect notion of the mechanism employed, and would serve very inefficiently for the development of the science of electricity. Or a person may walk without having studied anatomy. But if the process of walking becomes defective, as by injury to any part of the mechanism, it often requires that the surgeon shall be well versed in anatomy in order to apply the proper remedy. The great mathematicians, notably Descartes, Gauss, Plücker, Hamilton, and Riemann, sought all their lives to enter more and more profoundly into the understanding of the nature of the assumptions involved in the simple operations of calculating.

Riemann's writings on the subject are very abstruse and highly technical, but they have been productive of brilliant results in the Higher Mathematics.

We have already noticed the value of Descartes' system of dividing the

which we left at p. 131. The digression has been necessary in order to throw light upon the difficulties which had occurred, but now the solution will become clearer.

By the distributive property we have

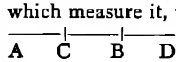
$$43 \times 25 = 43 \times (20 + 5) = (43 \times 20) + (43 \times 5).$$

So that we may first multiply 43 by 5; then multiply 43 by 20; then add the results.

In multiplying 43 by 5 we will find it advantageous again to make use of the distributive property. Thus $43 \times 5 = (40 + 3) \times 5 = (40 \times 5) + (3 \times 5)$. Similarly $43 \times 20 = (40 \times 20) + (3 \times 20)$.

Multiplying 40 by 5 we get 200. Multiplying 3 by 5 we get 15. Adding now according to the system of

straight line into parts. For one thing, since it enables us to define a line by the number of units which measure it, we are able to denote it by a symbol.

Thus if we have 

a straight line A B divided at C into two equal parts of length b, and produced to D, and if the length of C D be a, then we prove in geometry that the rectangle contained by the whole line thus produced, A D, and the part of it produced, B D—that is to say, the rectangle A D, D B—together with the square on half the line bisected—that is to say, the square on C B—is equal to the square on the line made up of C B and B D—that is to say, the square on C D.

The proof given by Euclid is somewhat long; but if we observe now that A D is equal to $a + b$; and B D is equal to $a - b$, and that $(a + b)(a - b) = a^2 - b^2$, as is familiarly known in algebra; then we have a comparatively easy proof of the proposition. For now, $(a + b)(a - b) + b^2 = a^2$; that is to say, re-translating the algebraical language, the rectangle A D, B D added to the square on C B is equal to the square on C D.

Considerations of this sort led Sir William Rowan Hamilton to form a calculus, which he called Quaternions, in which processes of multiplication take the place of geometrical transformations.

Hamilton, however, found it beyond his powers to devise a form of multiplication which should have the property of commutation, although he was able to secure for his Quaternions the associative and distributive properties.

Thus if p, q, and r represent three quaternions, we would have $(p\,q)\,r = p\,(q\,r)$; and $(p + q)\,r = p\,r + q\,r$, but not $p\,q = q\,p$.

This disadvantage has been severely felt in the Calculus of Quaternions, which, however, has become developed into a remarkably interesting and powerful instrument for dealing with complex problems in geometry. It is worth noting, therefore, that the questions we have discussed are of great interest in themselves, apart from the purpose they serve of determining, within their scope, the necessity and sufficiency of the Fundamental Processes we have set forth.

notation which we found so marvellously adapted to our needs, we obtain 215 as the result.

Multiplying 40 by 20 we get 800. Multiplying 3 by 20 we get 60. Adding these numbers together we obtain 860.

We must now add 860 to 215. We have thus five units, 7 tens and 10 hundreds, that is to say, 1 thousand. According to our system of notation we unite, 1075.

This manner of obtaining the result will bring clearly to view the great practical value of what in the higher branches of mathematics is called "elegance" in the manner of solution, and of which an excellent example is the ordinary rule of multiplication:

$$\begin{array}{r} 43 \\ 25 \\ \hline 215 \\ 86 \\ \hline 1075 \end{array}$$

We multiply the units together, 15. If the product exceeds 9 we "carry on," that is to say, we note the number of tens—in this case, 1. So that we set down 5 out of the 15 and carry over 1.

We then multiply the number of tens, that is to say, 4, by 5, and we get 20 tens; and adding the 1 carried over we get 21 tens.

Carrying out the same principle with regard to the tens, when we get more than 9 tens, we have one or more hundreds, and we place the number representing the hundreds in the third place from the right. We have thus 1 ten and 2 hundreds. For convenience of addition simply we place the product of 43 by 20 in the line below the first product.

As 20 represents 2 tens, the product, 20 times 3, which

is 60, contains tens. It is evident that such a product can never contain a series of units less than ten. Therefore there will be no number in the first place on the right in the second line. Therefore we always begin with the second place. It would be tedious to examine the process further in detail. A few observations, however, will make clearer how it is that a process really involving so much ingenuity has become so familiar. The "rules" of multiplication are still simpler than we have indicated in the last description. There is in the mind of the person who multiplies no thought of tens, of hundreds, or of systems by notation. He guides himself by a series of associations which by constant repetition have become "mechanical"; and the successive steps are also of this nature. Thus 5 threes are 15; 5 and carry 1; 5 fours are 20, and 1 are 21; and so on.

But in this particular case the result might have been obtained still more simply, though we anticipate an operation of division which is simple in practice. Thus for 43×25, we proceed:

Divide 4 (of the 43) by 4, set down 1; 3 being less than 4, set down 0, add 75; thus the total is 1075.

This is obtained without the result being previously known. The rule is simply: To determine the product of any multiplicand by 25, divide the multiplicand by 4; if at the end 1 be left over, we add in order 25; if 2, add 50; if 3, add 75. Thus $365 \times 25 = 9125$.

Under ordinary conditions there is no need for these shortened processes, but for a professional calculator they are often useful.

To multiply 45 by 35 the process might be reduced to this; square 4, diminish the result by 1, add in order 75.

Thus we have $45 \times 35 = 1575$. The reason of this will be seen if we state the problem thus: $(40+5)(40-5) = 1600 - 25$.

To multiply, say 67 by 99, write at once, beginning at the left, 6; then 1 less than 7, then the result of taking 6 from 9; then the result of taking 7 from 10.

Thus we have $67 \times 99 = 6633$. The reason of this will be seen if we state the problem thus: $67 (100 - 1) = 6700 - 67$.

Other examples may be easily devised by any one who has studied the meaning of the ordinary notation, and who is acquainted with the simplest algebraical formulæ.

With practice results of the kind may be written, or announced, with all the speed required of a "lightning calculator," and of course even by those who cannot render any explanation of the processes they perform.[1]

Referring now to the symmetrical arrangements of the objects on p. 131, it may be asked, What new element had been there introduced which enabled us to establish easily the distributive and the commutative properties of multiplication?

In the first place we see that there are seven in each of the three rows, by simply observing that the first row contained seven, and that each of the other rows contains a Unit associated directly with a Unit of the first row.

But in the exposition of counting we find that the simplest exercise is to associate one established series with another; and visual associations in this way are the most facile. We count the second row by using as counting symbols the units which we have counted in the first row.

When we commutate, we find the threes now ar-

[1] E. W. Scripture and Alfred Binet amongst others have studied the case of calculating "prodigies." Cf. Alfred Binet's "Psychologie des grands calculateurs et joueurs d'échecs," 1894. These studies, however, though interesting, throw into relief the superficially marvellous character of the performances rather than offer the veritable explanations

ranged in series, the series being vertical. In each one of these observations there is a new effort involved in each Presentation: and new Processes are involved in each Association. But by constant exercise and repetition these processes become extremely rapid, and unless attention be called to them specially they make little impression on the mind.

After having established the series of threes, each series is now taken as a Unit, and as the first horizontal row was counted seven, so each of those units (series of three) is counted by being associated with the corresponding original Unit of the series of the horizontal row.

Now we have already seen in the discussion of counting how the mind establishes the principle that the order of counting is indifferent, provided that in both instances all the units are counted.

The visual aspect of the arrangement of the units enables us, what otherwise would require considerable efforts of Memory, to decide that in both forms of multiplication, 7×3 and 3×7, all the units are counted, and no Unit is counted more than once.

But suppose that instead of this arrangement we had the units of the three series of seven distributed in disorder. Then we would find it difficult to revert for comparison to the other method of considering the total—viz. that of an arrangement of series of threes—because in the effort of Association we would meet with units which should not be taken into account, and we would have the unnecessary expenditure of energy in observing, and then by conscious efforts, such as we have before explained, rejecting certain units. Moreover, we would have to preserve the recollection of those rejected, and form the associations relative to the positions of the units involved, so as afterwards to take these units into account.

Moreover, Association is itself easier when objects are presented in some way, as for instance in straight lines, at regular intervals, which correspond to facilities of our movements, even to the slight, almost imperceptible, movements of vision, and which depends on innumerable experiences based on the physical condition also of the external world.

Considerations of this kind elaborated so as to apply to each one of the movements, associations, recollections, involved, will make clear why even for mental operations an orderly arrangement of objects is of great importance.

But another question arises: How does the mind arrive at the notion of this orderly arrangement? That is the result of a series of tentatives, combined with the associations and recollections of other orderly arrangements which have served useful purposes. By the nature of our lives our minds become storehouses of operations for securing orderly arrangements in the simple necessary occurrences of every day. If the arrangement required be more complex than these, the mind makes tentative efforts on similar principles, and when an orderly arrangement which serves its purpose is found in this way it is retained, while the others are rejected.

Already in the simple exercises of calculation which we have considered we have been acquainted with certain principles, the vital importance of which is seen throughout the whole range of mathematics. These principles might be referred to as good choice of symbols, good form of notation, orderly arrangements; symmetry where possible; knowledge of known forms and operations; "elegance"; tentative efforts estimated according to these standards.[1]

[1] Mathematicians will recognise that even complex problems, such as that, for example, of the solution of elliptic integrals, may be referred to successively under the headings indicated above. The celebrated quarrel of the followers of Newton and of Leibnitz turned to some extent on a question of notation. That adopted

Suppose that we proposed to define multiplication as some form of association different from that with which we are familiar; and suppose that we so selected our form of association that we could speak of multiplying 3 pigs by 4 cows, meaning thereby that to each of the three pigs we adjoined in some way four cows. There is no difficulty in such a view of the process. But this system would not be commutative, for if we multiplied 4 cows by 3 pigs, then to each cow we should adjoin 3 pigs; and the result would not be the same as in the previous case.

The peculiarity of the association of ordinary multiplication is, then, that the series of the multiplier takes the place of each unit of the multiplicand and so becomes in a manner substituted for that unit. The multiplicand does not become reckoned in the count of the result. Thus, taking the first instance, we would associate four cows to each of the three pigs; but we would not count the pigs, and we would count the cows only by referring them to a new series of counters, that is to say, associating them in a peculiar way with this new series.

Approached in this manner the ordinary process of multiplying seems less direct than that apparently fantastic one which we here first considered. And a similar remark applies also to the ordinary process of counting. It might appear, then, that a special effort of ingenuity had been required at a point of development where that ingenuity was of the greatest importance, and yet where that importance could not well have been foreseen.

Such a supposition, however, is not necessary. We

on the Continent was, for the development of the Differential Calculus, more convenient than that of Newton. It prevailed. The immense impulse given to mathematics by men of genius, such as Euler, Lagrange, Gauss, was therefore realised in forms that were a sort of foreign language to the Newtonians, and "English mathematicians stood still in reverence of their great countryman." More recently Newton's notation has been employed for special purposes.

find that by repetition the associations of any series may become very strong as compared with the associations of any one of the series with any other object. This is especially the case where the members of the series are mere symbols and the repetition very frequent. Hence a sequence is formed which at each step allows of an association with another object, and yet continues on in the regular succession of the sequence.

It must be noted that the association is that of symbol with symbol. For when we count a series of groups, for example, of four pigs, we consider each group as a unit, and the reference to the group as a unit is only possible by symbolisation.

We may now introduce a question which is somewhat recondite. How is it possible to establish such a sequence of symbols as enables us to perform the operation of counting? The answer may be that by repetition we have established a series of associations step by step so that these associations have become stronger than any other. But I do not think that this answer reveals the whole truth of the matter.

Suppose that we have three vertical columns containing four objects each, say for simplicity dots ⁞ ⁞ ⁞ This arrangement may be viewed as three times four. But suppose now that we regarded these as four horizontal rows each containing three dots. This arrangement would represent four times three. How do we know then that four times three is equal to three times four? The readiest answer is to say that this conclusion is obvious. And if the grounds of this reply be sought it will be said that there has been no change in the objects. If it were said, There is no change in the *number* of the objects, that would be assuming the truth of what we are seeking to establish. So that the further question arises:

Is there any assumption in this that can be definitely separated as an assumption? This question compels us to look narrowly into the actual process of counting.

Before entering definitely into that matter, it may be well to point out that in no case should the problem be regarded as idle. For if the dots represented voltaic cells, and these were in the first instance linked up in parallel circuit (that is to say, zinc to zinc, for example) in the vertical columns, and then these columns linked in series of three, we would get a different result from that obtained by linking the horizontal rows in parallel circuit and then linking these rows in series of four.

But it will be objected we have not merely left the cells in position, we have interposed an arrangement. But in counting, and in as far as the results of counting only are concerned, we interpose an arrangement. We must discover whether it affects the results; and if not, why not?

In counting the whole system we might begin with the first column, calling the first dot one, the second two, and so on. We would arrive at the number twelve. In the second place we might count the horizontal rows in order, calling the first dot one, as before, but the second dot of the row two, instead of, as before, the second dot of the column. Here then we have an essential condition that the sequence of the counting symbols must be so strong that it is not disturbed by any variation of the associations.

The next step is to inquire how far this may lead us. Can we, in fact, suggest variations of the association likely to disturb the sequence?

The dots may really represent other objects. Let us take them to represent the symbols of counting themselves, and let us replace them by these symbols. Further let us place the symbols so that the first column

begins with 2, and so reads, 2, 3, 4, 5 $\begin{smallmatrix}2&6&10\\3&7&11\\4&8&12\\5&9&1\end{smallmatrix}$ If then we count this column we refer the "one" of our counting to 2. If then we give attention to the fact that this object is really 2, the next association should be 3. But the next effort of the will[1] causes us to return to the "two" of our counting, which now we apply to 3 of the column.

Here is a case where the mind must proceed on one or other of these courses of association; it cannot proceed on both at the same time. So that in closely considering counting we find not only examples of Association, but we discover experimentally a fact of Association, that repetition increases the strength of a series; and we are also brought to a clear view of the character of symbolisation (that is to say, Generalisation under another aspect) as a Fundamental Process; and finally we meet with a proof of the position set forth of the conception of the Unit.[2]

If this last point is not quite clear another example may help to indicate it. Suppose that each item of the vertical row consisted either of an individual object or of a group, as for instance of men, and that the first consisted of one man but the third of two men. Then the counting would proceed normally to the third. If the group of two men was taken as one object, the counting would proceed normally. But if it were sought to recognise that the group was really formed of two, and that, without the intervention of a symbol, we desired to make the count under both suppositions at one and the same moment, we would find it impossible.

Thus we discover that our counting is conditioned by this limitation of our faculties. But counting we will see to be the base of all mathematics, even to the highest reach, for all these are but adaptations of principles of combina-

[1] This will be considered subsequently.

[2] Cf. note 2, p. 98.

tions which can be reduced eventually to the type of simple addition and multiplication.

Subsequently we will see that the whole process of Reason depends on limitations of Discrimination. The fact of experience that repetition strengthens the series of associations depends on conditions of organic growth. The study of counting therefore has brought us to the consideration of basic phenomena of thought.

DIVISION

Division presents little difficulty after multiplication, and after the consideration of inverse processes already discussed (p. 129). For example we wish to divide 21 by 3. The problem may be expressed thus: How many series of three must we take in order that the total of the units thus presented be 21?

But our use of multiplication implies a knowledge and easy recollection of certain results. We know that 7 threes are 21; that is to say, we know we must take 7 series of 3 to produce 21.

And this knowledge must be so familiar that (cf. Counting) the association of 7, 3, and 21 in a certain relation is stronger than the associations essential to any form in which the proposition may be stated. That is to say, apart from any set enunciation, we know that 7 series of 3 produce 21, in such a sense that, varying the form of words, we must take 3, 7 times to produce 21, or that 21 divided by 3 gives 7.

But suppose we had to divide 23 by 3. We know that $3 \times 7 = 21$ and that $3 \times 8 = 24$. Thus, although we have no associations immediately between 3 and 23, yet the associations between 3 and 21, and 3 and 24, are so strong that they enable us to make the tentative efforts to show that 7 is too small a number and 8 too great.

Now the principle of notation indicates that in any category as, for instance, the tens, the greatest number of complete tens, up to 9, is taken, and the remaining units are placed separately; and similarly for the other categories. Consequently we would here select 7. Now we have two units remaining.

As we are not tracing out the actual development of mathematical forms, but inquiring what processes are involved in those we consider in order, we may assume some acquaintance with fractions, the most facile of employment of which are decimal fractions.

With these the Unit is divided into ten parts, and each of these being taken as the Unit under consideration, we find that the 2 remaining as above are reckoned as 20 tenths, or 20 decimal parts.

Dividing by 3 we obtain 6 and 2 over. We treat these two remaining decimal parts according to the same principle, expressing them now as hundredths; and we continue the process of division. The method of arranging the notation in the decimal system is admirable and simple. A dot indicates that the places to the right are occupied by tenth parts, hundredth parts, and so on.

In this particular instance we will observe also that as at each step we have a remainder 2, the process continues indefinitely.

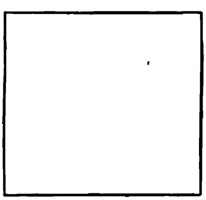
We will have occasion in several places to refer to infinite series, and to inquire as to their significance. Meanwhile we may express the result of division of 23 by 3, thus $23 \div 3 = \frac{23}{3} = 7{\cdot}666$, etc.

We may here leave the subject of division, for its discussion introduces no new Fundamental Processes, nor any derived processes which are new and different, except in application, from those already discussed.

CHAPTER V

OPERATIONS WITH SPATIAL RELATIONS

PROBLEMS involving spatial relations introduce questions that deserve separate consideration. If we take a square each side of which is equal, say, to 1 inch, and if then we say that the square is represented by 1×1, we are here performing a process which is not that of ordinary multiplication. The form, 1×1, as interpreted by the meanings we have already attached to these symbols, indicates taking a Unit once, that is to say, simply positing a Unit; and the Unit in this case is the side.



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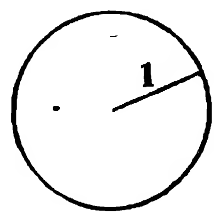
The operation of positing an area as a Unit arises from the Fundamental Processes involving Space. Certain aspects of the combinations of such processes are better discussed in the chapter on Externality. In this place, however, we may call attention to the various impressions produced in the exercise of vision in regard to length from one point to another, with all the associations derived from corresponding movements of the ocular apparatus; and the impressions in regard to length in a direction perpendicular to the first. We find Space as an Immediate Presentation, but we also find associations, which we may group together as those of inclusion, with regard to the impressions of lengths just noted.

Also in experience we may meet with many instances of spaces being divided into parts symmetrically adjusted.

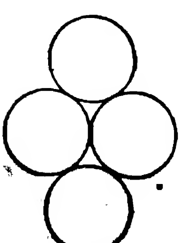
In this way we arrive at the operation of positing a certain defined area as a Unit area.

If the whole matter be considered in connection with that of the division of the line into units as already discussed (cf. pp. 124 *et seq.*), we see that in the adaptation of a Unit area to the measurement of areas we have an advance similar to that which led from the old Greek treatment by homologies and homographies to Descartes' system with its facilities of transference to algebraical forms.

To bring clearly into view the convenient character of the square as the Unit of space, we may inquire why the circle has not been so adapted. We might take a circle of Unit radius as the Unit of plane spatial measurement; and the sphere of unit radius as the Unit of measurement of space of three dimensions,[1] or what we call solid measure.



It is a fact that in an advanced stage of mathematical development, Gauss in a remarkable treatise on the Theory of Surfaces, and Hamilton in his development of Quaternions, found great use in conventions of a similar nature.



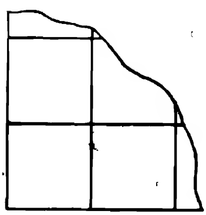
If, however, in tentative efforts we endeavour to comprise an area by means of a number of unit circles, we find a great difficulty in the fact that the circles, however small, cannot be made to fill up the area; and that the

[1] That is to say, according to the Cartesian system.

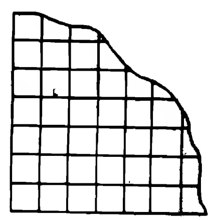
parts excluded from their areas are of a complicated character.

On the other hand the unit squares are exactly juxtaposed.

At the boundaries of an irregular figure we find that the squares do not become exactly applied. The smaller we take these squares the smaller will be the errors so



occasioned. We may then suppose our unit subdivided indefinitely, if only in the summation we take for one unit as many parts as those into which the unit was subdivided. Thus if we had taken the square inch as a unit, we might divide this into 10,000 parts, and use such parts for applying to our space to be measured. We would get a closer application to the boundary than if we measured by the original unit.



Then in estimating the total area we could take 10,000 of these parts for the original unit, and estimate spaces left over by the number of parts.

We have had occasion already to refer to a similar process in decimal fractions in the discussion of division.

Considerations of this kind led to the formation of the Infinitesimal Calculus, or Differential Calculus, as it is usually called.

There is a fascinating study in the inquiry as to the dominance of the straight line and the right angle in our simple geometrical process. We measure the circumference of a circle, for example, by straight lines, which are arcs of the circle. It is evident that the smaller these arcs are taken the less is the difference of their total

length from that of the circumference. Hence if we take the arcs small enough, and so make their number correspondingly great, we may have a length differing from that of the circumference by a deficiency as small as we please.

Here again we have a suggestion which, given forth by the older geometers, notably Archimedes, and developed later by Fermat and by Pascal, led, in accordance with notions arising from the problem of areas, to the Infinitesimal Calculus.

But it is possible to consider just as closely a straight line as made up of a great (unlimited or infinite) number of small arcs of circles.

There are reasons which might tend to make this method of estimation the more natural. Our physical movements are mainly directed by the pulling of muscles on the bony framework. The movements of this framework takes place at joints, or hinges.

Hence the movements of parts are in the first place circular. Certain of the joints, it is true, permit slight sliding movements. Straight movements are, however, obtained mainly by the combinations in various suitable ways, taught by experience, of circular move ments.

The eye, which is not attached to a joint, is moved by the combination of complicated movements, but the dominant character of the movements even in this instance is that of rolling around various axes.

In our contemplation of the external world we meet comparatively rarely with perfectly straight lines, but very often with lines whose deviations are on all sides of a straight axis. Moreover, our mental habits are greatly

influenced by the fact that the direction of propagation of light and of sound is in straight lines, and thus the impressions of our two most active and most discriminating senses are associated with straight lines. Then again, although our movements are compounded of circular movements round hinges, it is easier to draw a straight line than a circle; and, what is of greater importance, it is easier to note and correct the deviation from straightness than it is to correct the deviation from a perfect circle.

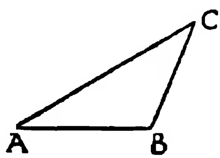
Hence the simplest of drawing instruments are straight-edges. It is important too that, though in the human body there are comparatively few straight lines, yet here again the deviations are round well-marked axes. If a man stand upright with his two arms extended and held sideways, then his body represents a structure round a vertical axis; his arms contain an axis at right angles to his body; his vision is on a line at right angles to both those axes. The direction of gravity is in the vertical axis.

It is in fundamental conditions such as these that we must seek the origin of the choice of certain standards, bases, and units that we constantly apply without reflection.

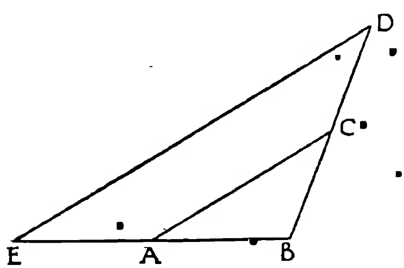
Once adopted, the straight line and the right angle prove their value in practical use, by reason of their general concordance with the conditions of the external world. To illustrate the meaning graphically one might suppose the intelligence of a man in the frame of a star-fish. It is possible he would then think not that there were two sides to every question, but that there were five.[1]

[1] Under certain circumstances we find such a special point of view helpful in our higher mathematics, where Cartesian co-ordinates become unwieldy in problems which are simplified by Polar co-ordinates, and still more strikingly by Plücker's co-ordinates. Mathematicians may compare the system of Grassmann's Ausdehnungslehre with Cauchy's Clefs and Hamilton's Quaternions.

In order to show how purely conventional it is to apply the symbol of multiplication to a certain conjunction of the two sides of a square in order to obtain the area, we may adopt a convention inconsistent with this, yet by interpreting the symbol in a manner consistent with itself obtain under certain circumstances valid results.



Thus we may say A B . B C = area A B C when the dot does not necessarily imply multiplication, and we then proceed to develop such an "algorithm" or system of calculation.



Call A B α, B C β; $\alpha \cdot \beta$ = Area A B C. Produce the sides B A to E, and B C to D, making B E = 2α, and B D = 2β. Then we would have $2\alpha \cdot 2\beta$ = Area E B D.

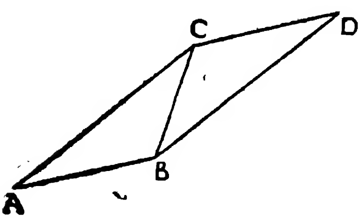
If in this formula we now assume that for the number 2 the process of multiplication is applicable, we have $4\alpha\beta$ = Area E B D. But $4\alpha\beta = 4$. Area A B C; so that the area of the triangle E B D is four times that of A B C. The result in this case is true; but there is an unjustifiable assumption in the use of the "algorithm" itself; for the area of a triangle cannot be expressed as dependent simply on two contiguous sides; it depends also on the included angle.

In the case we have considered the included angle remains the same for both triangles, hence a cause of error in the application of the algorithm disappears.

It is necessary therefore to examine any proposed algorithm not only to observe if its use be consistent

in itself, but also as to whether it is justifiable in regard to the object to which it is applied.

Certain assumptions are made in regard to vectors (see p. 122) which, though justified, must be carefully noted. Thus if A B + B C = A C, and we wish to apply this formula, with the faculty of commutation, we have B C + A B = A C. But beginning with B C, we can in this system add A B, beginning at the end of B C, by assuming that C D, which is equal and parallel to A B, is its equivalent in regard to the formula. Then A B + B C = B C + C D = B D. That is to say, B D is the equivalent of the vector A C; that is to say, is parallel to it, and equal to it in length. Hence with this notation we arrive at a demonstration of the proposition that the lines which join the extremities, in order, of equal and parallel straight lines are themselves equal and parallel. But in the posing of the algorithm itself this proposition is virtually assumed.



Similarly whereas Euclid lays down his axioms, such as that things that are equal to the same thing are equal to one another, etc., he assumes, in the statement of one of these axioms, what is less obvious, that all right angles are equal.

A brief summary of certain other mathematical forms is necessary, so that later we may review results in regard to the whole field with reference to the Fundamental Processes.

We have already incidentally discussed the use of fractions. Thus $\frac{3}{4}$ means that a unit is divided into four parts, and that three of these parts are taken to be represented by the symbol here chosen.

There is nothing new presented in the conception of such division, for we have already considered that question in discussing the Unit.

There is nothing new in considering each of these parts as a unit, and counting the number taken. The notation of fractions, as for instance in decimal fractions, the questions of addition and multiplication of fractions, and all other developments of arithmetic dependent on the use of fractions, will be recognised in examination simply to entail the application of principles we have already considered. Occasionally the devices of arithmetic in this respect are ingenious, and noteworthy for that reason, but never as introducing any process not found by combination of the Fundamental Processes applied to the objects that successively present themselves.

Thus $\frac{3}{4} = \frac{9}{12}$. That is to say, we have now twelve parts in the second of the fraction instead of four.

We have divided each of the four parts into three, since $4 \times 3 = 12$. We must now count three of these parts for every one original part. Therefore we must have three times three of the new parts, and $3 \times 3 = 9$.

Having now established that $\frac{3}{4} = \frac{9}{12}$, it is easy to form practical "rules" for reducing fractions. Thus we may multiply numerator and denominator of any fraction by the same number; or again, we may divide numerator and denominator by the same number.

The problems of squaring, cubing, etc., are simply special forms of multiplication. Thus the square of three simply means three multiplied by three. Expressing the formula generally, we have $a^2 = a \cdot a$. The only new fact is the notation employed, as in a^2.

Taking the square root is the inverse process of squaring, just as division is the inverse process of multiplication. If $3 \times 3 = 9$, taking the square of 9 simply

means obtaining 3 by considering the conditions implied in the formula.

The notation here is new. $\sqrt{a}$ means the square root of a. But this notation depends simply on a choice of symbols. In cases where the exact square root is not immediately found, it will be seen on investigation that devices are applied which, though introducing no new principles, exhibit the original number in classifications of thousands, hundreds, tens, units, decimals, and so on; or in algebra by other convenient categories. Such operations we have already seen exemplified.

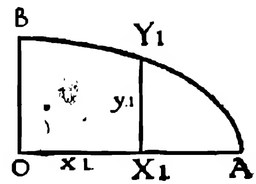
All applications of arithmetic to problems involving money, or weights or measures, depend on rules already indicated. The only new facts are the classifications of the units, as in pounds, shillings, and pence; and the conditions under which, for example, 45 shillings are classed as 2 pounds and 5 shillings.

Problems of arithmetical, geometrical, or harmonical progressions introduce nothing new beyond different modes of grouping series. Of combinations and permutations the same remarks, considered generally, are true.

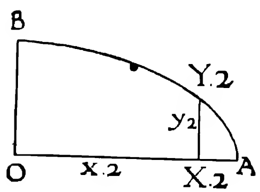
In short, if we examine the whole field of arithmetic we will find that though we meet with new arrangements and series of numbers, corresponding to certain requirements of calculation, and though certain of these are expressed by special notations, yet the whole development of the science is due to the combination of the principles already shown applied to various cases which arise in experience, or in inventions based on experience.

Algebra arises from a generalisation of arithmetic. Thus if we express that $3 \times 5 = 5 \times 3$, we limit our consideration to that particular case. But if we express that $a\,b = b\,a$ we state that the principle of commutation involved is true of any numbers whatever.

We may say that we make "abstraction" (cf. pp. 45 *et seq.*) of any peculiarities of the numbers 3 or 5, observing that the principle does not depend on these peculiarities. The word "abstraction" is often convenient to use, so that we here take further occasion to observe what it implies.



If we have two lines at right angles O A and O B, whose lengths are a and b, it is possible to have a curved line joining B and A and having this remarkable property: if any point Y_1 be taken in the curve, and from Y_1 a perpendicular $Y_1 X_1$ be drawn to O A, meeting it in X_1, and if we call O X_1 x_1, and $X_1 Y_1$ y_1, then $\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 1$.



If any other point be taken, Y_2, and if we proceed to a similar construction, and if now X_2 correspond to X_1 in the first construction, and if O X_2 be x_2 in length, and X_2 Y_2 be y_2; then $x_2^2 \quad \frac{y_2^2}{b^2} = 1$. Now as the property indicated corresponds to any point on the curve, we may posit the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; and say that corresponding to any point whatever on the curve, y is the length of the perpendicular from the point to the base line O A, and that x is the length of the line along the base from O to the foot of that perpendicular. A curve for which the above equation holds is an ellipse. The equation is called the equation of the ellipse.

Here we may be said to make abstraction of the particularities involved in any of the equations first

considered; but abstraction in such a case is simply composed of Generalisation and symbolisation; these, however, depending on Processes of Agreement, Association, Dis-association, and again Association.

Thus understood, the use of the word "abstraction" presents no difficulty, but there is danger in the notion that in abstruse reasonings, such indeed as may occur in mathematics, there is anything insecure, or baseless, in abstraction.

The equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ may be taken as the symbol of any one of the particular cases, x and y being applied as symbols of the lengths of the co-ordinates corresponding. It is the general equation, but a general equation is without meaning unless it be considered with regard to application to particular examples.

If we consider A———B, a line A B, we find that it does not correspond to Euclid's definition as that which has length and no breadth. It has breadth, or we would not be able to see it. It is a quantity of ink reaching from A to B.

Again, we might have a line whose visible representation would be a mass of chalk reaching from A to B.

But after the manner we have already discussed (cf. pp. 45 *et seq.*), we find that apart from the ink in the one case, or the chalk in the other, we have certain Immediate Presentations. One of these is the direct impression of Space. Then there are associations of length with various muscular movements (cf. pp. 46, 151; also see analysis of Externality).

We do not recognise these movements unless we pay particular attention to them, but when we have been familiar with the accidental aspects, such as due to the ink or the chalk, we recognise length as a real thing.

The process of abstraction is not so much the recognition of a whole and the deliberate subtraction of some incident associated with it, as that, following a principle discussed in regard to Discrimination, we devote less energy of thought to things already recognised, and so are freer to see more clearly what at first has been dimly perceived.

To make this more evident. Suppose that we have two eggs, very similar, one brown and the other white. We are immediately struck with the difference of colour, but afterwards we recognise the similarity of shape, the equality as far as weight is concerned, the similarity of feeling, the similarity of taste on eating. All these are positive things, and it is they which form associations strong enough to enable us to speak of "egg," without experiencing that the associations due to colour overpower and exclude the others (cf. pp. 46 *et seq.*).

In the case of a line the positive associations due to length seem less strong, and the things which we afterwards exclude are at first proportionately prominent; and, in accordance with these facts, it is not until after considerable exercise in thinking, and familiarity with lines of various characters, that we arrive at the "abstraction" of a line.

When an untrained person of little intelligence is told to measure a line, it is true that he ignores the actual object which presents the line materially, but it does not follow that he resorts to an abstraction; he simply applies a measure, such as a tape measure, between the extreme points, and reads off the number indicated. He is tracing out a certain series of associations which have been taught to him by aid of symbols.

The notion of length between two points does not necessarily imply that of a straight line. We arrive at the notion of straightness in various ways (cf. p. 46

and p. 151). At length we build up the conception of the length of a straight line between two points.

Thus in an abstraction there may be involved operations of combination, but these combinations are formed of the ideas, or presentations in Memory, of real things. For example: An artist may wish to paint a phœnix. He fancies, that is, builds up by combination of the ideas of things once known, an image of a bird which as a whole never existed, but of which every element corresponds in idea to an object once presented in experience.

When we have formed the notion of a straight line, then in order to make this clear to others, or even to ourselves, we usually again set forth a straight line materially represented.

In the case of such a conception as that of the straight line the abstraction is concerned with ideas so tenuous and so weak in associations, while the invariable experiences of a material line make such comparatively strong associations, that it is possible that we cannot form any presentation except in terms of some material line. The material aspect may be varied. It is figured as narrow as possible. There is an effort then of the mind to reject the Association formed by any particular material representation (Disassociation, see p. 35). In this case the operation of abstraction may be considered in connection with what has been said of subtraction regarded as an inverse operation.

The straight line of which we have formed the notion is considered "generally." Any proposition relating to the straight line is applied to any particular straight line we have specially to consider. Thus what is called abstraction is in such simple instances coincident with generalisation, which it must be noted is not the Fundamental Process of Generalisation, but is combined of this in connection with Association, Disassociation, Associa-

tion again, symbolisation, and hence through these is seen to be a complex formed of all the Fundamental Processes. At the extreme limit of simplicity, abstraction seems to coincide with Generalisation, or if not invariably to coincide with Generalisation, to be another aspect—partly inverse and therefore more observable—of that Process.

The process of abstraction that we meet with early in algebra is equivalent to generalisation, and the simple operation of employing a symbol considered "generally," such as a for a number, has produced extraordinary consequences in mathematical development, and hence in the whole course of our civilisation. For algebra often poses the problem of finding a number which is not given, but which is defined as being in accordance with certain conditions. Making abstraction by way of excluding everything except the notion of a number, we posit the number, and express it, usually, by the symbol x. We say: Let x be the unknown number. Translating ordinary language into the simple and expressive language, or symbolisation, of algebra, we free our minds from unnecessary expenditure of energy and confusion (cf. p. 140). Accordingly we are enabled to see more clearly into the essential conditions of the problems, to recognise the best groupings (cf. pp. 140–141), and at length, from a statement which implicitly determines the value of x, to arrive at a statement which explicitly declares the value of x.

The persistent reduction to simple forms by means of symbolisation is the characteristic feature of algebra.

For example, we have seen:

$$(a + b)(a - b) = a^2 - b^2.$$

But suppose we have:

$$\{(x + y)^2 + (z + w)^2\} \{(x + y)^2 - (z + w)^2\}$$

Let us call:

$$(x + y)^2, a; \text{ and } (z + w)^2, b.$$

Then this complex form becomes the less complex $(a + b)(a - b)$, which by virtue of more familiar associations we recognise at once as a known form. The reduction of the more complex problem is thus facilitated.

The science of trigonometry involves certain spatial relations which we have not yet discussed. Its development depends on a combination of these with expressions of algebraical form.

We have proceeded far enough, however, in the present discussion to give the suggestion that no really new calculus can arise, that is to say, a calculus that could offer a new faculty for solving problems, and which would be other than the continuation of those we have already discussed, and all that is involved in the multitudinous and complex forms of their applications, together with the necessary storing in memory of certain "known forms," and frequently repeated complex operations.

The Differential Calculus introduces new symbols which give at first the appearance of a new order of ideas, but it is really algebra applied to conditions where we deal with the addition of "infinite" series, of which each term may be the product of a certain quantity and a quality differing as little from zero as we please.

Now in algebra we apply the symbol Σ which is simply a form of S to indicate a sum.

Thus $a_1 b_1 + a_2 b_2 + a_3 b_3 +$ etc. might be expressed as $\Sigma\, a\, b$; that is to say, the sum of all the quantities, each term of which is of the kind of, or is generalised by, a b.

In the Differential Calculus we have the symbol $\int$, which is also a form of S, for sum, and this indicates the summation of such infinite series as are referred to above. $\int y\, dx$ means the summation of an infinite series, $y_1 dx_1 + y_2 dx_2 +$ etc., of which $y\, dx$ is the general term. It is understood that y is a "function" of x; that is to say, a quantity whose value depends on and changes with that

of x; and dx means the "increment," or increase of x, or in other words, the difference (hence dx) between successive values of x, when this difference is considered to be as small as we may conceive.

But we have already considered the resources of algebra in respect to the Fundamental Processes. If then the differential calculus involve as new only the conception of indefinitely large and indefinitely small numbers or quantities, we need only consider especially the processes involved in these conceptions (cf. chapter on Infinity).

We will subsequently, in regard to the purpose of this exposition, review the results which we have obtained. Meanwhile, we will consider another order of ideas though still in the realm of mathematics; for mathematics afford the most striking examples within our knowledge of long processes of ratiocination conducted on a basis of apparently few assumptions, and yet leading to surprising discoveries bearing applications of the highest practical value.

CHAPTER VI

THE AXIOMS

THE axioms of Euclid have generally been considered as the simplest basis upon which a series of reasonings may be built; and since Euclid starts with the statement of the axioms, the postulates, and the definitions to which he intends to make reference, it is assumed that the reasoning requires no further appeal to experience. We shall find, on examination, many grounds for regarding the whole matter in a different light.

Let us consider the axiom : Things that are equal to the same thing are equal to one another. The previous discussion of the Unit has led us to the position of considering that things may never be equal one to another; that each depends eventually on the Immediate Presentation of Unit, these Presentations being separate and distinct things; and that combinations of these, depending on various operations in which Association plays a large part, are never the same for two objects. Even that two units presented in succession may be classified under one generalisation depends, as we have seen, on the limitation of our process of Discrimination.

If we confine our attention to the magnitude, for instance, of the area contained within the boundaries of two things, then we have already made an abstraction (cf. pp. 44 *et seq.*, pp. 157 *et seq.*) which we have observed to be, in such a case, the result of the combination of several Fundamental Processes.

But even on this basis we shall find that certain further assumptions are necessary which may, or may not be, justified, but which in any case can only repose on our experience.

Before discussing the matter in this light it may be well to notice certain objections that may occur. For instance, there is ordinarily no hesitation in accepting the axiom as being both self-evident and basic, and moreover the theorems deduced by Euclid from such premises are found to be true.

But in the first place the axiom assumes what is here the point contested, viz., that different things may be equal. But if the equality be extended to absolute identity then the axiom becomes reduced to a barren expression. If there be not identity, then since there are differences in the two things considered we must exercise discrimination, and paying attention to the features of resemblance we perform Association, and we make Generalisation, and thus finally abstraction.

That we habitually proceed in reasoning on assumptions of this kind and arrive at valid results is consistent with what has been said, for our assumptions and our results may be both true within the scope of our consideration and within the limits of error of our Discrimination.

Thus we may say of a number of eggs that they are all of the same size; they may not be absolutely of the same size; that would be practically impossible. But within the limits of error of our superficial observation, and within the limits of precision imposed by the use to which we wish to apply them, we may say the eggs are all of the same size.

Or again I bowl a cricket ball. It is hit over a fence, and after a search found. I say it is the "same" ball. I may repeat this statement. But in the end it may be

battered out of shape, and have lost some of its substance. It is not absolutely the same ball; and as the change has been gradual it is evident that absolutely it was not the same ball at any two successive trials. Yet no confusion arises in the ordinary use of the language.

But in studying, in regard to its processes and its products, such a delicate affair as the mind, so elusive in its operations, and yet capable of producing such complex and wonderful results, it is necessary to pierce as far as we can to the foundations and to examine the necessary parts in the finest movements of consecutive working. In this way alone can we hope to understand its mechanism and to indicate its development.

We will see later, in studying the matter in synthetic illustrations (cf. pp. 338 *et seq.*), how it is that the mind may with sufficient correctness for most uses proceed to reason from assumptions which it has not completely investigated.

It should be remembered, moreover, that any reasoning depending on this axiom gains no force from the previous statement of the axiom. For the axiom acquires a definite meaning only when applied to a particular example. But it is easier to see the truth of the axiom in regard to the example, than first of all to express it in general terms, and then to particularise from the general expression.

Such a proposition does not hold in regard to all cases of generalisation, for the generalisation may be arrived at by the examination of a combination of properties in the examples it covers; and it may be more convenient to conduct this examination once for all, in such a manner as to make sure that it will cover all the examples, than to express the properties in a general form, that is, to establish our generalisation, and then

apply it to each particular case that arises (cf. p. 29 and pp. 160 *et seq.*).

But in the case of an axiom which should present no feature for discussion and combination the reasons just given do not apply.

It may be laid down as true that if any object be transported into another place it is not the same. A little discussion will make this clear, and it will be seen to be not a matter of futile metaphysical discussion, but finally of practical importance.

A body placed a metre higher than in its first position does not weigh the same as before. There was a time in the history of the world, and long after some of the most celebrated and elaborate systems of the Universe had been devised, when this proposition would have appeared absurd. To those to whom it might for some metaphysical reason appear not absurd, it would at least appear unimportant.

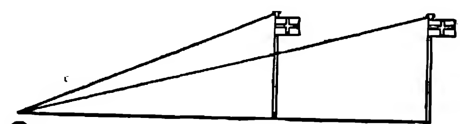
Yet the questioning of such possibilities led to various theories of gravitation, to which at length the genius of Newton gave a clear and correct expression.

Suppose that we asserted that the shape of an object changed when the object is transported, what means would we have of disproving the statement, or proving the contrary? To measure it would be beside the mark, for we now see that in the case of gravity our finest measure would have been useless, or deceptive, in regard to a fact which we all now accept. Besides, if the proposition were true the measure itself would change in the same proportion.

No philosopher regards a question as settled by appeal to what is called "common sense" or ordinary experience. What operation of common sense could indicate that two lights could produce darkness? The deep investigations of Thomas Young as to the veritable

nature of light, however, enabled him to devise experiments showing the phenomena of interference. Or again what ordinary experience could tell us of the existence of the X rays?

The proposition that the magnitude of a body changes with its position seems really more nearly in accord-



ance with actual experience than the contrary, for the farther a body is removed from the eye the less the angle which it subtends at the eye.

Moreover the conception of magnitude seems to depend more than that of weight on subjective impressions. Thus we appreciate the weight of a body by supporting it in the hand. But we also have a means of estimating its weight by observing its effects on a balance; and this experience is made manifest to us by different senses. Accordingly we gain a notion of objectivity in regard to weight.

This is less obvious in the conception of magnitude. Yet the complete conception of magnitude is formed by the combination of many elements of judgment brought from different senses (cf. chapter on Externality).

The direct visual impression alone, apart from the impressions of the muscular sense in the movements of the eye, as in direction and accommodation, does not give the notion of the externality of the object perceived. Common sense would not make us aware of that fact. In fact common sense is often invoked to prove the contrary, but the good fortune of experiment on persons

operated upon for cataract—beginning with the famous case of Cheselden—has placed the matter beyond doubt.

The idea of externality is derived from an elaborate system of co-operation with vision not only by the muscular senses of the ocular apparatus, but by the Feeling of Effort of the muscular system in approaching or receding from objects; by the sense of hearing, by the sense of smell, by the sense of touch, both in light touches and in impacts; all these combined in our experiences in all sorts of manners and degrees and repeated incessantly. Now the muscular sense is affected by the situation of the body, because the weight of the parts moved by the action of the muscles is so affected.

Therefore it is evidently not from direct impressions that we can know or deduce that objects do not change their magnitude with their situation.[1]

If we look at the question in the light of the analysis of all the processes concerned in the conception of space, and of what we know of "objective" conditions, the postulate that a body, or a figure, remains unchanged when its situation is changed, so far from being self-evident, seems rather in danger of losing all meaning. For any material body changes by the operation of gravitation; the objective standards of measure change; the immediate subjective impressions are altogether

[1] In Riemann's famous treatise on the Foundations of Geometry, which is one of the most abstruse discussions in the whole realm of philosophy, but which has nevertheless been fruitful of progress in mathematical science, the great analyst sends the plummet here and there profoundly. He treats as questionable assumptions the postulates, always implied in mathematical physics, of the persistence in regions infinitely great, or infinitely small, of the most universally accepted laws. He is not sure that it is right always to assume the possibility of an absolutely plane figure, or of lines indefinitely parallel. It appears to be with him that the notion originated, often quoted from the brilliant Clifford, that space may be curved. The germ of the notion, however, will be found in the speculations of Gauss as to whether a geometry might be built up which ignored the twelfth axiom of Euclid. Bolyai and Lobatchewski followed Gauss in developing the Pan-Geometry.

erroneous. Moreover, it may be questioned whether we can think of a shape except in association with a material body. Already in the case of a straight line between two points, the association of material forms seems predominant; but if in the terms of our proposition we suppose that any two points may change relatively in position, how can we apply a fixed standard?

Can we even form a conception of a point fixed in space? If we could we would be able to speak of absolute motion of a point instead of merely relative motion. Clerk Maxwell, whose profound and perspicacious intellect was greatly concerned with this aspect of the question, concluded that the only motion we could think of was relative motion.[1] The appeal must be made to one's faculties of introspection.

But if this be so, then it seems also that we cannot think of a point except in association with the presentations of material objects. For how otherwise can we define our point?

Abstraction in this case can get no further than the conception of external minute material things, with the faculty of throwing off from the mind the Impulse connected with the associations of any particular material thing (cf. pp. 46 *et seq.* and pp. 158 *et seq.*).

If we look at the question in the light of our scientific knowledge, we find that we are constantly moving in space, in movements compounded of our rotation around the earth's axis, of our movement in the earth's orbit, and of a number of perturbations; while again the whole solar system is moving in space. The idea of an abso-

[1] It is true that, as Clerk Maxwell points out (cf. "Matter and Motion," arts. cv. and cvi., also arts. xxix. and xxx.), rotation may be absolute, but this does not invalidate the proposition above. Rotation is a case of relative motion where all the parts concerned are under certain conditions that determine their relative movement to any point, as, for instance, to a point in the axis.

lute point or absolute motion vanishes. And to find a definition for points, relatively considered, we must resort to definite associations with external objects, though taking into account the faculty of abstraction already considered.

But adopting these conclusions, can we not form a notion of the permanency of a line, or shape, independent of its situation ?

We may study the question of weight in order to seek some guidance. Here the senses give illusory direct evidence, and no instrument that we can devise is delicate enough to detect slight differences of weight when a body is moved within a small space. But we have by a series of reasonings arrived at a theory of gravitation, expressed by a law, which has been found to give results always compatible with experience even with the most delicate measurements and in regard to distant bodies, the sun, the moon, and the stars. We make an assumption that the law of gravity applies to all bodies and in all situations. Our reasoning on the matter has no certitude beyond this assumption.

We therefore seek in the problem of shape, with regard to space, some constant "objective" condition. We might find this in the permanency of the laws of light. Thus, for example, theoretically we might suppose the length of a line measured in a certain situation. We might suppose it transported into another situation. Then we might by using the formula of trigonometry so arrange that we measure the length of the line in the second situation by means of angles and lines measured in the first situation; and therefore not involving the transport of the standards themselves. But even this assumes certain permanent conditions with regard to light, and as by the researches of Faraday, Clerk Maxwell, Hertz, and other physicists we are led to

believe in a close correspondence between light and electricity, at least in their objective condition, we begin to see at length under what limitations our "certitudes" are founded.

The object of the foregoing discussion has been to unsettle the mind a little in respect to the immediate acceptance of axioms, either as self-evident or fundamental. Regarded in the manner in which we now approach them they have quite a different aspect. Instead of saying: Things that are equal to the same thing are equal to one another; we would prefer to think in such terms as these: Suppose that in contemplating objects we make abstraction except in regard to magnitude, we find in experience that with different objects we may reach the limit of our faculty of Discrimination in this respect, and that the variations of Impulse arising from the two concepts, and of the associations formed, and of the directives to the subsequent movements of the mind along a course of reasoning, may be taken as beneath the limit of Discrimination for any purpose concerned.

Now all these considerations we may, if we please, express in the familiar form of the axiom, and then there will be no difficulty in its use. Thus, for example, in physics we meet with the term Centrifugal Force, due originally to erroneous ideas of the nature of certain forces; the error of these ideas has been perfectly exposed, but physicists still find the term convenient for reference.

It is evident that the consideration of the axioms does not introduce any new Fundamental Process of mind.

CHAPTER VII

NEW VIEWS OF GEOMETRY

THE postulates of Euclid are direct appeals to experience. The definitions are formed by choosing the most evident, or the most characteristic, properties of figures having many relations of a similar kind, those properties being chosen which are necessary and sufficient.

That the grounds of a proposition have not been established at a basis which marks the ultimate limits of our analysis, does not necessarily imply that the proposition is not true. We have in the propositions of Euclid themselves a striking example of a theorem believed to be true long before any valid "proof" of its being true was obtained.

This is the famous 47th proposition of Euclid, which was solved at length by one of those great Greek philosophers to whom we owe so much—Pythagoras. The proposition is, in short, that, considering any right-angled triangle, the square on the side opposite the right angle is equal to the sum of the squares on the other two sides.

One might try to prove this by minute practical measurement in a number of right-angled triangles of various shapes; and then, as the proposition would be always found verified, one might make this assumption of its generality.

This is a procedure which has been adopted in many

sciences, and with excellent result (cf. pp. 386 and p. 397 *et seq.*). But an objection immediately arises: Suppose that the measurements involved an error smaller than could be detected by the instruments at our disposal, in what then could we obtain greater certitude?

It was in investigating problems of the kind that the Greek thinkers were led from one proposition to another until at length they arrived at the axioms. In the course of their demonstrations they made many assumptions, but none that were likely to be seriously disputed.

The previous discussion has been intended less to question the use of the axioms and definitions than, proceeding from the basis of the Fundamental Processes of the mind, to inquire in what manner we may arrive at clear views of all that these axioms and definitions imply.

The analysis of Pythagoras has not been substantially improved on. Since his time certain different arrangements of the demonstrations have been devised, but the main principles of Pythagoras are adopted.[1]

Further on a proof is offered based on a different analysis.

In view of the development of modern mathematics it may be said that the Greeks limited themselves unduly in their postulates. They failed to arrive at the position expounded by Descartes. They had not reached the generalities of algebra.

[1] The joy of Pythagoras, who sacrificed a hecatomb of oxen—though fabricated on this occasion of milk and honey—is comprehensible, for he has left a work which is one of the most marvellous of the monuments of antiquity.

Hamilton in the development of his system of Quaternions offers a proof of the Pythagorean proposition, independent of the method of the first master, which is very interesting and instructive. It is ostensibly shorter, but it depends on assumptions the establishment of which from first principles would make the demonstrations considerably longer. Another very ingenious demonstration, which by rule of thumb test would appear simpler than that of Pythagoras, was devised in 1830 by Henry Perigal. This is capable of geometrical demonstration, but it has no advantage in this respect over the method of Pythagoras. The figure is cut on Perigal's tombstone at Wennington, in Essex.

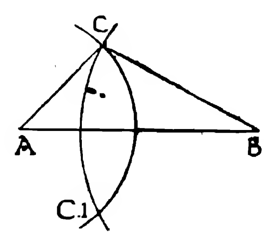
Suppose then that in the spirit of modern mathematics we approached the propositions of Euclid, and that we were, as Euclid was, early confronted with the problem of the triangle.

We observe that the triangle may be said to be known when the length of each of its sides is known, and also the magnitude of each of its angles.

With these elements we can construct or reconstruct a given triangle; and we assume that within our limits the triangle does not change with change of situation.

It becomes obvious in attempting the combination that it is not necessary to know all the elements mentioned in order to construct the triangle. We may therefore ask, What are the fewest elements we require to know?

Let us take one of the known sides, and posit it as A B. Then with A as centre we describe a circle with radius equal to another of the sides. With B as centre we describe a circle with radius equal to the third side.



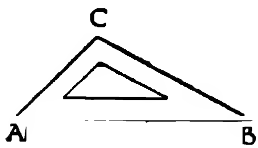
We assume that the two circles cut each in two points. This assumption may subsequently be examined in order to establish it on a sure basis.

The triangle A B C is constructed. The triangle A B C′ is also constructed with the same elements; but we make the assumption that the triangle is not changed when its situation is changed.

Hence we establish that a triangle is known or "determined" if we know the three sides.

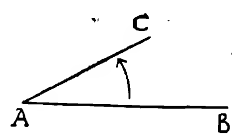
When we have three angles "given" we discover that this is not enough, for altering the scale of a triangle does not alter the shape, and therefore does not alter the angles.

The reason why the three angles do not determine a triangle is that the three angles are not independent; their sum is equal to two right angles, and therefore when two angles are given the third is also determined.

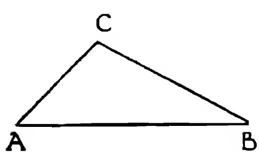


To show that the three angles of a triangle are equal to two right angles, an angle may be referred to as the amount of turning of one of its sides from the other, for example, A C from A B.

This is not a new idea, for it is really implied in Euclid's assumption that the magnitude of the angle is independent of the length of its sides; it is only a matter of convenience of expression.



Now if the triangle A B C be examined in this respect, it will be seen that the three angles are equivalent to two right angles. For if we turn, beginning with A B, till we get A C from A B; then C B from A C; then A B from C B; we have turned through all the angles in turn, and by the manner of turning we have therefore turned through their sum; and we have arrived at the line from which we started. But if a line be turned till its last direction coincides with the first it has been turned through two right angles.[1]



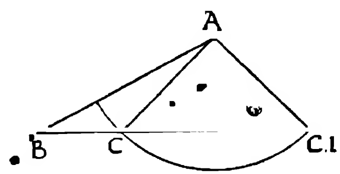
Next let us examine the case where two angles are known and one side. This is equivalent to all the

[1] If there be any difficulty in following this reasoning it will quickly disappear if we effect the operation with a material line, such as a straight stick, turning as indicated, round A, then round C, then round B (cf. pp. 44 *et seq.*, p. 157, and pp. 158 *et seq.*).

angles being known, and the scale of the triangle being given. The triangle is determined.

Next let us examine where two sides are given and one angle. If the angle be the included angle, we have only to carry out the construction indicated by the conditions to see that the triangle is determined.

If the angle be not the included angle, it will be opposite to one side and contiguous with the other. If the side A B be given we may take the other given side as A C, and the given angle as A B C, which implies that the direction of B C is given. The apex, or third point of the triangle, must therefore be somewhere on the line B C.



If now with centre A, and radius equal to A C, we describe a circle, the apex must necessarily also be somewhere on this side. The apex will therefore be where the line B C meets the circle.

We assume that a line that cuts a circle cuts it in two points. Therefore, generally, in this case we shall have two triangles A B C and A B C_1, fulfilling the necessary conditions.

If the line B C meets, and is a tangent to, the circle there will be only one triangle.

Now reviewing all the cases considered we find that when we have any three independent elements, sides and angles, given the triangle is determined; except in the case when two triangles fulfil the conditions.

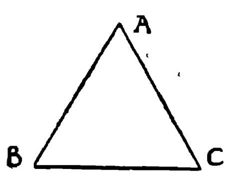
But with what we have here established we find that the propositions of Euclid referring to triangles are easily proved.

The first proposition of the kind we meet with is the

4th proposition of the first book. It is simply the first case (p. 177).

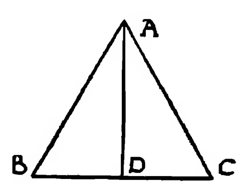
The 5th proposition of Euclid is famous as the *pons asinorum*—the bridge of asses—because it has proved a great stumbling-block to beginners.

We will examine it briefly, not by Euclid's method, which is unnecessarily cumbrous, but by the light of the preceding discussion. The proposition is, in brief, that if the side A B be equal to the side A C, the angle A B C is equal to the angle A C B.



If we had begun by defining "symmetry" in mathematical combinations, and pointing out certain consequences which easily flow from its consideration, we should have said at once that the truth of the proposition was apparent by reason of the symmetry of the figure. Such a form of proof is perhaps the best, as it affords the best mathematical discipline to the mind.

But we may proceed thus. If we suppose a line, at first coincident in direction with A B, to turn round A towards A C, then as the magnitude of the angle it makes with A B begins at zero and increases gradually until it is equal to that of the angle B A C, it is evident that there is a direction A D where the angle B A D is equal to the angle C A D.

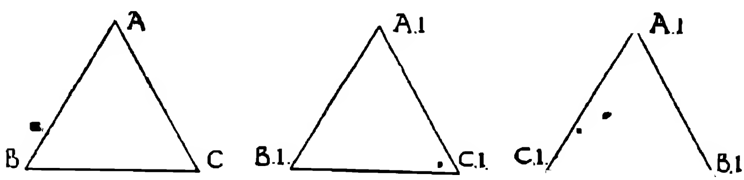


Here then we have the conditions of the 4th proposition. The triangle C A D differs from B A D simply by change of situation. The angle at C is therefore equal to that at B.

Here also, if we had discussed symmetry, we might, after establishing that an angle B A D exists which is

equal to C A D, have said that by symmetry the angle at C is equal to the angle at B.

Or without any construction we might have said that the proposition is simply a case of the 4th proposition. If this be not immediately clear, suppose another triangle $A_1B_1C_1$ to represent it, simply changed in situation; and that then we change the situation of $A_1B_1C_1$ simply by turning it over as shown. This leaves the angle

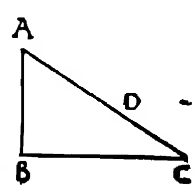


at A_1 which is equal to that at A unchanged. Then the angle C_1 (consider the first and third figures) is by the 4th proposition equal to triangle B. But C_1 is simply what C had become owing to the triangle first changing its situation. Therefore the angle C is equal to the angle B.

It is not needful here to trace out the course of development of Euclid's propositions, but it may be interesting to show an analysis by which the proof of the 47th proposition may be reached by a course quite different from that of Pythagoras.

Suppose we set ourselves the task of investigating the problem by a method involving at each step the mental effort which is the most direct and which depends least on ingenious devices.

Draw a line A B. Suppose B C to be at right angles to it. Then if from A we draw a line to any point on B C we have a right-angled triangle.



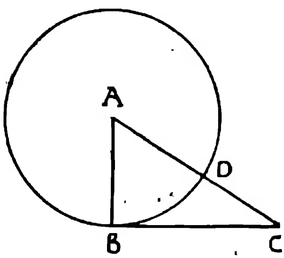
If we imagine a point, C, to move

along the line B C, beginning at the point B; then when C coincides with B the triangle disappears; the side opposite the right angle, the hypothenuse, coincides with A B; the horizontal side becomes zero. So that in this "limiting" position the proposition is seen to be consistent with the conditions.

It is often useful to examine limiting positions.

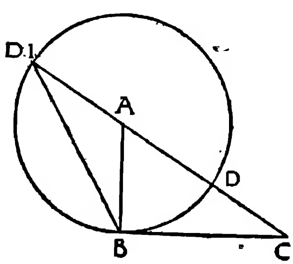
Now suppose the point C to move along the horizontal line. A B remains constant. The side B C now appears. So that we have to prove that the square of the side B C is equal to the increase of the square of the hypothenuse due to its having assumed the position A C as compared with A B.

We find the increase of the side A C itself by describing a circle with centre A and radius A B, thus A D is equal to A B.



We require to prove that the excess of the square A C over the square of A D is equal to the square of B C.

But the terms in which this is expressed suggest a proposition with which we have become previously acquainted (cf. p. 161).



If we complete the figure as shown, the proposition proves that the square of A C is equal to the square of A D together with the rectangle formed by D_1 C and D C.

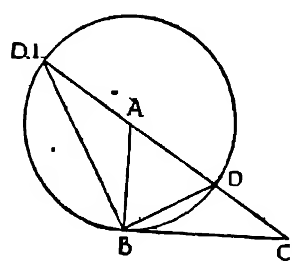
The proposition itself, we have seen, is easily proved by introducing algebraical forms in the study of geometrical problems. It is proved by Euclid independently of any assumption of the Pytha-

gorean proposition. Then we may make use of it here.

The excess of the square of A C over the square of A D, or its equivalent the square of A B, is therefore the rectangle formed by $D_1 C$ and D C. We have now therefore to prove that this rectangle is equal to the square of B C.

Let us for clearness of examination follow the principle already expounded and express the matter symbolically. We require to prove $D_1 C . D C = (B C)^2$. Or again, following a former hint (cf. pp. 136, 161), we may express it: We require to prove $\frac{D_1 C}{B C} = \frac{B C}{D C}$; that is to say, that the ratio of $D_1 C$ to B C is equal to the ratio of B C to D C.

This we shall be able to do if we prove that the triangles, D_1 C B and B C D differ only in scale, $D_1 C$ and B C being corresponding sides.



We prove this by showing that the corresponding angles are equal. We have the angle at C common to both. Therefore it is necessary only to prove that the angle D B C is equal to the angle B D_1 C; for the sum of the three angles of any triangle being equal to two right angles, if two angles in one triangle are equal respectively to two angles of another, the third angle in one will be equal to the third angle of the other.

Now the angle D B C if added to the angle A B D makes up a right angle. And the angle A B D is equal to the angle A D B, by the 5th proposition of Euclid (cf. p. 178). Therefore the angle B D_1 C if added to the angle A D B should make up a right angle. But if the angle D_1 B D be added to these two angles the sum is

two right angles, because they are the three angles of the triangle B D_1 D. Therefore we establish our proposition if we prove that the angle D_1 B D is a right angle, for then the other two make up a right angle.

Now by the 5th proposition of Euclid (cf. p. 178) the angle A B D_1 is equal to the angle A D_1 B, and the angle A B D is equal to the angle A D B. Therefore the sum of the angles A B D and A B D_1, that is to say the angle D_1 B D is equal to the sum of the angles B D_1 D and B D D_1. But the sum of these three angles is two right angles. Therefore the angle D_1 B D is a right angle.

We have here an analytical exposition of the Pythagorean problem. We have now only to retrace the steps of this analysis in order to present the proof synthetically. But so as not to interrupt the progress of the demonstration, we would begin by establishing certain propositions that we have required to state in the course of our analysis.

That relating to the "scale" of the triangles, or otherwise expressed, the proposition that in similar triangles, or triangles having their respective angles equal, the sides corresponding are proportional, may be proved in various ways.

It is easy also to prove that the line B C is a tangent to the circle.

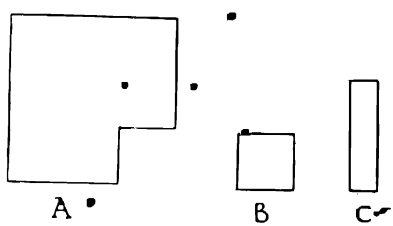
The proof here set forth is not short, but comparatively it is not long; for the demonstration of the 47th proposition as given in the first book of Euclid demands the establishment of previous propositions; and, as we have remarked, the analysis of the demonstration down to the basis of the first principles set forth in Euclid was one of the important factors in the foundation of that system of geometry.

We have now indicated how a system of geometry might have been built up on different conceptions and by a different course of demonstration. In the examination

of this course we have found no new Fundamental Process.

The second book of Euclid deals with equalities of squares and rectangles of which the sides are parts of straight lines divided in various ways. An examination will easily convince the student that no new Fundamental Process is involved.

Yet there are certain assumptions made that are less immediate than those of the axioms. For example, we meet in the course of the demonstrations with such virtual postulates as the following: that if B be a square one of whose sides is equivalent to a side of the square cut from A, then B placed in this situation will complete the square A.



This, it may be said, is obvious, but it implies an assumption already discussed that B does not change with change of situation. It implies also the notion of the division of a space, which is certainly not a notion offered in Immediate Presentation. Just as we found it convenient to use the term "abstraction" for a certain combination of Fundamental Processes, so we might refer to the process here indicated as that of Space-Association.

If now we proceed to speak of the equality of the figure B and the figure C, it is evident that we make a further assumption, for the two figures do not fit into each other. The only manner in which we can suppose

such equality is to form the idea of both B and C being divided into small areas which taken together complete the figures. Thus we have Space-Association, together with all the ideas involved in division and in counting or summation (cf. pp. 106 *et seq.*).

Here we are so far from simple equality, that we require even to form an abstraction, and to refer to the equality of areas. The operations involved are more complex than those which form the base of the system of Cartesian co-ordinates (cf. pp. 124–125).

The whole of the operations may be referred to as Space-measurement.

The third book of Euclid deals with circles and the relations between various lines defined in relation to them, and the areas of squares and rectangles formed on various divisions of such lines. The demonstration we have just discussed of the Pythagorean proposition has introduced us to problems such as are dealt with by Euclid in the third book.

The other books of Euclid deal with questions of proportionate relations of magnitude, all of which are much more easily discussed by means of algebraical forms. In none of these are new Fundamental Processes brought into evidence.

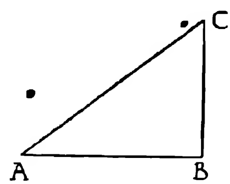
It is no part of the intention of this book to discuss the development of mathematics in detail, but as mathematics is the science of close reasoning *par excellence*, we must find some means of assuring ourselves that we have covered the whole field when we affirm that even in the discoveries of the higher mathematics we evoke no new and possibly hitherto dormant faculty or Fundamental Process. For if in this way we deal with mathematics, we shall find that the examination of the fields of the other sciences presents no difficulty. And if this be ascertained, then we may be satisfied that in the whole field of human

knowledge we can conduct all our operations by the aid of the Fundamental Processes we have set forth.

We observed in discussing the triangle that the triangle is determined, in all cases but one, when we know three independent elements of its construction, and that in that case we have a choice of two triangles which fulfil the conditions (cf. p. 177). From this it becomes easily suggested that the other elements may be calculated from any three.

Trigonometry proposes in the first place to establish the principles of such calculation.

Suppose then we study the problem from this point of view. According to the process of development which we have hitherto frequently seen in operation, it is better to begin with a figure which promises the simplest formula. We may easily take this to be the right angle.



Now as the angles are unchanged by change of scale of the figure, we would naturally expect that in the calculation we might meet with certain expressions regarding the angles, which are independent of scale. If we have, for example, the sides given as h, the hypothenuse; a, the side opposite the angle A; and c the side opposite the angle C; we would expect in our calculations of the angles to have to deal with ratios of the sides such as $\frac{a}{h}$, instead of other relations obviously more difficult to define.

Suppose then we take all such ratios, we have: $\frac{a}{h}$; $\frac{c}{h}$; $\frac{a}{c}$; and the inverse of these.

To these ratios, as they frequently occur, we give names, thus:

$$\text{Sine of A, written } \sin A = \frac{a}{h}$$

$$\text{Cosine of A, ,, } \cos A = \frac{c}{h}$$

$$\text{Tangent of A, ,, } \tan A = \frac{a}{c}$$

Now from the formulæ it follows that $\tan A = \frac{\sin A}{\cos A}$; also $\sin A = \cos C$.

Therefore calling the right angle $\frac{\pi}{2}$, we have $\sin A = \cos\left(\frac{\pi}{2} - A\right)$; or the sine of an angle is equal to the cosine of its complement, the complement being the angle which must be added to the angle considered in order to make up a right angle.

It is evident also that—

$$\cos A = \sin\left(\frac{\pi}{2} - A\right)$$

Also, by the Pythagorean proposition—

$$a^2 + c^2 = h^2$$

If we divide every term of this equation by h^2, we obtain—

$$\frac{a^2}{h^2} + \frac{c^2}{h^2} = 1;$$

or,
$$\sin^2 A + \cos^2 A = 1$$

This last result may be expressed thus:

$$1 - \sin^2 A = \cos^2 A;$$

or,
$$1 - \cos^2 A = \sin^2 A.$$

We have already reached the possibility of establishing numerous unknown relations, if certain other relations are known.

Thus if we know all the sides we have $\sin A = \frac{a}{h}$; and from this we know sin A.

The process by which the angle A is determined, when sin A is known, is shown in a highly interesting analysis in special works of mathematics, the analysis having been developed by the labours of many great mathematicians, of whom Euler was one of the most illustrious.

Ordinarily, however, it is not requisite for practical purposes to be acquainted with this analysis. Tables have been prepared from which corresponding to any value of sin A the angle A is given. Hence as above we find the angle A. Then $C = \frac{\pi}{2} - A$ is at once determined, for $\frac{\pi}{2}$, expressed in degrees, equals 90°.

For example, if A be 40°, then $C = 90^\circ - 40^\circ = 50^\circ$. It will be seen that since $a^2 + c^2 = h^2$, we obtain all the sides when any two of them are known.

Again, if either of the angles A or C be given; then since the angle H is a right angle, all the angles become known. If then any one of the sides be given, it is easy to apply some one of the formulæ given.

Thus if h be given, we have $\sin A = \frac{a}{h}$. Therefore $a = h \sin A$; a becomes known.

The area of the triangle is half the area of a parallelogram of which a and c are the sides.

This may be easily seen, by symmetry, if the parallelogram be drawn. Therefore the area $= \frac{ac}{2}$.

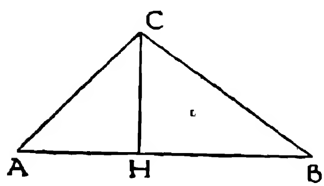
Let us take another step and study a triangle, A C B, which is not right-angled.

Since we have learnt to deal with right-angled triangles, it would soon be suggested to form relations by means of right-angled triangles. Thus if C H be

drawn at right angles to A B, we have two right-angled triangles.

Suppose now that we know A B, C B, and the angle B, and we require to find the length of A C.

In the right-angled triangle C H B we know one side and one acute angle, therefore all the angles, since the right angle is also given; therefore we may determine all the other elements. Thus C H becomes known; and H B known. And as A B is supposed known, and H B is now known, therefore A H becomes known.



And $\overline{AC}^2 = \overline{AH}^2 + \overline{CH}^2$; and therefore becomes known. Also all the elements of the triangle A C H become known. Therefore, finally, all the elements of the triangle A C B become known.

The development of trigonometry introduces us to many problems, such, for example, as obtaining sin (A + B) when sin A and sin B are known; but into these developments it is not necessary to enter.

Enough has been shown to indicate the general nature of its problems and of its methods. The uses of trigonometry are obvious; for example, it is readily to be seen how from the measuring of a selected base line, and the observation of two angles at its extremities, it is possible to ascertain the distance of an inaccessible object.

Spherical trigonometry differs from the plane trigonometry we have considered in that the lines are supposed drawn on the surface of a sphere.

The spirit of its development is not dissimilar to that of plane trigonometry. Spherical trigonometry forms one of the foundations for the science of astronomy. It

is observed, on examination, that no new Fundamental Process is here involved.

If now in order to deal in a general way with a vast subject, we seek for some highly general principle of classification, we may observe that all the questions of mathematics hitherto discussed involve either groupings of series of numbers (the basis of the science being counting); or the relations between spatial objects, such as lines and areas (the basis of the science being Space-Association) (cf. p. 183).

A vast and highly interesting field of mathematics has been developed in the latter of these classes with but little recourse even to simple forms of arithmetic. But we have already seen that problems that belong originally to that class soon, by virtue of Space measurement, become capable of being dealt with by the methods of the first class.

In this respect we find some of the best applications of the Differential Calculus, and the groundwork of its wonderful development.[1]

Let us again search for some deep trenchant division in order to find a manner of survey of the range of objective science. It will have appeared to students who

[1] The Differential Calculus has branched forth in many subjects which form absorbing special studies; the science of Determinants shows modes of dealing with certain complicated algebraical series and groupings, so that it is, as Sylvester called it, "algebra upon algebra"; Laplace, Lagrange, Cauchy, Hamilton, and Grassmann have in various ways built up a Calculus where operations themselves are combined, with certain conditions, after the manner of algebraical magnitudes; the studies of Abel and Galois into the nature of equations have a profoundly psychological caste; the researches of Abel and Jacobi in elliptic functions demanded imaginative brilliancy; the studies of Gauss and Riemann with regard to the hypothesis of geometrical science are amongst the most abstruse but also the most productive in the whole range of philosophy; the later development of mathematics in various directions, in the hands of Plücker, Klein, Sophus Lie, Darboux, Picard, and Poincaré, has given us an instrument which, as Darwin had already remarked, seems almost to bestow another sense; yet in all this we have nothing beyond the continued application of principles we have already discussed, except possibly in regard to "imaginaries" and the problem of infinity which we have reserved for discussion.

have pondered over the question, and who have pursued their analysis persistently, that the tendency of sciences such as chemistry, electricity, heat, light, is more and more to seek explanations which finally resolve to matters of mechanics.

Newton, in his "Principia," laid the foundation of our modern treatment of mechanics.

In the "Principia" he sought to base his exposition, as formerly Euclid in his geometry, on the fewest possible assumptions.

The science of mechanics consists in the study of material objects with regard to forces and motions. The application of this science is to the whole objective world considered in this respect. In the application appeal must be made to experience, but the experience is reduced to its most direct and simplest expression. The science is the *schema*, which with our experience supplying the concrete representation, applies to all the external world viewed with regard to forces and motions.

Now a deep division separates the animate from the inanimate world. How do things comport themselves in the animate world in especial?

The tendency of all modern sciences such as physiology or biology is, again, to seek ultimately physical explanations as far as possible.

The science of physiology, for example, is compounded of chemistry, mechanics, and the complete study of anatomy with all that it should include; and the action of low forms of life, either forming part of our bodies and controlled by an elaborate nervous system, or simply within our bodies.

Apart from the world of sensation, ideation, thought, which we have expressly placed beyond these spheres, most modern physiologists seek to find ultimately in vital action nothing more than what we might include

n the most general sense amongst mechanical causes. Thus apart from phenomena purely mental we may adopt a conclusion expressed by Kirchoff, who declared: There is only one science: mechanics.

In considering this expression a certain allowance must be made for aphorismal form. In this respect it is comparable to the philosophic epigram of Buffon: There is only one animal.

Kirchoff did not mean to imply that all science could be expounded from a basis of mechanics, for after all we have not yet reached that point with regard to certain sciences in which the truth of the aphorism is most apparent, chemistry, or even electricity.

Kirchoff simply asserted the mechanical action of the elements whose combinations give us the varied phenomena which constitute the field of the sciences. The limitations of the application of the principles of mechanics are simply the limitations of our knowledge and of our powers of mathematical expression.[1]

[1] I have met with suggested mathematical expositions of animate nature in quite unexpected quarters. Sir Ronald Ross, in a recent work, "The Prevention of Malaria" (1911), writes: "As a matter of fact, all epidemiology, concerned as it is with the variation of disease from time to time or from place to place, *must* be considered mathematically, however many variables are implicated, if it is to be considered scientifically at all."

Here, however, we are far indeed from the direct application of such an aphorism as that of Kirchoff, which indicates something practically impossible at our present degree of mental development and in the present condition of our command of the Forces of Nature. The only way in which it seems feasible to apply mathematics to the study of diseases is by means of the science of statistics. Karl Pearson has distinguished himself in such researches. Other notable names are Farr, Brownlee, Goring, Greenwood.

A suggestion for the application of mathematics in a region still more unpromising is that of Prof. Haret, of Bucharest. The title of his book, "Mécanique Sociale," already implies something formidable. He deals with questions of sociology by means of applied methods of statics and dynamics, and we meet with terms such as "social space," "social force," "social motion," "social rest," "social vector." A writer in *Nature* (G. U. Y.) has, however, pointed out that a system of Cartesian co-ordinates cannot be so adapted: "The three axes chosen do not represent qualities of the same kind or dimensions."

The theoretical dictum of Kirchoff must be conceived of as becoming realisable at a basis far more profound.

In regarding the whole field of biology, with its various subsidiary sciences, we behold many problems which are those of the adaptation of the organism to its environment, that is to say, the adaptation to physical conditions by virtue of the interplay of instincts, feelings, emotions, thoughts; that is to say, finally, we see all the factors included under the classification of mechanics and psychical phenomena.

But when we pursue our inquiries into the mysterious questions of embryology, of cell division and multiplication, into origins, who can say and offer proof that we have nothing here but mechanics?

The solution of such a question would not affect our present view. In studying even such a science as bacteriology we do not require to pierce to the ultimate nature of organisms. We deal with bacteria as substantial living things, and the difference of our study of bacteria and that of worms, for example, is due simply to the fact of the bacteria being so small that special means are necessary for observing their operations. Often we know them only by the results they produce, their appearance in "colonies," their facility of cultivation on various media, their vitality at different temperatures, their gas-forming properties, or their capacity of living without oxygen; all of which properties may be determined without a single organism having been defined under the microscope.

Thus throughout the whole range of the organic sciences our knowledge is built up only on what we can know by experience, and of what we can apply from other sciences. It will be seen, then, that the study of these sciences can introduce no new Fundamental Process.

All the other objective sciences, that is to say, those dealing with inanimate nature, we may include under the classification: mechanical.

Here we have observation of material things, rearrangements for experiments, all that we may in short sum up as experience; and the application of mathematical science.

The science of mathematics we have therefore discussed minutely in its basal operations, and we have examined the principles of its development.

The science of mental phenomena we have examined step by step in that discussion, in as far as they became exemplified, and in the whole course of the exposition we will keep observation on these phenomena.

The way is now cleared for entering upon the discussion of problems, involving the infinitely great or the infinitely small, which we had previously reserved.

CHAPTER VIII

PROBLEMS OF INFINITY

THE question of motion may now be considered. The discussion will be found to involve that of infinity.

We know of motion only by the change of position of something, A, which having been at one moment in position a, is discovered at another moment in a position, a′. Thus we find in the consideration of Motion conceptions of Time and Space associated with those of the moving object in a peculiar way. The conception of Motion therefore is not fundamental in the same way as those of Time and Space.

The old problem of Achilles and the tortoise is most instructive, for far from being worth only a smile it poses a problem which demands the subtlest delicacy of thought. The tortoise has, let us say, 10 yards start, but Achilles runs ten times as fast. The old sophists asserted that it was impossible to think that Achilles could catch the tortoise, for when he reached the distance of 10 yards, the tortoise would be 1 yard ahead, and when Achilles reached that spot, the tortoise would be one-tenth of a yard ahead; and so on *ad infinitum.*

If one follows closely this kind of reasoning, and keeps within the limit of a conception of motion formed as of a continuous progression, and if one must, as the sophists did, tacitly assume that this continuous progression can be described by means of small, even

infinitesimal intervals of Space and Time, having certain relations determined by the conditions under which the problem is offered for study; then there is no escape from the conclusion: Achilles cannot be thought of as overtaking the tortoise.

In reference to the conditions offered by the sophists, it must be remarked that as the interval between successive points becomes diminished, and the time diminished accordingly, the time is diminished at a rate such as precludes Achilles overtaking the tortoise within any interval considered. Thus suppose that Achilles runs at the rate of 10 yards a second, and that the tortoise has a start of 10 yards. We next consider the situation at the moment when Achilles has covered the 10 yards interval. One second has elapsed, but this is too short a time to consider in view of the start the tortoise had. Similarly when Achilles has travelled another yard, we have to consider the relative positions after the lapse of one-tenth of a second from the previous moment. The interval of Space between Achilles and the tortoise has been lessened, but the Time necessary to produce the next positions which we are asked to consider is again too short. And so on, as Achilles approaches the tortoise correspondingly smaller periods of Time are offered for our consideration. The solution of the problem is impossible on these terms.

But suppose Motion to be effected in this way: An object reaches a position a, then is suddenly annihilated. It is restored to its entity in the position a′. The fundamental conception of Space is here involved, and also that of Time, but it is in their primordial aspects. a′ may be any distance we please from a, and the time required for the operation may be anything we please.

We have thus a new conception of the motion of a body from a to a′, and if the limits of Space and Time

involved in the operation be such that they do not conform to the notion of the infinite subdivision supposed by the sophists, then we should have disposed of the difficulty of Achilles and the tortoise.

Certainly we meet with a new difficulty, for we have evaded the conception of *continuity*, that is to say, continuity beyond even the limits of our infinitesimal concepts. But that is the crux of the whole question. Our minds cannot conceive of continuity in that sense. The mental operation involved in the conception of Motion is the same as if the objective process were, as described, a disappearance and reappearance of the object.

It is true that we do not commonly think that the object so disappears; but we are aided in our conceptions by the limitations of our visual organs. Thus we know from physical and physiological sciences that an impression lasts on the retina one-tenth of a second, and that the same period is necessary for a clear impression to be formed. So that if in verity the object did disappear for an extremely small part of a second and reappear, our visual organs could tell us nothing of that event. Thus in the familiar experiment of whirling a lighted stick in a circle we know that the lighted part does disappear from one position to reappear at another, but our direct visual sense would indicate that the lighted part occupied the whole circle and that it remained there continuously.

It is not here contended that the object actually ceases to exist from time to time, but that whatever be the condition of the objective reality, the conception of Motion cannot be formed otherwise than as that of a series of appearances at points at intervals from preceding points. Thus the answer to the sophist is that apart from the question of the decreasing time limit already discussed, he has no right to assume that Motion is ultimately continuous; further, the mind is not capable of representing

this hypothetical continuity, by means of infinitesimal divisions. The sophists, in short, ask us at any moment to consider a finite image, whereas the division of time imposed by the conditions must be infinite; and moreover they assume continuity which cannot be represented even by infinite division, in so far as what we call infinity, in this respect, is a veritable concept of the mind.

Let us look at the matter from another point of view. It has in recent years been determined that the old problem of squaring the circle is insoluble.[1] In other words, circular motion cannot be represented by a series of small rectilinear motions. With this falls our conception of tangents, otherwise than by approximation. And thus the basis of our mathematics, as far as applied to these subjects, is tentative, that is to say, it marks the limit of our notion of things in our endeavour to conceive their ultimate reality.

We seem thus to be brought face to face with Kant's Noumena, or things-in-themselves. But if our tentative efforts mark the limit of our powers of conception, it would seem that likewise we can never know the meaning of things-in-themselves.

As the question of Motion and that of Infinity which arises from it are amongst the most obscure in the whole range of philosophy, it is well to seek light from any quarter that promises to offer a solution, and to re-examine even at the risk of repetition and insistence. Let us make the problem of Achilles and the tortoise simpler by taking it to assert that whether Achilles be pursuing the tortoise or not, it is impossible for him, setting out from a point A, to arrive at a point B.

[1] By Lindemann: cf. Klein's Lectures at the University of Evanstown; cf. also "Œuvres de Hermite." The proof is valid only in as far as it assumes that the problem can be approached by the method of the infinitesimal calculus, and not by any new algorithm, if such there be possible, free from the difficulties that here arise.

This is really the paradox of the old sophist who denied that there was such a thing as motion. A body must at each instant of time, he said, occupy the space where it is, but if it occupy the space where it is, it is at rest in that space.

All these paradoxes will be seen to depend on the meaning given to such words as instant, moment, point, infinity, or infinitely small.

Diogenes refuted the sophist[1] by getting up and walking. This was, it seems to me, a perfectly valid refutation, and not one that depended merely on superficial appearances. To make my meaning clearer, this may be contrasted with an apparent refutation of a statement. A man asserts, for example, that an object cannot be in two places at once. Another may whirl a lighted stick round his head and profess to show him the object in several places at once. Now what is true here is that there is a phenomenon occupying a certain space at a given moment, but it is not admissible to assume that that phenomenon is the lighted stick.

But in the case of motion, when motion may be defined as the change of situation of one body in relation to another, and where it is sought to deny fundamentally that there can be motion, it is sufficient in any one instance to exhibit a change of situation of two bodies.

The sophist is thus answered, for the appeal is made to tests as fundamental as those which he advances; but we may pursue the question further and ask what was the origin of the sophist's paradox.

Suppose then we consider Achilles proceeding from A to B, and let us suppose the distance from A to B subdivided into intervals, I, II, III, etc.

Now if we consider Achilles traversing interval I,

[1] This paradox appears to have been long in honour among the sophists from Parmenides to Xeno the Eleatic, who is said to have propounded it to Diogenes.

then since no one has contended that a body can be in two places at once, Achilles occupies an appreciable time in passing from the beginning to the end of interval I. Similarly with interval II; and so on; so that if his rate of progress were uniform, and the spaces equal, then to arrive at the end of interval XX, he would require XX times the period of time required to traverse interval I; and to arrive at the end of interval N, when N is any number however great, he would require N times the time spent on interval I.

But now the divisions of the entire distance A to B may be as small as we please, and even when Achilles had traversed N of these intervals, we might consider that his progress was limited to confines as small as we pleased. And this would also be the case when N was made as great as we pleased, greater than any assigned number; we have only to make the intervals of the original distance small enough in proportion. If then instead of merely employing an empiric dogma, as of the Common Sense school, we try to obtain a clear vision of Achilles' progress, we fail.

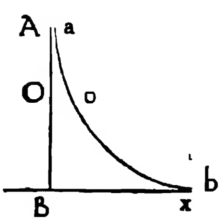
The underlying fallacy of the paradox is that whereas in attempting to visualise the progress of Achilles, we assume an appreciable measure of time employed, and this is brought into stronger relief by the fact that our mental operation occupies a considerable time—yet we assume the division of the distance of A to B as infinitesimal.

If we actually fall short of this and consider the division of the distance to be yet expressible finitely; and if we assume that Achilles actually traverses the first interval, then even though a lifetime may not suffice to consider him traversing each interval in succession, yet we become satisfied that the problem is soluble. And making the calculation that for a certain extremely

small, yet finite interval, we would require a finite though extremely small period of time; we arrive at the common-sense conclusion that in a certain finite, and not extremely small period of time, he will traverse the distance between A and B.

This mode of reasoning fails if the distance be infinitely divided. But in that case the assumption also fails that any appreciable and therefore finite period of time is occupied in traversing any one of the intervals. We are unable to form an appreciation of such an interval at all.

Before asking, however, what is the meaning of infinity, or in what way we can form associations with this notion, it may be well to consider certain other cases.



Suppose we have a certain horizontal base line B x; and that we have a straight line A B perpendicular to B x, and that further we have a curve a b asymptotic to B x, that is to say, which continuously approaches B x, and which may be continued until it is less than any assigned distance from B x, but which does not, at any finite distance from the point B, meet B x.

Suppose A and a to be at the same height above the base, B x, and suppose two points O and o, to move, one along A B, the other along the asymptotic curve, in such a manner that they are always on the same level; that is to say, so that the height of O above the base is equal to the height of o above the base.

Now as O moves along A B it may be supposed to pass through all the points of the line A B. To each point occupied by O in A B will correspond a point occupied by o in the curve.

Suppose first that O moves with uniform velocity, then as the arm of the curve directed towards b becomes more and more near to the horizontal, the velocity of o will increase, and it can be thus increased above any assigned limit. But the point o is continually approaching the horizontal line B x, and yet it does not reach it; so that the point o may be approaching the line B x continuously, and may be moving with a velocity as great as we please, and yet may not reach B x.

On the other hand, if o moved with uniform velocity we would have the velocity of O continuously lessened, but O would nevertheless be continuously approaching B. In that case we would have a point O moving in the shortest path towards B, and continuously approaching it, yet never arriving at that point.

But reverting to the case of uniform velocity of O along A B, we find that every point in A B, however close to B, will have a corresponding point in the curve. But eventually O arrives at B; where then is o to be found? It cannot be at any finite distance from the line B x, for the point which it occupied would then correspond to a point marking the position of O on A B, distinct from B.

It will also appear that when O reaches B, o cannot be at a finite distance from the point B. In a region of Space therefore beyond any appreciable or conceivable distance from B the point o reaches the line B x.

If we suppose Space, or in this case the line A B, to be indefinitely divisible, then it will be impossible to appreciate, or conceive, the point immediately preceding B, say B_1, for if that were the case we could conceive the line B B_1, and according to hypothesis we could divide that.

This shows that we cannot arrive at a notion either of infinity, or of an infinitely near point, by proceeding

continuously from finity, however great we may make the finite.

But also we cannot, by way of insoluble paradox, ask the question: Is it possible to conceive of infinity? since indeed we actually employ the term. The question that is here appropriate is: What do we mean then by infinity, or in what way do we arrive at the notion? It is evident from what has preceded that we arrive at the notion by a Process of Disassociation, followed by a Process of Association, which is repeated, uniting now new and suitable associations. The Process of Impulse is here called into play. More popularly we should say that we arrived at the notion of infinity by a leap.

Let us consider another curious example cited by the Hon. Bertram Russell in his work on "The Principles of Mathematics." It is that of Tristram Shandy, who took ten years to write the story of the first two days of his life. Supposing that he lived for ever, Mr. Russell says that at that rate he would eventually get all the story of his life written. That is a paradox that requires a little circumspection. On the one hand, it is evident that as his life proceeds he must leave the story of more and more years untold, and by suitably selecting the date we can make the number of years, which at that date will be unrecorded, greater than any assigned limit.

Certainly if we fix attention on any finite period of Tristram's life we shall find it possible to complete the story; but therein the character of the paradox comes to light—the substitution of a finite quantity however great for an indefinitely great or, as we say, infinite quantity.

In this case also we cannot arrive at infinity by continuity from finity. We have here again the series: Disassociation, Impulse, Association. If now we revert to the case of Achilles and the tortoise we say that the paradox is there based on the circumstance that we are

asked to arrive at infinity by a continuity through finity.[1]

The notion of degrees of infinity may be made clear by simple examples. If there be two curves such that the ordinates of one have always the same ratio to the ordinates of the other, then we may suppose the curves extended so that the ordinates become infinite, but the ratio will remain unaltered. Thus we may have $\frac{\infty}{\infty} = k$, where k is some finite quantity.
Similarly we may have $\frac{0}{0} = c$.

In mathematics indeed it is not possible to carry on operations by positing infinity absolutely, or zero absolutely. We cannot multiply by 0 absolutely, except by removing the factor in question; multiplication has no sense in such a case; neither rightly has, of course, addition or subtraction. Hence no mathematical process has the usual sense in such a case.

But we may multiply, say, by a quantity, which may then be understood to vary, and it may vary to infinity, or to zero. In this way we are informed of the manner in which it approaches infinity or zero, and we can deal intelligibly with symbols such as $\frac{\infty}{\infty}$, $\frac{0}{0}$, or $\infty \times 0$.

It must be especially brought to mind that the use of the term "infinity" is merely a convention. It is only gradually and with difficulty that we entertain the notion of infinity. Therefore it is a matter of which the factors depend on experience. But of their synthetic result we can have no experience. No one has ever seen a line infinitely long, no one has ever lived for an infinite

[1] Considerations more difficult, but having a great practical outcome, arise in the study of mathematics, as, for instance, when integrating round a curve enclosing a point where the function becomes infinite. The problem seemed hopeless until Cauchy threw light upon it, and showed how in each case a correction of a certain residue was alone necessary. Herein was made apparent that there are degrees of infinity; and that the degree of infinity is determined by the rate at which the function approaches the critical points.

period. Moreover, it does not follow that because we are able to conceive a complex, or synthesis, of known elements, or to form a symbol of such, that these have any existence in the objective world. We might talk of a million microbes, and we might say that half a million would occupy half the space. And such a conclusion being consistent with regard to the numbers, we might substitute cows for microbes; but it might be impossible for the cows to exist in the space that sufficed for the microbes. Our Processes of Association, Disassociation, Impulse, Generalisation, yielding symbolisation, do not imply the objective reality of the results.

Has this discussion any application to the larger question of infinity? It might well be asked: Have we any right to assume as a reality the indefinite, or infinite, extension of Space? That is a question that has seriously occupied the minds of the greatest and most fruitful of those thinkers whose study of mathematics has led them to deep philosophical speculations, notably Gauss, Riemann, and, in England, Clifford. Clifford propounded the notion of Space being curved. This is not the place to enter into a discussion of what that implies, or in what way he was led to the notion. Moreover, before Clifford, it was expressed by Riemann. Suffice it to say that the fact that such a position has been considered tenable is alone sufficient to disprove that the notion of indefinite extension of Space must be something both fundamental and inevitable. Riemann, moreover, in one of the profoundest studies in the history of philosophy, "The Foundation of Mathematics," seems to question also the reality of the infinite divisibility of Space.

How does the question of infinite divisibility of Space look from the standpoint of objective sciences?

We must beware of preconceptions, or of such arguments as that of the "Sufficient Reason." Suppose

before the discovery of the law of Universal Gravitation philosophers were discussing, as they did, the mutual spatial relations of bodies. They would say that if there were only two bodies they would preserve their distance because there was no Sufficient Reason why they should move closer or further. It was considerations like this that induced Pythagoras, or his disciples, to believe that the planets moved in circles at uniform speed.

From preconceptions arising from like sources, I—and I suppose this is the case with most—was unable to think otherwise than that Space, and hence solids occupying Space, must be capable of infinite division. One says, if a thing have form we can suppose the form divided. And the old fallacy then intervenes of setting up the finite as symbol of the infinite.

But according to the atomic theory matter at least is not infinitely divisible. We come at length to the molecules, which are now understood to be complex systems. If we divide one molecule we do not get homogeneous parts, we get resultants from the break-up of the system. If we divide the molecule of water we get parts of Hydrogen and Oxygen which in no wise resemble water.

In another sphere, if we multiply, or diminish, the stimuli of the senses beyond certain limits we do not get increased or diminished sensations. We may get no sensation at all.

Here it may be objected that we cause the subject to intervene. That is so, but so does the subject intervene in the notion of infinity. That is built up from considerations which if analysed lead down to the base of Fundamental Processes, and these have meaning only relative to a subject.

But it may be urged: Let us come to the pith of the

question of motion: Is it possible to conceive the passage of a point between A and B that does not imply the passage through all the points between A and B? I reply that it is impossible to conceive a point as defined at all, and that we can only employ the term as a convention by virtue of a synthesis of Processes resulting in abstraction.

Waiving, however, that discussion, I say that it should be shown by the propounder of the question that there are between A and B necessarily an infinite, or indefinitely great, number of points.

Finally I say that even if there were, I can conceive that a body might pass from A to B without passing through these points, for the transference might be that it ceases to exist on leaving A, and that it reappears at B. And that is not merely a metaphysical subtlety. I could demonstrate it by a gross, material experiment; for I could electrolyse a drop of water at A, and synthesise the resultant gases into water again at B.[1]

Now the assumption is of metaphysical origin, that any portion of Space, as, for example, a straight line, is built up through continuity of infinitesimally small parts. The onus of proving that should rest with the metaphysicians, for it certainly has no claim to be considered as arising fundamentally, or inevitably, in Psychology. I do not suppose that the usual way for a drop of water to move from A to B is to vanish at A and to be reconstructed at B, but I do believe that the mental operation corresponding to our appreciation

[1] Here it may be objected that the elements pass from A to B. I could overcome that difficulty by a process, which is indeed considered to be the veritable process in electrolysis, that of interchange of molecules with successive drops. It might then be said that the resultant drop at B would not be the same drop as that we first considered at A. To this I reply that if the drop were electrolysed at A, and recompounded from its elements at B by another electric current, it would not be the same drop, and neither would it be the same drop finally if it were simply transported (see pp. 167 *et seq.*).

of that movement involves a break of continuity, or, expressing the matter in its fitting terms, there is found a certain Process of Association in reference to the object and the place A; then there is a Disassociation, and with new elements of Time and Space there is Process of Association again at B.

This explanation may give the impression of a certain imperfection in the working of the mind, but if we say simply limitation, we will find nothing inconsistent with our rational conclusions. We have found already in dealing with the senses that limitation is the condition of their existence, and we have found that our faculty of reason depends on limitations, as, for example, of Discrimination. Were it not for these limitations we would not build our knowledge laboriously step by step, but we would behold at once the conclusions which Reason dimly gives us.[1]

Reason reconstructs as a man deprived of sight reconstructs a landscape which with that sense he would take in at a glance.

Does any problem still underlie all this? Yes. The mind is not satisfied with its own limitations. The problem that still underlies the analysis of infinity is not remote from that of "things-in-themselves."

I have known, for example, a psychologist ask, "How do I know that the red of my sensation is like the red of the table-cover which gives rise to it?"

This question has an aspect of sheer futility. Looking at the matter with the guidance of physical science, we find that the sensation red is one result, and a remote one, of certain activities of matter in the table-cover.

If the Undulatory theory of light be correct, there proceed from these activities regular disturbances of the

[1] We would "see as a God sees," to quote Keats in one of the marvellous flashes of his genius.

ether which reach our bodies, and create therein, and especially in the organs of sight, a series of other disturbances so intricate that we can only guess at their nature; and then comes the sensation red.

But, it may be repeated, the undulations of ether produce on a sensitised plate a photograph. And in a phial containing Hydrogen and Chlorine they produce an "explosion," of which the result is a new substance, Hydrochloric acid, formed by some combination of the elements. If we could imagine an animated photographic instrument it would inquire not what was really the colour of a table-cover, but what was its actinic strength.

We are thus brought again to the question of Kant's Noumena, the things-in-themselves.

But in what way can we make known to ourselves things-in-themselves, except again by their results as they impinge on our organs of sense?

I do not mean here to imply that at any particular stage of knowledge one must cease investigation simply because one is confronted with a paradox; I believe, moreover, that from the standpoint of Idealism, in its most illuminated aspect, it is possible to gain a clear notion of objectivity, and that the essential character of that objectivity is the containment of unknown phenomena and forces some of which may become revealed to our persistent searching. But any results obtained must be capable of enduring the closest scrutiny, and they must be consistent with what we have ascertained to be the necessary conditions of our understanding.

Suppose, however, we were dealing with a problem much more circumstantial, such as that of algebraic equations. We might under certain conditions say that the problem was not completely soluble, because all the necessary data had not been made available to us. The

character of the insolubility would be indicated. That is the method of the great analysts. But if a writer, ignoring these limitations, offers a solution in incomprehensible terms he does not thereby show himself a superior geometer, but simply an incapable analyst. And this is the manner of the Transcendentalists when they speak of things-in-themselves.

Here the examination of infinity has been carried to sources as fundamental as my powers of analysis permit. Is it possible to obtain still clearer lights on this subject? I cannot tell, but I certainly would not desire to see the door closed in that direction.

Whenever we meet with a paradox, we may be assured that, in the end, some way out of that paradox will be found consistent with the nature of the Fundamental Processes of the mind. The assumptions here are that what we call space is capable of infinite extension and of infinite subdivision, and that it is continuous, and that objective reality corresponds to such terms. It is generally assumed that to deny anything of this is absurd.

At one time, when many suggestions arose that seemed to point to the earth being round, it was asserted, as a paradox, that this would involve the conception, if that were possible, of antipodes. But eventually not only did the conception of antipodes seem possible, but that conception, leading to the discovery of universal gravitation, prepared the way for the enlargement of the whole scope of human understanding.

Again, when doubt was thrown on the theory that heat was a substance the paradox arose that then it must be something imponderable, intangible, immaterial; and this seemed inconceivable.[1] Such a notion, however,

[1] I remember that when, as a boy, I first encountered this notion I thought it most wonderful and entrancing, because it seemed to have come from the realm of the unattainable.

was at length seen to correspond to well-ascertained principles of Nature, and here again the removal of a paradox was found prolific of important results.

Similarly, it may be found with regard to the positing of infinity, or of the infinitely small, as ordinarily understood, in spatial relations.

A NOTE ON IMAGINARIES

The problem of imaginaries, so called, in mathematics really does not introduce anything new; that is to say, we can account for the use of this method by the reasonable application of principles already known.

The term Imaginary, and the symbol $\sqrt{-1}$, together with the surprising results in facilitating the operations of the calculus in the hands of great mathematicians, all have tended to give to this subject an air of mystery and of some sort of transcendental achievement.

The use of imaginaries has been the subject of many memoirs, and I do not think that the theme has been yet exhausted.

It would lead too far from the present intention to discuss in detail the explanations that have been offered; suffice it to say that, in mathematics, the question resolves itself into a matter of convention in the designation of operations.

Briefly the paradox arises thus. The signs + and − in mathematics signify addition and subtraction. It is also found that if we multiply composed quantities, as, for example, $a-b$ by $c-d$, then we must give to the product

of $-b$ and $-d$ the sign $+$, in order that the correct result may be obtained by the convenient rule of multiplying each term of the multiplier by each term of the multiplicand, and setting the terms so obtained in order with the appropriate sign. Now if we square a quantity, whether its sign be $+$ or $-$, as $(+a) \times (+a)$, or $(-a) \times (-a)$, we must accordingly have a result $+a^2$ in each case.

And as the symbol $\sqrt{}$, for example in $\sqrt{b}$, implies ascertaining the quantity of which the square is the quantity affected by the symbol $\sqrt{}$,—in the case given, b,—then it is evident that such an operation $\sqrt{-b}$ can have no meaning, according to our previous principles.

But in the form to which a reduction is permitted, $\sqrt{-1}$, we find great advantage in employing the symbol. Now it is not reasonable to say that we have made mathematics perform an impossible operation and obtain true results. The question must be posed thus: What interpretation can be given to the symbol $\sqrt{-1}$ so that the operation shall be consistent with the principles of mathematics? When the matter is so stated the paradox disappears.

Thus suppose we have a horizontal vector, or line of definite length and definite direction, considered as directed to the right hand. This we might call the positive direction, and mark by the sign $+$, and if the length were taken as the unit we might indicate the vector thus, $+1$; a similar vector, but oppositely directed, might be marked by the sign -1. Now we might interpret a vector of equal length, but marked by the sign $\sqrt{-1}$, as a vector which has been obtained from the first vector by rotating it, the rotation being such that, if the operation be repeated, we obtain the vector marked by the sign -1. The condition would be fulfilled by a vertical vector.

This, which is the interpretation usually given in mathematical text-books, is not the only reasonable interpretation. The subject soon becomes technical, but those who are interested may consult the writings of Maurey, Argand, Cauchy, Hamilton, Bellavitis; and these will lead the way to an extensive consideration of the whole matter.[1]

What is here important to note is that we do not find any operation in mathematics that cannot be explained on the basis of those already taken into account, and hence on the basis of our Fundamental Processes.

[1] I do not think that even in the writings of these celebrated mathematicians the last word has been said on this subject. On the contrary, it will be found necessary for the complete understanding of the problem to re-examine it in the light of the analysis of the operations of multiplication that has been already given. It will then be found that the usual explanation, as just indicated, and the suggestions of the authorities cited, refer only to special cases of the application of general principles, and that these special cases are not the most obvious, nor the most consistent, nor in all circumstances the most fruitful of results.

CONCLUSIONS FROM STUDY OF MATHEMATICS

We have now reached a point from which the ground covered may be surveyed, and certain results indicated.

In regard to the inquiry into numerical mathematical operations, reserving the question of infinity, we showed in what way the system of mathematics depended on multiplication, addition, and thence on counting in conjunction with the Fundamental Processes, and thus finally entirely on these Fundamental Processes; we have now examined infinity, and shown how its use in mathematics is only explicable on the basis of these Fundamental Processes; and again all the mathematics of spatial relations are the combinations of mathematics of numbers and the representations of space, which in ultimate analysis are also expressed in terms of the Fundamental Processes.

Hence the Fundamental Processes are necessary and sufficient to explain all the operations of mathematics whatsoever. But all things that may be considered in any way at all are capable of numeration; and, apart from their quantitative relations, they have only qualitative relations such as can be made known to us by the medium of the senses. And all our knowledge of the combinations of such qualitative relations have been shown to depend on the Fundamental Processes.

Accordingly these Fundamental Processes are necessary and sufficient for the interpretation, as far as it is possible to our minds, of the external world.

But in a previous part of the exposition we made the division between what we may indicate as the External or Objective, and the Internal or Subjective, provinces (see p. 24 and pp. 74, *et seq.*). We have also seen in

what way the Subjective movements or affections of the mind can be analysed to show dependence on the Fundamental Processes.

Therefore, finally, the problem which we posed as the basis of this work is solved in these terms: That is to say, that the series of our Fundamental Processes is necessary and sufficient for dealing with the whole scope of what comes within human consciousness; or again, that by the combinations of these Fundamental Processes in conjunction with actual experiences we build up the whole containment of our knowledge.

CHAPTER IX

EXAMINATION OF FECHNER'S LAW

THE researches instituted by Weber, extensively carried out by Fechner and his school, and which indeed have mainly formed the foundation of Experimental Psychology, show that the visual sense does not even roughly measure in proportion to the objective quantity, but that as the quantity is increased the perception of illumination fails to indicate an increase to the same amount[1] (cf. pp. 50, 52 *et seq.*).

It should be noted that, as we learn from physiology, change of illumination affects the pupil, which contracts when the illumination is augmented; and the change in the pupil, which is simply an aperture in the curtain formed by the iris, depends on the balancing of action of minute sphincters supplied by branches of the third nerve and of dilator muscles, running radially, innervated by the sympathetic nerves. The reaction of the pupil to light varies with different eyes, and in some forms of disease it is absent. It would be interesting to know in how far subjectively one has appreciation of those processes.

The estimation of Space is much more imperfect by

[1] Weber's Law, or, as it should be called, Fechner's Law, may be graphically expressed thus: that, representing the objective stimuli by abscissæ, or horizontal ordinates, and the subjective estimation by vertical ordinates, the curve uniting the extremities of the vertical ordinates resembles the logarithmic curve. This is understood to be true, however, only in a part of its course.

direct vision than it may be made by other means. Thus, in a fundamental way, it may happen that when two objects a and a′ are brought near together at an interval, depending on the distance from the eye, the condition of the eye itself, and the amount of illumination, the eye may be unable to detect an interval between them.[1]

Notable examples are those of double stars, which to our unaided vision look single, as, for instance, *a* Centauri, one of the pointers of the Southern Cross.

Certain functions of the visual apparatus in regard to Space are better discussed in connection with Externality, but that of magnitude may be here referred to. The perception of difference of magnitude seems to be fundamental, but that of estimation of ratio already introduces complexity. If one thinks, for example, of two lines of different lengths, or two squares of different sizes, it will be found impossible to form a notion of ratio without the application of one object upon the other. And the conception of ratio thus involves that of counting, even if the counting be only 1, 2.

Ratio involves a standard, though, in a particular instance, one of the objects may itself be made the standard. If we adopt another standard, common to

[1] Moreover, the frequencies of ethereal vibrations range from 20 millions of millions per second to 40,000 millions of millions per second, but the extreme visible rays range only between 392 millions of millions per second and 757 millions of millions per second. The wave-lengths of radiant heat, light, and actinic radiations range from $\frac{1}{200}$ cm. to $\frac{1}{1800000}$ cm., but the visible limits are only $\frac{1}{13069}$ cm. and $\frac{1}{25126}$ cm.

In this respect our outlook on the Universe is discovered as very limited, for the eye is the farthest reaching and one of the most delicate of our natural instruments. If, for the purpose of illustration, we borrow a fine passage from Coleridge—

> "And what if all of animated Nature
> Be but organic harps diversely framed,
> That tremble into thought, as o'er them sweeps
> Plastic and vast, one Intellectual breeze,
> At once the soul of each, and God of all?"—

we find that the soul's harps are very incomplete, a few strings suspended in a Universe of myriad chords, and these strings vibrating only to a very limited number of the undulations that reach them.

both, as for instance an inch-measure, then we estimate the ratio between this and the first object, and then between it and the second object. Thus we obtain two countings, which we may symbolise by the final numbers. The establishment of the ratio between the two objects involves the comparison of these two countings symbolised by their numbers.

It must be remarked that there is no absolute standard of size in the subjective operations. Thus, even if two persons measure an object by the same standard, yet the visual impression of that standard is not the same for both. It may be said that we form a conception of the standard by the aid of other senses, as by feeling, and by the Feeling of Effort in muscular activity; but the scale of these also differs in different persons: and moreover the relation of the muscular sense to vision may be different in different persons. So that the subjective estimations of each individual depend on personal standards, not only in any particular sense but in regard to the combinations of impressions formed by the senses.

Questions of ratio enter into the consideration of Fechner's Law. Thus we have seen that the simplest form of comparison, viz. that of one line with another, demands an effort of superposition, with the subsequent operation of counting. But in the effort of superposition we have other elements involved besides those of the direct visual. It is impossible to think of such a superposition without the aid of impressions brought from the motor apparatus, particularly that of the eye. It may be doubted if it is possible to form the notion of such superposition unaided by a previous experience of the like kind.

In what way could the mind form the subjective notion of a double illumination? It may be doubted,

again, if this could be possible without previous experience. The previous experiences are in general inexact, being formed from various degrees of difficulty of seeing objects under various illuminations, not well defined, or by observation of the dilution of shadows, or, in cases of great illumination, by a sense of strain or of pain. Consequently in the tests for Fechner's Law the mind really tries to identify a present state with a badly remembered state, and the law is supposed to take note of the discrepancies involved. But in all this it will be found that there is some physical basis for these speculations, for the discrepancies are always of such nature as to make the subjective impressions indicate a less ratio than the reality.

The matter can be better thought out with regard to the Feeling of Effort in muscular work in supporting weights. Suppose that one holds a pound weight in the hand and pays attention to the muscular sense. Then, at a short interval, one holds two pounds, again paying attention to the muscular sense. Then, if the experiment be repeated, it will be found that the person may estimate much more closely than according to Fechner's Law the correspondence of subjective feelings and objective stimulus. But suppose, then, that some months had elapsed without any experiment of the kind, the estimations would again lack in precision. But the persons selected for the determination of Fechner's Law are not those having any direct experiments to aid them, so that the standards of comparison they possess are derived from vague estimations of the innumerable various objects they have handled.

When the objective stimulus becomes excessive then new factors enter into the estimation, viz., for example, those of fatigue demanding the aid of other parts of the system, as, for instance, a more rapidly beating heart,

bringing also its Feeling of Effort.[1] Moreover, the question of pain adds to the complication. Fechner's Law therefore, it is admitted by its upholders, fails after a certain limit. It is subject to variations depending on individual experience within those limits. Its quantitative estimations are therefore not of great value; but it serves to indicate that ideas of ratio in connection with the senses are dependent on actual experiences.

Another question arises in connection with Fechner's Law. In any act of attention not only the actual sense is involved, but the whole system is affected in correspondence. So that the objective stimulus is not merely the object A, but also the call to attention, B, implying diverse associations. And the subjective response is that partly noted by the sense immediately involved, a, but also by the multiplex impression from the whole constitution of the experimenter, and this may be indicated as b.

Now the general factor involved in the call to attention remains but little altered in increasing, for small amounts, the objective "stimulus." And the correspondence $A + B$ to $a + b$ is not in great discrepancy with that of $2A + B$ to $2a + b$. Thus, in order to indicate the meaning, let us take as the factors $\begin{matrix} A = 10 \\ B = 5 \end{matrix}$, and as the subjective response, $\begin{matrix} a = 10 \\ b = 5 \end{matrix}$.

When A and a are doubled, both B and b will increase in the new conditions say by $B_1 = 2b_1 = 2$. Then $A + B = 15$, $a + b = 15$.

$2A + B + B_1 = 27$, $2a + b + b_1 = 27$; whereas if B and b, and consequently B_1 and b_1, did not enter into the matter, we should have had in the second series $2A = 20$, $2a = 20$, and therefore double on each side; but

[1] Cf. "Change of Heart Rate with Attention," by M. L. Billings and J. F. Shepard (*Psych. Rev.* 1910).

as it is we have 27 and not 30, which is the double of the subjective (a + b).

So that even if the objective stimulus, as applied to a certain sense, be doubled, yet, even in the case where we suppose subjective impressions to measure accurately objective stimuli, the total subjective reaction is not doubled. Herein is indicated one source of fallacy in attempting to apply Fechner's Law to any sense considered separately.

In the case of comparing the length of one line with that of another we are really dealing with the subjective reaction to objective stimuli, but the number of experiences is greater in this series than in any others, and the experiences are generally more precise. Hence we find that, with practice, we can judge fairly accurately that the length of a line is really double the length of another.

And if we carefully noted the subjective effort of the illumination of a single candle, and then the effect of a double illumination, we would eventually be able to form an estimation far more accurate than in the first trials.

Here we really depend on Memory and Association. A certain subjective effort is recalled as being associated with a certain standard. This standard may even be measured by another sense, or it may be a symbol, as, for instance, a number. A certain other effect is recalled as being associated with double the previous standard. And if it be objected that this is not a fair test of Fechner's Law, it may be replied that, from the moment our experiences begin, we cannot get pure conditions for the test of this law.

The conditions are in general much more complicated than those we have discussed. For example, a certain illumination at one time makes far greater effect than

at another. A familiar instance is that of the setting sun, or, it may be, the rising moon. Various explanations have been given of the apparent greater size of their bodies just above the horizon. A certain factor appears in the mechanism of accommodation of the eye. When the moon is high in the heavens there is generally no object intervening between it and the eye, so that the muscles of accommodation are entirely relaxed; but when the moon is just above the horizon the eye sees the objects of the landscape, even near objects. Now in general the apparent size of an object is determined by the angle which it subtends at the eye; or, since the eye does not have direct cognisance of angles subtended, the apparent size is determined by the area illuminated by rays from the object. But this judgment of size is modified by various factors, among which is that of the estimation of distance furnished by the sense of accommodation of the eye. Sensations are, in general, more lively at the occasion of change, and thus, as the eye traverses the space between it and the moon, noting in its movements various objects of the landscape, it has the impression of seeing the moon at the extremity of a long perspective. Hence, for a given angle subtended, the impression of size is greater than if these factors had been absent.

There are other factors involved, and one that seems to me important is this: When the moon is beginning to rise, the general illumination of the atmosphere is small. Accordingly the pupil of the eye is dilated, so that the pencil of rays which converges to the retina is larger, and, as there is always some dispersion, the effect is to increase the impression of size. There is also dispersion of rays from the objects in the perspective and perhaps also in all these cases an "overflow" of stimulation on the retina. This may have a bearing on

the apparent clinging of the disc to the horizon. Moreover, the problem is complicated by the fact that the rays which reach the eyes when the moon is in the horizon are formed in unusual proportion of the rays in the red region of the spectrum.

That the question of general illumination enters into this problem has been made convincing to me by various observations of the size of a small gas-flame as seen at night with no other illumination, and again under conditions when the room was otherwise illuminated.

Various other considerations also enter into the matter, such as the state of fatigue of the observer, diminution of general stimulation; for these affect the contraction of the pupil, the mechanism of accommodation, and also the general receptivity to impression, such receptivity again being modified differently for different kinds of impressions. There is also an effect of comparison, when the moon is high in the sky, with the vast round of the heavenly hemisphere; while, with the moon at the horizon, only a portion of this is seen. Still another consideration arises in quite another order of things. The rays by which we perceive the moon overhead are approximately normal to the spheres formed by the successive layers of the atmosphere of varying density; the rays arriving to us from the rising moon are more nearly tangential. Hence the psychological problem becomes involved in complex physical problems; and questions of refraction and reflection afford a clue to an explanation of part of the phenomenon.

To estimate all the factors that enter into this question would demand a treatment beyond the limits of this book, but the whole discussion has tended to show how circumspect one must be in appreciating such a psychological position as Fechner's Law.

Recently a good deal of attention has been devoted

to this interesting problem of the apparent size of the sun and the moon on the horizon, and with each new study the problem appears more complex. An article of E. Lechalas in the *Révue Philosophique*, 1888, "L'aggrandisement des astres à l'horizon," and another by M. Blyndal, in the same review, "Une Association inséparable," may well be consulted. In *Pflüger's Archives*, 1904, appear two articles, one by A. Lehmann, who advances the theory of irradiation; and the other by R. Mayer, who lays stress on the factors of the long intervening perspective. It is found, however, that if the image of the moon be reflected on a screen at the distance, say, of fifty metres, its size does not seem diminished.

A new theory is advanced by Ed. Claparède in a remarkable paper in the *Archives de Psychologie*, 1906. He examines in turn, and finds insufficient, all former explanations derived from considerations of refraction, pupillary dilatation, relaxation of the crystalline lens, comparison, contrast, direction of vision, overestimation of angles, weakness of peripheral vision, apparent longer perspective. His own view is that the appearance is due less to intellectual perceptions than to the influence of affections. As a proof he points to the fact that, in a picture, the moon looks larger at the horizon than at the zenith, although the angle it subtends may be the same in both cases. But this is not a pure test, for the judgment of the long perspective, for instance, would arise as a factor, and there would be the effect also of memories of former impressions.[1]

[1] J. O. Quantz, who studied the influence of colours on apparent size, found that red seemed to increase size. Götz has written interestingly ("Phil. Stud.," Bd. V. H. 4), "Ueber die scheinbare Grösse Gegenstände und ihre Beziehung zur Grösse der Netzhautbilder." A. Müller, who has recently (*Zsch. f. Sinnaphysiol.*, 1910, and *Archiv. f. d. Ges. Psych.*, 1910) studied the question, has considered

These citations serve to indicate how involved may become the study of phenomena that at first appear relatively simple.

the effect of twilight on the telescope field, and the character of judgment in regard to the heavenly dome.

Some of the experiments described in the literature of this subject are very ingenious. Those of Stroobant and Plateau deserve especial mention. Stroobant found that the image of the moon at 51 metres appeared as large as the moon itself. Plateau found that throwing a light into the eye and so causing a diminution of the pupil made the moon appear smaller. By the use of a mirror at 45 degrees from the vertical he obtained an image of the rising moon as if at the zenith, but it still looked enlarged. Electric sparks seen horizontally appeared larger than when seen vertically. Since writing the above, however, it has occurred to me to obtain accidentally very good conditions of observations. In the train from Paris to Marseilles I observed the moon rising with the usual circumstances of delightful mysterious pomp. The clinging of the disc strongly suggested irradiation or overflow in the retina as an explanation. When the moon rose higher it happened that frequently clumps of trees in the near distance intervened. I tried, therefore, the effect of accommodation by fixing intently these trees. I could detect no certain difference. The "clinging" here was almost, if not entirely, lost. Finally, the moon emerged clear and high, but with little or no apparent alteration of size as compared with its appearance behind the trees. Reviewing all the factors, therefore, I concluded that the most important were really the objective or physical—questions of refraction and reflexion. Hence this psychological problem has conducted us to considerations of Meteorology.

FORMAL EXAMINATION OF FECHNER'S LAW

The plan of this book does not necessarily include a consideration of Fechner's Law, but, as we have been already led to it by diverse routes, it may be well to enter on a more formal examination of the subject.

The problems that Weber set himself were straightforward and relatively simple. A typical example is that of the estimation of weights. He lifted a weight by the use of certain muscles, and then he endeavoured to ascertain the increment of the weight which, in the lifting operation, was just appreciable. He found that within certain limits the increment required was proportionate to the previous weight.

Fechner, who followed Weber, bestowed the name of Weber's Law on the expression of such relations. To this law, however, he gave extensions never contemplated by the original investigator. He applied it to other senses, and he gave it a more technical and recondite appearance.

The whole history of the origin of Experimental Psychology, and of Psycho-Physics, which is but one branch of it, is very curious and interesting. It dates from a period long before Fechner or Weber,[1] but Weber's laborious experiments and the exactitude of his results laid the main foundation of the subject. Fechner was not only a student of Psychology, but also of

[1] The names before Fechner, or contemporaneous but independent, include Bessel, Gauss, Gerling, and Prazmowski; and, in regard to special studies of illumination, Bouguer, Masson, Arago, Foucault, Herschel, and Steinheil; in physiology, Johannes Müller, Dubois-Reymond, Helmholtz, Donders; and, proceeding from Helmholtz, at a later period and up to the present time, Exner, von Auerbach, von Kries, G. F. Lipps, and G. E. Müller. In France, Binet Beaunis, Bourdon, and V. Henri; in Italy, Mosso, Sergi, and de Sarlo are representative names in Experimental Psychology, though where so much brilliant work is being done it is almost invidious to mention even these distinguished names.

Physics, and he was well versed in mathematics. There arose in his mind, therefore, the desire to fuse these studies together, and particularly to dower Psychology with some advantage of that instrument of mathematics of which he had seen the exercise in admirable form in physics.

The process was not facile, but Weber's figures gave a basis of operation. Briefly, his reasoning was of this kind.

Calling the psychic effect, or sensation (Empfindung) E, its increment or difference might be expressed as ΔE; calling the external stimulus (Reiz) R, its increment would be ΔR.

Weber found, within certain limits, facts that might be expressed in this language as:

$$\Delta E = c\frac{\Delta E}{R}$$

c being a constant or quantity that remained unchanged as R and ΔR varied.

In the introduction to the Differential Calculus one finds symbols of this sort; and in the development of the subject the symbols ΔE and ΔR become reduced to dE and dR, the meaning now being that dE is an infinitely small difference of sensation, and dR an infinitely small difference of stimulus.

Fechner, in order to employ these symbols, made two assumptions which may well be criticised. He assumed that the increment of sensation ΔE, when the stimulus and the increment were large, was equivalent to ΔE when these were small. And then he assumed that the law would hold when the increments were infinitely small.

Hence he arrived at the formula:

$$dE = c\frac{dR}{R}$$

But this is a simple differential equation, and the solution is obtained by integrating both sides. Thus we find $E = C \log R + A$, where A is some constant that depends on the conditions of the problem.

The equation may be expressed more elegantly, though without essential change, thus:

$$E = C (\log R - \log a).$$

When $\quad E = O$, then $\log R - \log a = 0$;

or $\quad \log a = \log R.$

This is the condition when no sensations exist, and at that point, R, the corresponding stimulus becomes a. This stimulus may be called the unit stimulus. Thus we have $a = 1$; but $\log 1 = 0$, therefore $\log a = 0$.

So that at length we have $E = C \log R$.

From this formula we can construct a curve, by representing the stimulus R along a horizontal line, and by calculating at adjacent points along this line the value of E, and by setting off these values along vertical lines arising from those points, and finally by joining the tops of these vertical lines. Thus, corresponding to any stimulus, we have our sensation rendered tangibly.

Here we have a very remarkable result, and one that at first sight did not appear compatible with the nature of things, for how can we measure sensation, emotion, feeling of any kind? It is true one sometimes hears an expression: I was ten times as happy before; but this is due to rhetorical associations; for how could happiness be supplied by measure?

But consider attentively Fechner's assumptions. If Weber supported a weight of 30 oz., then he felt the difference when 1 oz. was added. If he supported 60 oz. then he felt the difference when a weight of 2 oz. was added. Fechner takes it that the second feeling is equivalent to the first, and that it may be expressed by the same numerical quantity. But here we enter into the realm of

pure fantasy. We are ostensibly engaged on a process of obtaining exact measurement for something we feel to be vaguely quantitative; and, to obtain these measurements, we make a connection of equivalence between two things so intangible that we cannot even divine the extent to which this assumption involves what we seek to prove. But what we do know on all grounds is that the assumption is false.

Consider the matter by means of an illustration. We have, let us say, thirty minute voltaic cells in series, and these produce an effect on a needle. We add another cell, in series; we observe a change, just perceptible, of the position of the needle.

If now we take another series of thirty similar cells and a needle, we observe here also a change on the addition in series of another cell.

If we bring both systems of cells together and make the change indicated, we find in the system of the two needles a just perceptible change. We might suppose that, everything else being unaltered, one needle was placed vertically under the other so as to be undistinguishable from it.

Now the change in this case is not the same as that of the change in the first case. We have, in fact, a relation of cause and effect easily understood as far as the addition of the two systems is concerned; and, on the basis of this result, it would be unscientific to build up such an expression as that of Fechner's Law.

Fechner's second assumption has really no valid meaning whatever. It has been entirely suggested from mathematics, where it is customary to "proceed to a limit."

But that there is no correspondence to this in sensation was by no one better recognised than by Fechner in other investigations. For he was at great pains to estab-

lish the "threshold" of sensation, beneath which, though the external stimulus might vary, nothing of that variation could become known in consciousness.

The acceptance of Fechner's Law entails certain consequences which enable us to criticise it from another standpoint. Within certain limits it will be found that, if the stimulus be quadrupled, the sensation should only be doubled. But what is a doubled sensation?

If we consider a straight line of a certain length we shall find, in a manner explained in the chapter on Externality, that our knowledge of the line depends on the consensus of a great number of perceptions. If we take a line of equal length and place it in continuance of the other, then we have a line of double length. In what other way could we say we had a line of double length? If Fechner's Law were valid, we might obtain a line of double length by a process of direct estimation of the sensations. But we could only be satisfied with such an estimation if it responded to the test of doubling the length of the line as before. A similar principle holds in regard to weights, or anything else which can be measured.

But, for sensation, we are told we form an estimate of the sensation having been doubled. But what is the meaning of doubling? How can we take a similar sensation and apply it to the other and observe the result? We can imagine a sensation similar to another; but that is not what we are here required to do. We are asked, with no previous experience, to estimate, as a sensation, the double of a certain sensation.

It is necessary to insist on the condition "without previous experience," for if we require previous experience, it follows that we must rely on memories, more or less vague, of sensations referred to objective stimuli in cases where such stimulus in one case has borne some

estimated relation to the stimulus in another. There is no escape from this.

But there is no method "without previous experience," or *a priori*, of estimating a doubled sensation; the meaning of doubling, involving that of coalescence into a new Unit, can only be known through experience.

Let us consider a case that might seem to be in contradiction to this. We have a mile at sea marked by two buoys. Then we have a second mile indicated by a third buoy. We might be inclined to say that the second mile looks less than the first. But that is a matter of judgment, and of judgment that may be rendered more correct in proportion as our acquaintance with the sea, including our knowledge of sea-effects, increases.

In our ordinary life and for objects near at hand that judgment has been well cultivated. We see a stick near at hand, and we see it some distance off, and we say that it is of the same length. But in the second case it may subtend an angle less than half of that in the first case; but we make due allowance by taking into account, even though not deliberately, the appearance of the intervening space. But when we are asked to say that a mile at sea looks small, we are virtually invited to form a notion of a mile, mainly from the angle subtended by a mile in the neighbourhood, and then, keeping that constant while everything else varies, to apply our judgment. That is clearly not scientific. Thus eventually we find that, to the experienced eye, a mile at sea looks a mile.

But the appearance of the second mile contains elements different from that of the first, and the appearance of two miles is not that of two appearances in succession of one mile. The difference is a matter of experience. What we really seek for to satisfy ourselves in regard to the second mile is that, if we stood in the same relation to it as to the first mile, the effect, in as far

as length is concerned, would be similar. But that simply means that we require that the objective stimulus should be doubled in order to provide a measure of doubling the subjective effect.

And so with light. It appeared to Herbart quite natural to double the illumination, and to consider the resulting sensation to be defined as double of the first. And if we can use measures in this way at all that is really the best way to obtain a definition. If it be said that the second sensation does not appear to be double the first, the answer is that it is the sensation corresponding to a double illumination and is therefore, as far as such terms are applicable at all, the double sensation.

Even the suggestion to think otherwise could only arise from vaguely remembered comparisons with other experiences, based ultimately on objective measurements, these being not necessarily related to the same sense.[1]

It is curious that, though Fechner's Law is now regarded in some Universities as a sort of article of faith, it was not well appreciated when first announced. Volkmann of Prague called it a paradox; Brentano said that it was contrary to common sense; and Helmholtz and others criticised it adversely. An interesting discussion will be found in *Mind*, 1876, over the signature of James Ward, who finds Fechner's arguments unconvincing. To me Fechner's law seems a scientific toy.

Yet, an explanation should be sought for certain underlying facts, such as the apparent diminution of subjective effect at a certain limit compared to the effect when the objective stimulus is moderate. The whole

[1] For example, if a child got a cut with a cane, and then another which was much more painful, this second result might be due to other factors being involved; and the notion of repetition of pain would react on the whole emotional character. If, then, the child received a slightly painful impression from illumination and were told to expect another, the expectation aroused might be so exaggerated that the real effect of the second illumination would seem inadequate. But that would not be a case of measuring a sensation.

matter is far more complex than Fechner supposed, and the following suggestions will indicate how extensive are the factors to be considered. Fechner himself has well studied the lower limit at which the objective stimulus gives rise to sensation. There is an upper limit at which further additions to the objective stimulus do not make an impression on consciousness. This limitation of Discrimination has been already referred to in various places; it will eventually be found, in the course of the exposition, to be essential to the foundation of reason as we know it. It depends ultimately on our actual physical constitution. Now, by the very fact of the Conception of Unit, as well as by Association, we find that the stimulation of one sense affects every other, either by inhibition or by stimulation of Association, or by some secondary process. Again, the range of Discrimination varies with the senses. So that, as the objective stimulus is increased towards its limit, we have an immensely complex interaction of all the senses, and also of motor activities expressed by particular attitudes, and by the balancings and inhibitions necessary to secure these in equilibrium.[1] The matter becomes further involved with memories, instincts, effects of past experiences, and by vague judgments.

Even if the unwarrantable nature of the assumptions of Fechner had not been shown in the preceding discussion, it would be now evident that all question of law as referred to a single sense must be abandoned. All that remains, viz. the failure of proportionate response to increased stimulus at certain levels, depends on the limitation of Discrimination of each and all the senses.[2]

[1] Cf. Ribot on Attention: "Its constitutive elements are motor"; and Setschenow: "No thought without expression."

[2] Fechner's Law has proved so important in regard to the modern development of Psychology that a few additional references may be interesting. The conclusion of Descartes that the pineal gland was the seat of the soul arose out of the desire to find a nexus between thought and the central nervous system, but

Descartes was not the originator of such speculations. Servetus, who was burnt in 1553, believed the soul resided in that part which we call the aqueduct of Sylvius. Varoli, in 1575, declared the cerebrum to be the organ of mind. In these studies we find the beginnings of Phrenology and of Localisation of Functions. Haller (1708–1777) appears to have been the first to measure the time required for a psychical process. He stated that one-third of a second was necessary for the full perception of an idea. Helmholtz, in 1850, measured the velocity of transmission of nerve-effect. But the impulse towards Fechner's experiments seems to be more directly traceable to the remarks of Maskelyne in 1795 as to the discrepancies of the observations of himself and his assistant. (For this one may refer to Gonnésiat, "Récherches sur l'équation personnelle," or to E. C. Sandford, "The Personal Equation" (*American Journal of Psychology*, 1889). The writings of E. W. Scripture ("The New Psychology") and various others deal with the broader question.) Bessel's experiments have already been referred to in the chapter on the Unit. Gauss suggested to Gerling to devise an artificial transit in the form of a pendulum (1838). In 1854 Prazmowski gave an account of experiments with a luminous point instead of a star. Special studies in photometry, with regard to the law of Relativity, were carried on independently by the scientists referred to in a previous note. Weber was followed in his own manner by Ludwig, DuBois-Reymond, Funke, Hering, Hermann, and others. Modern developments are to be found in the numerous schools of Experimental Psychology in Germany, America, France, England, Italy, and elsewhere (cf. V. Henri, "Les Laboratoires de Psychologie expérimentale en Allemagne," 1893). Cf. also G. E. Müller, "Die Gesichtspunkte und Thatsachen der psychophysischen Methodik" (1904). Only two out of the great number of eminent names seem to call for special mention here on account of opening new paths. Kräpelin has studied the effect of drugs on psychic functions, and a large school of investigators have followed him. F. C. Müller (*Archiv. f. Anat. u. Physiol.* 1886) put forward the theory that Weber's Law was only a case of a larger "neuro-physical" law.

Finally, the unreality of Fechner's Law does not detract from the value of the facts ascertained by experiments in the attempts to sustain it. In the chapter on Association it is indicated how Fechner himself was led to the formulation of this "law."

CHAPTER X

MEMORY

MEMORY is a Fundamental Process of the mind. No mental life could be carried on without Memory, and on the other hand its elementary phenomena are as direct, spontaneous, and unanalysable in themselves as the Immediate Presentations of our experience.

But it is well to inquire into various circumstances on which Memory depends. We have already investigated conditions for the production of sensations; such as, for instance, in the broadest manner in the case of vision that the physical apparatus shall be in good order, and that an object be in the field of vision.

There is a Feeling of Effort involved in the Process, yet that does not depend on the will. This must be clearly understood; it may require an exercise of will, for example, to open the eyelid, but when the conditions are obtained the Immediate Presentation follows. We are not aware of the mechanism by which the result is being brought about; and we are not able, when we have fulfilled the conditions, to control the operation.

The conditions within our control are a part of the precedent physical conditions, but another and an essential part of these is beyond our control.

As Memory is a Fundamental Process, it will be readily understood that we cannot control the pheno-

mena of Memory by an exercise of the will, except by endeavouring to alter the physical conditions and to influence Associations according to intention.

That the physical conditions are of importance becomes evident when we proceed to extreme cases. Thus, if a man be knocked down by a violent blow on the head he may be altogether deprived of Memory for a time. Again, if he be telling a story, and then suddenly discover that his coat is in flames, he will have the current of his recollections changed.

The exercise of the will may modify the conditions under which Memory works, but under given conditions it cannot influence the order of the ideas that arise in consciousness.

It is often most profitable to examine a structure in its breakdown, and to consider a process wherein it makes default.

Therefore, in the study of Memory, I will begin with some observations of failure of Memory, and I will then proceed to the review of certain experiments in Memory, some of which extended over long periods of time. From all of these it will be possible to draw useful inferences. The observations and the experiments were all accurately described at the time, with dates and hours noted, and circumstances which seemed to have a bearing on the case mentioned. It does not seem necessary, however, here to reproduce these notes in the formal manner in which they were first set down.

I had occasion to recall the name of Fresnel, but, at the moment I required it, it "slipped my memory." I then resolved to study attentively the manner in which I should recover it.

The name had slipped my Memory because something accidental had arisen at the moment of recalling it which diverted my thoughts. This circumstance created a

stronger bond of association with the previous chain of ideas than did the name itself.

Therefore every time I pursued the same route I found my train of associations divided. Every successive change in associations not only implied a facility in the new route of ideas, but also an inhibition of other associations which I could yet "feel" as tending to assert themselves.

In such circumstances the mind cannot usually recover a name simply by persistent thinking along the old track. It is often necessary to allow a certain time to lapse, so that the association which has led us off the path, and which may not be a permanently strong one, may lose some of its force. Then the permanently strong association of the name with the facts to which it has reference find easier play.

I have presented the matter graphically by an illustration (cf. pp. 95 *et seq.*) which is at least true as regards the physical basis of the phenomena antecedent to thought. The elaborate network of the nerve-strands in the brain, with all their conducting lines, their stations of exchange, and their inter-communicating reticulations, forms a system which, though complex, is yet governed by a simple principle which is reproduced in various forms in all human works. The service of an extensive telegraphic system, a very elaborate system of canalisation, the graphic representation of systems of classification in a large and varied field, all have analogies with the system of the nervous network of the brain. These analogies have their origin, in part, in the nature of things, in part also in the circumstances that our minds move according to that principle, so that everything that we create for reducing complexities to order is founded on Generalisation, classification, and all that these processes imply.

But in the brain this whole system is living. Its activities are affected by various forms of stimuli, by the supply of blood, by the quality of the blood, by the healthy development brought about by exercise, or by the disabilities produced by fatigue.

Again, from what we know from neurology of the localisation of various "centres" in the brain, and from the fact that the mind holds in consciousness one Unit at a time, and that the progress of the mind is by a series of associations, one might suppose also that at any given moment only a comparatively small part of the brain is in activity, or at least in such activity as produces clear ideas in consciousness. This last point becomes evident when we consider the incoherence produced in uneducated persons by causes of excitement which disturb more areas than those normally stimulated but which produce only ideas revolving round a few "fixed ideas."

This digression has been thought advisable in order to make clear the course of the exposition. It will be seen to imply nothing beyond what is accepted in neurological science. It should be especially noted, again, that however accurately we represent the physical basis of our minds, thought itself is something quite distinct, and unimaginable in terms of this physical basis, though the physical basis affords a good graphic representation for referring to movements of the mind.

This I had in view in making the study we are considering.

So as to avoid the influence of disturbing associations (cf. p. 28 and p. 105) I thought quietly over various circumstances connected with Fresnel. I became satisfied that the name began with the letter F.

The fact that the Memory of the initial letter often persists when nothing else is recalled is due to a variety of causes. The initial letter is made a capital letter, and

thus it becomes doubly prominent as compared with the other letters. Moreover, being the first letter met with in the change of attention from ordinary words, as in reading, the associations formed round it are stronger by reason of the freshness of the attention. An extreme case of the like principle is that of the strength of the associations created by a joyful surprise. Moreover, the initial letter is often used, isolated, to indicate the person, and so a habit of mind is formed tending to impress attention on the initial letter.

Still endeavouring to recall the name, I formed in my mind a clear image of the bust of Fresnel which I had seen in one of the halls of the Institute at Paris; and I endeavoured to recall the name by associations with the notions I had formed in regard to the bust: the deeply thoughtful, absorbed look, the regular, classic features, the air of nobility of mind, the finely formed head, symmetrical and well-balanced, the well-arched dome, the large though not disproportionate forehead—all this recalled the man, but not the name. I then thought of his work: the rapidity of his progress in mathematics; his invention of reflectors for lighthouses; his intercourse with Thomas Young; how their work was on similar lines; and how certain results reached by Fresnel which filled him with a sense of triumph had already been attained by Young.

All of these circumstances brought a multitude of associations, but none of them produced the name. I then thought of him in connection with Foucault and Fizeau, but to no immediate purpose.

These efforts and observations I continued at intervals for three days. I then began associating the initial letter with various letters of the alphabet in order; and when I reached the letter l, the name Fresnel came to my mind followed immediately by a series of new associations,

prominent among them being that of Fresnel's rings, which in turn recalled the experiments of Newton, and various other matters in connection with the theory of light. Not only that, but I found that, by repeating the name Fresnel, the previous associations were made more vivid, and even the image of the bust became clearer.

I find this note appended to those set down at the time: There was also vague association with Napoleon, but I am quite unable, at the moment of writing, to add a single fact to this association or otherwise to define it.[1]

Now in this case the name Fresnel was not at all unfamiliar; but at the moment of desiring it I had a vague apprehension that I might not remember it. The slight emotional factor had an influence in inhibition. At the point I had reached then—using the graphic representation (cf. p. 95 and p. 236)—the way was barred, and the movements of my thoughts were diverted in other directions.

In seeking other associations I was "trying back," seeking to get beyond a junction so that I could reach the name by paths that did not pass through the barred road. But in cases of the sort there seems to be often not merely a barred road but a veritable inhibition of associations with the name, as if the natural flow of associations was reversed (cf. pp. 117 *et seq.*).

I succeeded eventually because the inhibition was due to associations not deeply based, and the influence of these in diverting the current of associations diminished more rapidly with time than associations deeper and of longer date.

The fact that the letter L served to recall the name is interesting; for it seems to indicate that, in a name, each

[1] Some years afterwards, re reading this passage, I remembered that the association with Napoleon was that I had once formed a list of men of genius with the clear-cut, classic features, and eager, emaciated countenance of Bonaparte when young; this included Fresnel and Keats (see p. 627).

letter of the name forms its own associations. No doubt the endeavour to find the name during the three previous days had facilitated many paths close to that related to the name, so that some new slight stimulus was all that was required to complete the union. Thereupon the effect was comparable to that of an explosion of stimuli, from a centre corresponding to the name, out along many adjacent paths.

We may have occasion to conclude from a minute study of such cases that not only every letter of a name, but every element of a letter which we observe, and in short every unit clearly recognised, forms its own associations; that is the manner of its existence in the mental life.

About the same period as the last observation I tried to remember the name of a great French neurologist, Duchenne, with whose writings I was to some extent acquainted. I had first met his name when studying the work of Sir William Gowers on the nervous system. Dr. Gowers spoke of the "elaborate hysterics" of the French women, and referred to Duchenne as an authority on that subject.

The chief association with that reference that remained in my mind was that of hysteria causing such a tension of the muscles in some cases that the patient was able to rest on the head and the heels—*arc-boutée*.

I had other associations, for I had read a comparison of his work with that of Charcot, some of whose lectures at the Salpetrière I had attended, and I had also read a biographical account of Duchenne in connection with a monument erected to his memory at Boulogne. I had been struck also by the Christian name, although it was not an unfamiliar one. Some years, however, had elapsed between these several associations.

Though unable to recollect the name, I was yet able

to classify it to some extent. Thus I knew it was a typical French name, that it was not an unusual name, that it was of medium length.

It will be observed that these elements belong to broad lines of classification, and they are useful, therefore, in indicating how the mind forms its associations. The word Duquesne, on account of the peculiarity of the pronunciation in regard to the spelling, occurred to me frequently. I had been struck with it in glancing, not long before, at a book of Fenimore Cooper's.

In this case, however, I was unable to identify the initial letter, and I believed that the letters most likely to be found in the name were m and l. These facts I take from notes made at the time when I was still trying to recollect the name.

The degree of recollection was evidently less than that in the case of Fresnel, and no doubt the name was less familiar on the whole. The difficulty was partly due to the fact that, since having had occasion to note Duchenne's work, I had been engaged in other studies which had greatly occupied my mind. This produced a certain force of inhibition with regard to studies less immediate.

It will be found, by experience, that if the mind be engrossed on certain matters of study, then subsequently find other absorbing studies, the recollection of the first will have become more difficult, with a sense of remoteness, than if there had been no severe studies intervening.

For a week I tried to recollect this name, finding new series of associations each day, but the inhibitive force was too strong. Afterwards I met with the name accidentally, but even then the new associations provoked were not so vivid as I had expected. It would seem that, all along, I had associated what I had known of the man with his personality rather than with the name itself. This may easily happen with foreign names.

We have now had an instance where the first letter was recalled with certainty, and an instance where there was doubt about the initial letter. I have notes of another where I believed the name began with a letter which was really not the initial letter. This was in the case of trying to recall the name Hesse, with whose work in the higher mathematics I had associated ideas of "elegance" of solutions. The name had escaped me, and in seeking, as in the previous examples, for associations I became convinced that the name began with F. At the same time I knew that it was a short name, and that, though the man was a French mathematician, the name was German.

The following is especially instructive. Late at night, while very tired after arduous work, I tried to recall the name of Berthelot, the great French chemist, a conversation with whom I remembered perfectly well. The name was obscured in my mind, but I resolved to recover it. After thinking for a considerable time I arrived at Berthollet. This satisfied me for a moment; but immediately afterwards I recollected that Berthollet was the name of another great chemist, and slowly, gradually, and by persistent thinking I found the right name Berthelot.

During this process a name had often seemed dimly to suggest itself, but before I ascertained this name I knew it was that of the founder of the anthropometric system. It was really Bertillon.

I knew the name was like Berthelot and Berthollet, but it required a considerable amount of patient searching before I recognised the name clearly—Bertillon. In this case the name satisfied me immediately it was recovered, and its recollection brought in its train a number of associations.

This example indicates that each element of the

name to be remembered has its own Memory, and that there are associations formed between the parts themselves, and also between the parts and external things independently associated.

The unusual names are the most likely to fall out of memory. Here is a note regarding an uncommon, though not foreign name. I tried to recollect the Christian name, for I had the surname, of a young man whom I had met and whom I had heard sing—Dermot Sidford. Since I had seen him I had had my mind occupied with serious studies and much hard work.

I determined to remember the name by effort, and to observe the manner in which it would return. I felt sure the name would soon appear. That is a fact worthy of particular note. One has a distinct feeling in this respect which suggests the graphic representation of the name being held down by a network which is being distinctly agitated. Much might be explained by a scientific study of these presentiments—*Ahnung* of the Germans—extending from a single case like this to that vague but powerful impression which a scientist may have that he is on the verge of a discovery.

In seeking the names, suggestions came to me of Murdock, Mudford, and names having an assonance like them. I knew with certainty that these names contained some element of the real name, but that they lacked something characteristic. I then thought of Bernal and Bertrand, but on close attention I became convinced that these names arose from associations with that of Bernard Shaw.

It will be noted that the names Shaw and Sidford have in common only one element, the sound S. This would suggest that, on the basis of our graphic representation, we should place the area for Si near that of Sh.

But as it is in the highest degree improbable that

there could exist any organic provision for storing complex symbols of artificial construction, as not only our words but also the letters of our alphabet are; we are led to analyse to its elements all the combinations that go to form a letter; and we become convinced that each element of the letter, including not only its sense of sound, but its force, impact, anything of which we can form a distinct conception as a Unit, has its physical correlative (cf. pp. 95, 237).

It will be seen from this how marvellously complex are the foundations of ordinary speech.[1]

Returning to my desire to remember the name, and acting on a suggestion (see p. 237) regarding the physical condition of brain activity, I determined an increased flow of blood to the brain by rubbing vigorously the back of my head, and applying my hands to my ear and bending down. The first effect of this was to obscure my question by introducing too many associations connected with it.

Then after a time I recalled circumstances in which I had seen Dermot Sidford, had listened to him singing, and had heard his name pronounced. I repeated the name Sidford several times in quick succession and in an eager way, recalling all the circumstances as vividly as possible.

After a time there was a sense of the name arising. This was really curious. I gained no additional information at all with regard to the letters of the name. It was only just a sense of a veil rising and floating intangibly, and yet that this would uncover the secret (cf. pp. 240, 242). I knew it was near it, though in actual information I had gained nothing.

[1] This observation may make clearer some of the remarkable phenomena found in aphasia, where certain elements of the alphabet are retained, while others are blotted out.

I tried again in the same way. Suddenly and with an air of surprise, as of a sudden dawning, the name was there in complete light—Dermot.

Another study of a similar kind presents a peculiarity in the way in which the name returned. I had occasion at a later period than in the observation last mentioned to refer to the composer Mascagni. I had not used his name for years. I could not recollect it. After some reflection it seemed to be coming, vaguely. It presented the letter M. I then thought of the opera "Cavalleria Rusticana"; of an occasion on which I had had an interesting conversation with him; of his appearance; of his voyage to the United States.

The name seemed at times as though ready to spring out all at once, but I could not quite attain it. I "concentrated" my thought, and I had the sense of applying energy with fervour. Then I made one or two tentative efforts. Then Magnardi seemed like it.

It was noticeable that before I recovered the name I knew that it was not longer than Magnardi, not so long as Magnardino.

Suddenly, quietly—and in a manner to suggest the explanation: not in the place at which I was looking—came the name Mascagni, naturally, freshly, familiarly, as though it had been disengaged from some encumbrance and were now floating easily.

I have also notes made at the time of a case where there was a persistent error in regard to the first letter, but where, nevertheless, the letter wrongly taken played a prominent part in the name. I had occasion to recall the name of M. Doumer, the eminent French statesman, whom I had heard delivering an address in Paris, but of whom I had not heard for some years. The names that became successively suggested were those of Mariéton, Moron, Merlou, Remon.

In the midst of other occupations I reverted from time to time during several days to my attempts to recover the name, but without seeming to make much progress. I had, however, acquired the certitude that the letters r and m were in the name. I inquired if I could find any others. The name of Drumont occurred to me, and with a sense that the real name seemed to be flashing near as though escaping from some encumbrance.

Then while walking in the afternoon I repeated as rapidly as possible, and in an agitated manner, several names such as Drumont, Remon, Durmer. Then suddenly, and with ease, I lighted on the name, Doumer. When the name was recalled there came with it a feeling of entire confidence, and many new associations were recalled.

Another case in which the Memory due to each letter becomes manifest I find recorded in my notes with dates. I desired to recall the name of a photographer in Paris, Branger, with whom I had once spoken. At first I could only form tentatives, Briaschi, Brigi, Brass, but with the knowledge that these were incorrect. The name did not seem French. I knew it began with B, and that it consisted of one or two syllables.

I thought persistently on the name, but not till two days afterwards did I find a dim suggestion of n in the middle. The following day I made the tentatives Brianchi and Braschi, but recognised that they were wrong. On the next day I met the name Branger, accidentally, and became persuaded that it was the name, though there was lacking the bright flash of certain discovery. The explanation seems to be that I was under the impression, having in the first instance only heard the name pronounced, that it was an Italian name like Branji. It will therefore be seen that the

efforts of reproduction were consistent with the hypothesis that a distinct Memory appertained to each element, that is to say, not only to the letters, but to their forms of combination, and to associations so determined.

One of the most instructive examples is that wherein the failure of Memory was partly produced artificially. I set down a note at the time: I was recalling the name of a firm, Swan & Edgar, being at the time fatigued with mental work. I would have had no difficulty in recalling the name, as it was familiar to me, except that I purposely followed for a moment another train of thought. On returning to find the name I could not recover it, the tendency being to pursue the new path artificially opened. I remembered that the first name began with an S, and the most frequently recurring tentative was Snow. Here it may be pointed out that this name contains the same number of letters as Swan. We have observed that each letter and element of a letter has its Memory. It would appear also that the Feeling of Effort associated with the expression of the name has its Memory.

The tentatives in regard to the second name were less certain, Eglington and Elbridge being among them. The first association Edg was the most vivid. It will be noticed that the a of Edgar does not appear in any of the tentatives, but the pronunciation of the name is really Ed-gr. Every element in the name, visual, aural, sense of Space, Feeling of Effort, frequency of Association, vividness of Association possibly on account of some Hedonic quality, all help the Memory.

The question in reference to each element having its own Memory is so important that I add the notes of another case which has remarkable features. After a tiring day, involving much writing and ending in a

long sitting in Parliament, I went to bed after midnight. My note taken at the time says: "Very fatigued."

It occurred to me to try to recall the name of Chantemesse, the distinguished bacteriologist, whom I had known in Paris, and whose name, although not often seen in the meantime, was familiar to me. I had artificially interposed the name of another bacteriologist, Calmette. I tried in vain. My mind seemed dulled and lifeless. Calmette came forward every time. Here it may be remarked that the French phrase, "*Ma langue se fourcha*" (My tongue forked), expresses well the physiogical process, the stream of stimulus being diverted to a wrong path.

I thought of various circumstances in which I had seen or spoken to M. Chantemesse. This produced no effect. I thought of mutual friends, but again without effect. Then artificially I tried associations of letters. I felt certain that C was the first letter. I was also certain that there was a t; and that the name in sound and style was somewhat similar to Calmette, and that I should find it lying under and lying longer than Calmette. This impression was remarkable. Suddenly, while trying combinations, there occurred to me the name Chassemelle. There was a quivering as of undergrowth, a stirring as if the whole would come to life. But the sense of fatigue came on again.

However, being convinced that the secret lay there I kept trying by repeating the name Chassemelle, but with searching here and there. I felt sure that the ending was of the form ette, elle. Then suddenly the name came easily, and with a feeling of entire confidence —Chantemesse.

It may be observed that though transposed the letters had in the main been recalled, c, t, a, ss, e. This associa-

tion with the name was stronger than that of the sequence of the letters themselves (cf. p. 118).

I do not intend to convey that the process of recovery is always, or usually, in accordance with those recently described. These examples have been chosen to show that each element of a complex, such as a word, has its own Memory. In most instances, however, it will be found that the associations between the parts of the word are so close that the whole word is recovered at once.

I find a note which illustrates this. Having laid aside certain studies involving great concentration I had occasion to recall the word "lumbrical." At first I had a vague impression, fairly persistent, of a sequence of letters, m, l, b; but I could make no progress. I then recalled the form and the function of the lumbrical muscles. Then I recalled other associations. Suddenly the word came, "lumbricals," and with an unmistakable impression of energy.

This impression of energy is remarkable. Consideration of this phenomenon will give some light on certain of the thousand factors that go to form taste and style in poetry.

It may be noted also that in such cases there may be no Impulse towards any action in sequence. The effect of recovery satisfies the intellectual and moral needs of the moment. This point should be remembered in the discussion of Will, for if Will could be analysed to mere relations of sequences and associations, there would not arise such a distinction of cases where Will is not marked from those where Will is well marked.

Another illustration of the manner of return of former impressions I find thus recorded: I was thinking of persons known when I was a schoolboy in Australia, and whom I had not met since. Most of the names came

readily. Some lagged, though I felt certain of remembering them. I tried to think of one whom I had seen as a young lady when I was a small boy—Ellen Moseley. I remembered the style, appearance, beauty, various circumstances. At length I became aware that the first name was Ellen. I believed that the second name began with M. I thought of Mooney, but I knew it was not Mooney, that it was not so familiar as Mooney, and yet that it was not altogether an unusual name. I went for a walk. Then suddenly with no marked Impulse the name was there all at once—Ellen Moseley. There was present a feeling of certainty and repose from the effort of searching.

About the same period I wrote down the notes of another case which contains peculiar incidents. At half-past seven in the evening I tried to remember the name of a notable operatic manager, Hammerstein. I had met him once, but had not thought of the name for more than a year. The name of Heinemann, the publisher, came up before me, and I could not recover Hammerstein. I went to my study, and engaged in absorbing mental work, followed by writing, until half-past eleven. I then went to bed, not intending to think any more, but I could not throw off the prompting to find the name. Suddenly there was a glimmer which I felt sure contained the name. I felt sure also that very little was wanting to bring the name to my mind. When I say "glimmer," I am endeavouring as nearly as possible to indicate the actual impression in my mind. It seemed as if in a certain region there was a flash, long in form compared to breadth, and remotely suggestive of a flash of lightning along the horizon.

I am particular in regard to this description, for I believe that a study of such mental imagery may yet bring forth some curious and important facts. I then

thought steadily on the name, but there were merely broken suggestions of H as the initial letter, with M to follow; and vague associations with the German character of the name.

I thought energetically, but felt that that was not the way to progress. I became convinced, however, that if I changed my position, as by turning over, I could recover the name. I did not do so at once, but allowed my thoughts to subside till there was a general blank.

Then, on account of other causes, I gave an involuntary turn, and suddenly, as if in the midst of broken, semi-transparent hurdles, I found the Hammerstein, not as presented vividly to the visual sense, but appearing distinctly as a thing within my knowledge.

This manner of the name arising should be compared with what is said of the simple case. It may be noted also in connection with a remark I once heard made by a physiologist, an Irishman, whose ardent pursuit of science had not blunted the wit of his race. He said that it was sometimes best not to look directly at a problem, but to observe it sideways, for then the solution would occasionally come out "unbeknownst."

Still another observation of a similar kind is noteworthy on account of a circumstance connected with the recovery of the name. I tried to remember, in conversation one evening, the name of a charming young actress I had seen in Paris some years before—Louise Bignon.

At first, although I remembered clearly the appearance of the actress—the colour of her hair, her manner, the circumstances under which I had seen her—yet the name was completely blank.

On the following morning, when my mind was fresh, I tried again and again to remember the name. These efforts continued at intervals for three days. Gradually a dim adumbration of the name seemed to arise—a sug-

gestion of Marigny, but also with the certitude that that name represented only a few of the elements. I could not recollect the initial letter.

On the following days I made but few efforts. Finally, about a week after the first attempt, I had occasion to look at one of the evening papers, *La Patrie*. I was not then thinking of the actress. Suddenly the name Bignon became quite clear. Immediately afterwards I recollected the whole name Louise Bignon, and it seemed quite familiar.

A curious feature of this experience is that on reading *La Patrie* I observed the name Bignon in the news referring to a municipal councillor. The name is not a common one; I had not noticed it before in the paper. It is, however, though improbable, yet not impossible, that I had really seen the name in the *Patrie*, and that though it did not attract my attention it yet produced an impression which later and in another way made itself manifest in consciousness. But I think that I had not seen the name, and that its appearance was simply a coincidence.

The above was written about two years previously to the note which I now append, thinking it best, as I have no certitude in the matter, to allow both to stand.

On reconsidering the Bignon case I now incline to believe that the name must have caught my eye in the *Patrie*, otherwise the coincidence would be extraordinary. One may well ask, then, why did the name not immediately attract my attention? It will be observed, however, from the previous note, that my mind was intent on some other subject. Now it is possible for an impression to be received in consciousness, to be forgotten, and to become revived on some other occasion without the associations of the first experience being vivid enough to produce again an effect in consciousness.

I will state certain incidents which appear to have a bearing on this problem. At one time, while a student in Melbourne preparing for an examination, I dreamt that a horse named Flying Dutchman won a race at a place near Melbourne called Kensington Park. I had not even known that such a horse existed, and I was too occupied at the time to take interest in the races. I looked at a newspaper in the morning, and there I found that Flying Dutchman was entered for the race. I was surprised, but having no belief in any special interposition, I did not pursue the matter. As a matter of fact, Flying Dutchman did not win.

Reflecting on the matter afterwards, I concluded that during the day preceding the dream I had read in a casual glance that Flying Dutchman was one of the horses engaged in the race. Other matters more important to myself then occupied my mind. During sleep, however, new associations arise, and in cases of great fatigue those due to forced pressure may disappear. The name Flying Dutchman reasserted itself and with vividness possibly due to new associations produced in dreaming.

About the same period also I was walking along Collins Street in Melbourne, absorbed in thought, when a friend of mine called me by name. He said that I walked on about twenty yards and then suddenly turned round. Here the circumstances form a sort of intermediate stage. I was thinking out a problem. My friend called my name. I heard him call, and the impression struggled with the tenor of my previous thoughts. During the walk of twenty yards I had reached a conclusion which gave me a resting-place, and then the call of the name entered with full effect.

Recently I had another experience which shows how a body of associations may—to use a graphic representation—be covered over by later growths, and how they

may slowly work their way to consciousness. On January 24 I had looked into an old box. I found some old letters and verses of a young Australian poet, R——, unknown but veritable, who had been a friend of mine. These made a peculiar impression upon me. On the following day I was thinking for a long time over certain characteristics of Byron's poetry. In the early hours of the morning following I dreamt that while I was in company with R—— Byron came along and we had a conversation.

Still more recently I have record of an experience which seems to confirm my last interpretation of the Bignon case. At breakfast I read in The *Daily Chronicle* a review of a book of Dr. Poumiès de la Siboutie, "Recollections of a Parisian under Six Sovereigns, Two Revolutions, and a Republic."

In this he tells a story of Horace Vernet, the painter, leaving Paris for Algeria on account of a slight put upon him by Napoleon III. In some way I formed the impression that Horace Vernet chose Algeria on account of the economy of living, and I read the paragraph again and again to account for that impression, but without result.

As I was putting the newspaper away my eye fell on a heading, "Where to Live Economically," which, it would seem, I had noticed previously to reading about Horace Vernet, and of which I had forgotten the exact associations.

Another curious little case may be added, as it allows one to observe with certainty the influence of associations which have not become realised in consciousness, but which have decisively affected the conscious life. I was looking over some type-written manuscript, reading quickly, when my eye fell on the name Romeford instead of Rumford: I struck out the e, and felt my attention called in a somewhat puzzled way to the first syllable. Here

it may be mentioned that there is a place in Essex called Romford. The error did not make itself known to me as an error, and I passed on reading rapidly. In the next page, however, I met the name Rumford correctly spelt, and instantly, before I could express the matter deliberately, I turned to the first name and struck out the o and inserted u. It appears to me that had I not seen the name for the second time I should have forgotten the error overlooked in the first name.

Re-writing all these observations I think they throw light not only on the recollection of the name Bignon, but also on many subjects—such as premonition, telepathy—not within the immediate scope of this treatise.[1]

THE MANNER OF FAILURE IN MEMORY

On a previous occasion I had, in a letter, written a phrase from Shakespeare; but, as I had not referred to the passage for years, I misquoted it: "The labour we delight in lessens pain." Immediately I recognised that the word *lessens* was wrong.

I tried to think of the right word; sometimes it seemed ready to spring out, but again it vanished, leaving a dull impression of forgetfulness.

Then I tried the effect of stimulating neighbouring paths. I read various passages of Shakespeare, and I looked over a glossary. This seemed to stimulate recollection, but without producing the word. I knew, however, that it was an unusual word, conveying a sense of diminishing pain by some drastic process, Having other work to do, I decided to send my letter with the remark that I had not got hold of the exact quotation. I then put the letter aside, and gave my attention

[1] Cf. F. Paulhan, "Histoire d'un Souvenir," *Journal de Psych.* 1907; E. Goblot, "Un Cas d'Association latente."

to other studies. Suddenly I remembered the exact quotation: "The labour we delight in *physics* pain."

In examining these cases we see that the most persistent associations are those connected with the import of the term. The strength of these associations corresponds with the manner in which such a term would arise in the mind of the author; for example, the term "lessens" not only fails to form the due associations in respect to its structure as a word, but it also diminishes the peculiar force and savour of the sentence.

It frequently arises that the sense of momentum, impact, and general effect of a phrase becomes shown forth to the mind while in the fervour of composition before the actual word corresponding is attained. It may happen that there is no word in the language available which gives the meaning desired, and yet fulfils other conditions of length and quantity. It is an accident of language that the word "physics" should exist in the form and with the expressiveness demanded by this phrase.

It is this felicity of phrasing that constitutes one of the charms of poetry, and that also masks the difficulty of composition. Those who have studied not only the final forms of the great poems, but also the erasures, the tentative efforts, the notes of some of the masterpieces, will observe that occasionally the word makes default. The rhythm, the force, the character of the phrase is already formed, and, if the accident of language be not propitious, a search is sometimes made in vain for a word already boded forth.

The famous line of Keats—

> A thing of beauty is a joy for ever—

was originally expressed in a different form:

> A thing of beauty is a constant joy.

Here the formal metre is better maintained than in the accepted reading. All the conditions of length, meaning, and quantity had been attained, but Keats, while not knowing how to replace the word, felt that it had not fully expressed the light, salient spontaneity of his thought. The associations thus formed were the deepest, and, by virtue of the principle of association which we will have occasion to discuss later in complex and in remote forms, the phrase at length arrived that fulfilled his desire, and we have the immortal line:

> A thing of beauty is a joy for ever.

By piercing deeply down into the nature of association and of the manner in which the Memory reproduces these associations, we gain a glimpse also of principles that should explain æsthetic appreciations. The true poet, in the force of his original quality, finds the "form" dependent on and adapted to his thoughts; and even if a word in this manner be a little forced from its usual meaning it often gains strength and freshness; but an artist, by dint of mechanical working at the "form," at the instruments of expression themselves, cannot, however skilful, produce associations of strength beyond that with which the words have already been endowed by others.

It will be found on examination that all poetry, especially in long poems, consists of passages of "inspiration" followed by other passages which serve to hold the sense and to link the thought to the next inspirational movement. The "connective tissue" is less striking in form than the fine passages of poetry where the words often have the air of being inevitable and unchangeable.

If, then, long passages be learnt by heart, and then

allowed to fall partly from recollection, it will be found that the Memory has unconsciously analysed the poem with respect to the poetry. The essential, the characteristic expression of a passage is retained, and the accidental is forgotten.

I have been able to observe this in regard to the poems of Milton, for example, learnt by heart perfectly and subsequently not referred to for long intervals.

In "L'Allegro" I remembered the opening:

> Hence, loathèd Melancholy,
> Of Cerberus, and blackest Midnight born
> In Stygian cave forlorn.

Then my mind was blank until the new movement:

> But come, thou Goddess fair and free,
> In heaven yclept Euphrosyne.

Then followed, in perfect recollection, the whole passage down to—

> Come, and trip it as you go,
> On the light fantastic toe.

After this for a period I found no strong sequence, but various suggestions—

> Nods, and Becks and wreathèd Smiles,
> . . . . .
> Laughter holding both his sides.

Examining these circumstances, I found that my Memory had preserved the really characteristic inspirational passages. It seemed to me that the introduction to the poem was an afterthought, laboriously wrought by Milton with a sense of artistic effort; as, for instance, in regard to contrast.

Then followed the felicitous lines of the veritable introduction to the Allegro passages. Then followed a

more or less artificial search for images of Mirth and Jollity.

Then I found that my Memory preserved, even more vividly than the form of the words, the "atmosphere" of the poem, the succession of delightful visions of English scenery, the spirit of cheerful meditation which is the characteristic of the poem.

Each of these scenes was marked by a phrase which formed its most apt reference:

Right against the eastern gate
Where the great sun begins his state.

The clouds in thousand liveries dight.

While the cock with lively din
Scatters the rear of darkness thin.

And every shepherd tells his tale

Under the hawthorn in the dale.

The upland hamlets will invite.

Mountains, on whose barren breast
The lab'ring clouds do often rest.

The Cynosure of neighb'ring eyes,
Rain influence, and judge the prize.

Then to the spicy nut-brown ale.

She was pinched and pulled, she said,
And he by the Friar's lantern led.

And the busy hum of men.

If Jonson's learnèd sock be on,
Or sweetest Shakespeare, Fancy's child,
Warble his native wood-notes wild.

In certain instances there was no sense of the order being preserved. But all this represents fairly well, no doubt, how such a poem would be composed. There is no sense of consecutive "argument," there is a brief recital, in fresh and glowing colours, of various scenes that at various times had impressed the poet's mind by a

sense associated with the spirit of "allegro," and which at the time, no doubt, also were represented in his mind by many of the phrases afterwards woven together.

It is usually easier to remember passages of rhymed poetry than passages of prose, because the similarity of the endings forms associations which recall the corresponding line directly, and also indirectly by virtue of the previous associations with that sound. The rhythm, the manner, and general atmosphere of the verse aid, and so may also peculiarities of expression forcible or felicitous.

I tested certain operations of Memory by first composing, and learning by heart, step by step, a prose story of over thirty thousand words, none of which was at that time written. I observed meantime the manner of the composition. First of all the general intention, then the selection of the principal characters, then the selection of the principal scenes, and the general course of the story; then the elaborations of the most striking or the most characteristic of these scenes; then the gradual appearance of the words, sometimes shown forth in striking phrases, at other times more connectedly; then the searching for a beginning, then the gradual weaving together of the words, led on from point to point by the course already marked out, and by certain passages already elaborated.

At first the progress was fairly rapid, but when half the work was completed the labour of retention and composition together produced a greater sense of effort. When the story was finished I found it possible to repeat it easily in three hours, and subsequently, after frequent repetition, in less time every day.

Some months elapsed before I wrote the story, and then I did not refer to it till three years afterwards. I then found that my recollection had become very uncertain with regard to the words. Examining the matter

closely, I discovered that the gradual failure of the Memory traced out in reverse order the manner in which the story was composed. That is to say, the less striking passages disappeared first. This broke the continuity of the verbiage. There remained some of the more striking phrases first discovered. The surest factors were those first determined, the intention and general course of the narrative, the principal characters, the main scenes, the most characteristic expressions. The rest was perceived as through a fog.

About the same period that I undertook this work I used to repeat every day a great number of lines mainly arranged in verses, observing day by day that the repetition became easier. The whole series included extensive mnemonic systems for remembering facts in astronomy and physiology, in which numbers were represented by letters, thus enabling me to form phrases corresponding, for example, to food-tables; a few passages from Shakespeare; verses from Milton's poems, including a passage from "L'Allegro," a passage from "Il Penseroso," two passages from "Comus," a passage from "Samson Agonistes," a passage from "Paradise Regained," and several passages from "Paradise Lost"; also eleven sonnets.

The effort of memorising, with slow recollection and hesitations, occupied me at first the greater part of the day. Subsequently, with continual repetition, I could repeat the whole series in half an hour, that is to say, at a far greater speed than I could have spoken the words.

With practice, I found that less and less effort was required. The image that was suggested to me was that of pulling a fine tape through my brain at a speed which corresponded to the necessities of my recollection, a speed which, though great, had no unpleasant feature. At length I was able to perform this operation so as to

be sure, at least, that in no part was my recollection faulty, while listening to something else. Having arrived at this facility, I ceased the experiment.

Less than five years afterwards, not having referred to this series in the meantime, I re-examined the matter and found that my recollection was blank concerning a considerable part of it. The mnemonics had apparently left but little trace except the general principle of their formation, and the dim knowledge of the facts they represented. I had forgotten the poetry in so far that I could not repeat it in a consecutive manner, but I had a clear recollection of its general intention, its inspiration and atmosphere, the more striking phrases, and, particularly with regard to "L'Allegro" and "Paradise Lost," clear mental pictures of what the passages had evoked. I here offer some extracts from my notes referring to these experiments when at length, having in the meantime had my mind constantly occupied with other matters, I endeavoured to recall these passages.

> Of man's first disobedience, and the fruit
> Of that forbidden tree, whose mortal taste
> Brought death into the world
>
> . . . . .
>
> Sing, heavenly Muse.

Here, in addition to the parts well remembered, I had a notion of the general sense of what I had missed; that is to say, I would at once have recognised the words as being correct if I had read them again; and, if I had seen a version in which extraneous words occurred, these would certainly have attracted my attention and would have suggested rectifications.

In this instance, therefore, it will be noticed that I had retained what seemed to me the expression of the poetical movement. The parts that had fallen out had rather the air of afterthoughts, or literary amplifications.

The general "argument" had remained in my mind, but only in broad outline and not always in sequence. On the other hand the spirit, the "atmosphere," of the poem was quite fresh in recollection; as also the clear sense of what I thought to be particular beauties, intentions, revelations of personal feeling, or warm inspirational passages; while many of the phrases were recollected as being associated with what I had noted of a peculiar tendency to paradoxical expression.

I found myself often lamentably unable to quote, even where I knew that I could "put my finger on" a quotation required, with the precise appreciation of its meaning. The inability to quote arose from having forgotten the sequence of words employed; in some cases of fairly clear images I did not seem to remember a single word.

Short phrases came to my mind which seemed to sum up the picture of a scene: "fretted gold";

> His looks and thoughts were always downward bent.

> Deep on his front engraven
> Deliberation sat, and public care—

which I associated with Strafford; and "yet he pleased," of Belial. Also appeared readily a remarkable phrase attributed to Belial, in a manner which I thought little consistent with his character: "Those thoughts that wander through eternity."

Proceeding, and noting here and there: a wondrous image of the ship, hung in the skies, of which the full passage is:

> As when far off at sea a fleet descried
> Hangs in the clouds, by equinoctial winds
> Close sailing from Bengala, or the isles
> Of Ternate and Tidore, whence merchants bring
> Their spicy drugs; they on the trading flood
> Through the wide Ethiopian to the Cape
> Ply, stemming nightly towards the pole; so seemed
> Far off the flying Fiend.

I had remembered also a fine instance of *curiosa felicitas* of expression, although I had absolutely forgotten the words, which might be thought to be the very essence of that example:

> A violent cross-wind from either coast
> Blows them transverse, ten thousand leagues awry,
> Into the devious air.

Though I had forgotten the words, I had a good impression of the very effect they produced in relation to the meaning intended to be conveyed.

Then, again, for a beautiful sense of the soft impress of flowery words; then the liquid melody of luted syllables:

> The pleasant dales of Sibma clad with vines.

> From Elealé to th' Asphaltic pool.

The splendid invocation commencing, "Hail, holy Light," was fairly well retained, and especially the sense of the salient swerve and movement of the first verses.

I had great difficulty—owing to lapses of words here and there—in recalling a passage with which I had been particularly familiar, because it seemed to me to be, if not the finest poetical flight, at least the most fervent, and the sole warm and unrestrained expression of the poet's inner feeling.

> But chief Thee, Sion, and the flowery brooks
> Beneath that wash thy hallowed feet . . .
> Nightly I visit.

That was as much as I remembered, though with the knowledge that this was only a note and that it did not represent the verse; and also with the recollection that in the passage there was a peculiar movement of rapidly

swirling flow of verse laden with light and flushed with bright surprise.

The verse is:

> Yet not the more
> Cease I to wander where the Muses haunt
> Clear spring, or shady grove, or sunny hill,
> Smit with the love of sacred song; but chief
> Thee, Sion, and the flowery brooks beneath
> That wash thy hallowed feet and warbling flow,
> Nightly I visit.

Throughout the poem various phrases remained, each having some striking characteristic:

> All night the dreadless angel unpursued

> Pomp of Graces.

> For contemplation formed, and valour, he;
> For sweetness she, and soft attractive grace.

> The fairest of her daughters, Eve.

> So easy it seemed once found which yet unfound
> Most would have thought impossible.

> Ramping the lion . . . and in his paws
> Dandled the kid.

Then the fresco-like descriptions:

> With clang despised the ground.

> Onward they came in dance,

> Pretty atheists.

In all of these examples it seems to me that the impressions retained, and the passages recollected, were those most nearly corresponding to the first dawning of the poem in the poet's mind; and then to the peculiar happy finding of rare and wonderful phrases, and of inspirational movements of verse.

In the "Paradise Regained" I remembered the whole course of the argument more clearly and consecutively than in the "Paradise Lost," and I recognised the reason,

precisely in this form of high philosophic discussion, that made Milton prefer it to "Paradise Lost."

Of "Sampson Agonistes" I remembered best the general spirit of the poem, the sense of its being an autobiographical relation in a more direct manner even than the other poems. Of individual passages I recalled the remarkable description of the coming of Dalila:

I wrote without reflection:

> . . . What thing of sea or land
> . . . With tackle trim
> . . . And in her train
> . . . Comes this way steering.
> . . . 'Tis Dalila, thy wife!

I was well aware of the character of the gaps, and six days later I had a clearer recollection of the words.

The whole passage is:

> But who is This, what thing of sea or land?
> Female of sex it seems,
> That so bedecked, ornate, and gay,
> Comes this way sailing
> Like a stately ship
> Of Tarsus, bound for the isles
> Of Javan or Gadire,
> With all her bravery on, and tackle trim,
> Sails filled, and streamers waving,
> Courted by all the winds that hold them play,
> An amber scent of odorous perfume
> Her harbinger, a damsel train behind;
> Some rich Philistian matron she may seem;
> And now at nearer view, no other certain
> Than Dalila thy wife.

"L'Allegro" has been already discussed.

Of "Il Penseroso" I remember the impression of its being an afterthought to make a comparison poem with "L'Allegro." I remembered it mainly for special phrases.

"Lycidas" left me cold in respect of its qualities as a

passionate expression of friendship. The recollections that remained were those of a supreme example of essay-writing according to a high classical model; and then turns of phrases, and touches of description.

> Once more, O ye laurels.
>
> . . . Forced fingers rude.
>
> He knew
> Himself to sing, and build the lofty rhyme.
>
> To sport with Amaryllis in the shade.
>
> Fame is the spur that the clear spirit doth raise.
>
> . . . . .
>
> To scorn delights and live laborious days.
>
> To-morrow to fresh woods and pastures new.

Of "Comus" I remembered the atmosphere, a certain sense of noble poetical feeling overspread by artificial manner. Here and there fine passages; the movement, for example, of inspirational passages. Thus the actual words—

> Fragrant melody
> Rose like a steam—

took the place of the following verse:

> At last a soft and solemn-breathing sound
> Rose like a steam of rich distilled perfumes,
> And stole upon the air, that even Silence
> Was took ere she was 'ware, and wish'd she might
> Deny her nature, and be never more,
> Still to be so displaced.

Yet in all these, even when the gaps seemed formidable, my recollection was sufficiently sure that, on re-reading, I recognised that I had known before all the passages that rose above the ordinary level; and at any part of any of the poems I would have noticed had an

error crept in of a manner of expression inconsistent with Milton's style.

With regard to Shakespeare similar remarks apply. I will only here refer to one passage which, forming part of the series already referred to, was at one period repeated so often as to pass through the mind with extraordinary facility. Less than five years afterwards I was only able to reproduce it thus:

> What you do still betters what is done,
> When you speak, sweet, I would have you do it
> Ever;
> Laugh, laugh so, . . .
> Buy, so buy.
>
> When you do dance I'd wish you a wave of the sea,
> That you might dance ever.
> Each your doing . . . crowns,
> In each particular . . .
> That all your acts are queens.

The correct passage is as follows:

> What you do
> Still betters what is done, When you speak, sweet,
> I'd have you do it ever: when you sing,
> I'd have you buy and sell so; so give alms,
> Pray so; and, for the ordering your affairs,
> To sing them too: when you do dance, I wish you
> A wave o' the sea, that you might ever do
> Nothing but that; move still, still so,
> And own no other function: each your doing,
> So singular in each particular
> Crowns what you are doing in the present deeds
> That all your acts are queens.

It will be seen, in the comparison, that the memory was clear as to the general import of the passage. It was also clear with respect to the most gracefully turned of the phrases. The impression was vivid of a light and sparkling, somewhat fantastically turned, manner of

compliment; and also, with regard to the phrases lost, of more artificial and less felicitious expression.

Of Byron's "Don Juan," once read with such keen attention that the whole sequence was vivid, several passages having been learnt by heart, I have a note over fifteen years after having read the poem. The opening is recalled thus:

> I want a hero, an uncommon want,
> Since every day brings forth a new one,
> Till, after cloying the gazettes with cant,
> The age discovers that he's not the true one.
> . . . my old friend, Juan.

The verse really runs:

> I want a hero: an uncommon want,
> When every year and month sends forth a new one,
> Till, after cloying the gazettes with cant,
> The age discovers he is not the true one:
> Of such as these I should not care to vaunt,
> I'll therefore take our ancient friend, Don Juan—
> We all have seen him, in the pantomime,
> Sent to the devil somewhat ere his prime.

From that point the recollection is lit up only by a series of passages and pictures, but with no determined sequence.

The most lively impression is retained of the spirit of the poem, its verve, its sparkling wit, its turn of humour, its *diablerie* of enjoyment even in shocking the prejudices of his countrymen, the sense that it is the most characteristic of Byron's poems, that in which he spoke most truly for himself, and without regard for the frowns or the applause of others.

There was also a recollection of the manner of digression, like the variations round a central theme, and also that these digressions gave the poem its charm. Then, in regard to peculiar artistic effects, there was the

impression of the growing passion of Adeline for Juan, first lightly touched on; then, after each digression, more deeply shaded in, with a sense of inevitableness; yet all without insistence.

Throughout the poem there were a great number of phrases which, for one reason or another, had struck my attention.

> Though young he was a Tartar,
> And not at all disposed to prove a martyr.
>
> Then some leaped overboard with dreadful yell,
> As eager to anticipate their grave.
>
> A wary, cool old sworder.
>
> Dusk as India, and as warm.
>
> He glanced like a personified Bolero.
>
> And seem to say, Resist us if you can,
> Which makes a dandy while it spoils a man.
>
> Proud with the proud, yet courteously proud.
>
> Excel
> The serious Angles in the eloquence
> Of pantomime. . . .
> As a poplar proportion'd or a pole,
> A handsome man, that human miracle,
> Yet there was something wanting on the whole
> Which women—certes, the sweet souls—call soul.

There was an understanding that this last verse was not correctly quoted, but no suggestion as to wherein it was faulty.

With this recollection of "Don Juan" I will compare that of a passage in which the closely continuous strain of deep thought and feeling, and the finely woven character of the verse aid remembrance:

> He who hath bent him o'er the dead
> Ere the first day of death is fled,
>
> . . . . .
>
> Before decay's effacing fingers
> Have swept the lines where beauty lingers,
> And marked the mild angelic air
> The rapture of repose that's there.

Here my recollection stops. The lacunæ are precisely those of the afterthoughts of artificial developments. There, as in most analogies, the associations between the lines and their application is strained.

Accordingly, although in some respects the recollection is more perfect than of passages in "Don Juan" formerly as well known, yet the associations are less varied, and there is less distinct impression of the place of these lines in the whole perspective of the poem.

The influence of beautifully woven verse, pleasant cadences, and good rhymes is well illustrated in a recollection of Longfellow's Prelude to the "Voices of the Night," learnt by heart and tested nearly twenty years afterwards.

Pleasant it was when woods were green
  And winds were soft and low,
To lie amid some sylvan scene,
Where, the long drooping boughs between,
Shadows dark and sunlight sheen
  Alternate come and go.

Here follows a lacuna. But throughout the poem passages are recalled with clear consecutiveness.

A contrast to this is Walt Whitman's. Here the original thought is striking for its amplitude, its profundity, its originality; so that the poem—such as "Starting from Paumanok"—brings with it a multitude of suggestions. Yet the form is faulty, although here and there vigorous expressions surge up which may astonish by their force, or even seduce by their beauty. Consequently, after having learnt many passages of Walt Whitman by heart, I find in my recollection ten years afterwards a clear idea of his intention, of his style, of

his place in literature, of his value; but I can only repeat his words in disjointed phrases:

> Starting from fish-shaped Paumanok,
> A woman waits for me.
> Tarrying and talked to by tongues aromatic.
> Have the elder races halted?
> Camerados!
> I listened to the learned astronomer.

Of Shelley I had at one time learnt by heart a considerable number of passages.

Amongst these was the "Ode to the Skylark." Fifteen years afterwards I remembered most distinctly the spirit of the poem; its form, its fine seizure, its sudden, brilliant flight, its "rain of melody." I also remembered that withal I thought certain of the verses rather far-fetched, and not in the spirit of inspiration of the rest of the poem. I remembered in sequential form thus:

> Hail to thee, blithe Spirit,
> Bird thou never wert,
> That from Heaven or near it
> Pourest thy full heart
> In profuse strains of unpremeditated art.
>
> . . . . .
>
> What thou art we know not;
> What is most like thee?
> From rainbow clouds there flow not
> Drops so bright to see
> As from thy presence showers a rain of melody.
>
> Higher still and higher
> From the earth thou springest;
> Like a cloud of fire
> The blue deep thou wingest,
> And singing still dost soar, and soaring ever singest

Then followed a verse which I remembered because, though beautiful in itself, it struck me as being a some-

what artificial fancy, a conceit, afterwards adjusted into the form of the poem:

> Like a rose embowered
> In its own green leaves,
> By warm winds deflowered,
> Till the scent it gives
> Makes faint with too much sweet these heavy-wingèd thieves.

Then, finally, the touch of self-consciousness:

> Teach me half the gladness
> That thy brain must know;
> Such harmonious madness
> From my lips would flow
> The world would listen then as I am listening now.

Of Keats I had learned a number of passages, including the opening of "Endymion." Many years afterwards I found my recollection of the passage down to "Pouring into us from the Heaven's brink" intact. The reason of this was, partly, that though I had not re-read the passage, I had often reverted to it in my mind.

This explanation applies also to many facts and circumstances which we seem to remember because of their importance. As a consequence of their importance we revert to them from time to time, and so continually freshen the recollection.

Referring to Keats's celebrated sonnet, "On first looking into Chapman's 'Homer,'" I find that I had at no time learnt it by heart; but I had at a certain period read it over frequently, and I had examined it part by part, line by line, and word by word, and by comparison also of successive versions. Then for three years I did not refer to it. I found my recollection thus:

Much had I travelled in the realms of gold,
And many goodly states and . . . seen;
. . . been told
That broad-browed Homer ruled as his demesne;
Yet had I never breathed its deep serene
Till . . . Chapman spake out loud and bold.
Then felt I like some watcher in the skies
When a new planet swims into his ken;
Or like stout Cortez when, with eagle eyes,
He stared at the Pacific . . . all his men
. . . wild surmise—
Silent on a peak in Darien.

The above fragment, imperfect and faulty, should now be compared with the original:

Much have I travelled in the realms of gold,
And many goodly states and kingdoms seen;
Round many western islands have I been
Which bards in fealty to Apollo hold.
Oft of one wide expanse had I been told
That deep-browed Homer ruled as his demesne;
Yet did I never breathe its pure serene
Till I heard Chapman speak out loud and bold:
Then felt I like some watcher of the skies
When a new planet swims into his ken;
Or like stout Cortez when, with eagle eyes,
He star'd at the Pacific—and all his men
Look'd at each other with a wild surmise—
Silent, upon a peak in Darien.

This sonnet, one of the most wonderful poems in all literature, came to the mind of Keats, accustomed as he was to the sonnet form, in a rare moment of inspiration bodying forth not merely its general atmosphere, but its salient images, and certain of its magical expressions, in associations of rapid succession already partly formed, but awaiting the luminous flash which gave them life.

But in all such poems there is much work of mere craftsmanship, much adjustment so as to secure the

necessary rhymes, and to give certain due proportion and sequence to thoughts, expressed under the conditions of a certain form. And here I find that the parts which I thought to be the essentials, the inspirational effects, were those that my memory retained with the greatest certitude.

In foreign languages I found the discrepancy most pronounced between the recollection of what is common to all languages and what is peculiar to the language itself. Thus at one time, being then quite familiar with German, I read Goethe's "Faust" carefully, and learnt by heart some of the passages.

Fifteen years afterwards I found my recollection of the general character of the poem, the delineation of the characters, the philosophic intention, quite clear; and this recollection went so far as to assure me of the qualities of the verse itself, the expression, the full-sounding phrases, the "tonicity" of Goethe's style. I had, moreover, a clear appreciation of various attempts at translation, including Shelley's, and I could tell in what way they fell short of the original. Yet of the actual words of "Faust" my recollection was very limited.

I remembered only two passages well; these by reason of frequent mental references in the meantime:

Die Sonne tönt nach alter Weise
. . . Sphaeren Wettgesang
Und ihre vorgeschrieb'ne Reise
Vollendet Sie mit Donnergang.

Then, in quite another vein:

Es Könnte kaum ein herziger Närrchein sein
Er liebte nur das Wandern all zu-viel
Und fremde Weiber, und fremdes Wein,
Und das verflüchte Würfelspiel.

Of Schiller's tragedy, "Wallenstein's Tod," once read carefully, I found, fifteen years afterwards, recollection of the actual words very limited, only a few short phrases such as:

Ich habe kein' Sohn mehr,

which I remembered from the intonation of an actor of the Herzog von Meiningen's Company which I had seen in the play at Antwerp; and the expression "Pflichtvergess'ne Kerls," which contained a point of irony not usual in German literature.

Of Camoëns' epic, "The Lusiad," read in the original with the help of good translations, my acquaintance with Portuguese being very meagre, I had, three years later, a lively recollection of some of the characteristics and a full appreciation of the justice of Humboldt's eulogy; yet, though I had committed passages to memory, only two lines remained, one being the famous:

Por mares nunca de antes navigados.

Of Dante's "Divina Commedia," read under similar circumstances, I remembered absolutely nothing verbal.

Of Virgil's "Æneid," read with care, I remembered, twenty years afterwards, not a word. I knew that "Arma virumque cano" occurred therein, but only by association with George Bernard Shaw's "Arms and the Man."

Of the "Ode to Melpomene" of Horace, learnt by heart, I found that twenty years afterwards the general "argument" remained, and a few of the words, in disjointed style:

Quem tu, Melpomene,
Semel nascentem videris
Illum non labor isthmius
Clarabit pugilem. . . .

But these I had frequently referred to mentally in the meantime.

Of Homer's "Iliad," read with attention, I remembered, twenty years afterwards, only two words, although whole scenes of the poem were clear in my mind.

On the other hand, certain phrases of German which I had learnt from hearing when a child have not only remained clear, but in such a manner I can remember a great variety of details referring to the circumstances under which I heard them uttered. In these cases attention was particularly devoted to the sound of the words themselves.

That the actual words should generally be apt to slip from recollection is easily understood. The words are, as we have seen, held in Memory by complex webs of association, and they are in themselves not generally important, but only the symbols by which we refer to the important things. In the first stages of learning a foreign language, however, the words themselves are not only of great importance, but also of a certain emotional interest; and thus many new associations are formed about them.

Amongst the most tenacious recollections are those of reasoning, and especially that form—geometrical reasoning—in which the actual objects of the reasoning are not of emotional interest, but the style of reasoning itself is very conspicuously displayed.

Having at a certain period been engaged in studies in the higher mathematics, I had occasion to revert to "Euclid," which I had read last seven years before. I had entirely forgotten what the first proposition of the first book of "Euclid" referred to, and I was obliged to consult the book.

It will be observed that the first book of "Euclid" is mainly devoted to relations affecting triangles, right-angles, and parallel lines; and the first proposition,

which is a problem, has no essential place in the theoretical considerations here involved. The series of the theorems begins with the fourth proposition, and from this I had a clear remembrance of the course of demonstrations throughout the book. This was not derived from the recollection of the forms or words used by any particular book, but simply by what I had observed of the lines of argument on which the whole chain of reasoning was conducted.

In the discussion of Reason it will be found that the base of the recollection also in the case of geometry is Association, the Memory being especially tenacious because the associations are formed through operations in which the natural manner of our experience involves continual repetitions of the Fundamental Processes of the Mind.

In regard to Memory involving principally the sense of sight, with sense of direction, and various emotional associations, I select the following example: I inhabited a house in rue Cambon, in Paris, for a fortnight. I made no special note with regard to the house. Nearly two years afterwards, and under somewhat similar conditions, I again inhabited the house. On this occasion I noted my impressions on entering. I had forgotten the number of the house, and alco the name, although the name sounded more familiar after I had recalled it than did the number. I remembered best the broadest characteristics, even some which were difficult to define because they were composed of many factors. That is to say, I remembered the general "atmosphere" of the house—for example, that it was a quiet house, well inhabited, not far from what might be called the centre of Paris, the Opéra, that the people were honest, the prices reasonable, and that one could live there with comfort and without molestation.

I remembered, also, that there was a post office close by on the opposite side of the street; that the street was busy with regard to wheel-traffic, but quiet otherwise; that nearly opposite was a dressmaking house where young girls used to sit near the window working busily but occasionally talking and laughing; that horses fell in the street frequently, and that the drivers seemed rather incompetent and cruel.

Regarding a great number of details my recollection was confused and altogether blank.

I could remember the general situation of the street with regard to other streets, but the names of some of these other streets had disappeared. With regard to some streets I remembered their names, and I also had a notion of their being near, but they seemed to float vaguely in my mind. I had forgotten the way by which I should proceed to them.

I knew that the charges were not excessive, but I had forgotten the exact prices.

I could not remember exactly how the apartment was furnished, though I had a clear impression that it was furnished in bourgeois taste, with nothing specially interesting either good or bad.

I could not remember how the apartment was heated, nor could I remember the places of things, pictures, furniture, or the like. I knew there were some pictures, and I remembered that there was a family resemblance in this style of decoration that one finds in apartments of the kind; but I could not recollect the subject, or appearance, or situation, of any picture.

I could not remember whether there was electric light in the apartment, or what was the style of the tables and chairs; I had forgotten whether I entered the apartment by a latch-key.

I remembered that the person in charge, who may

have been the proprietor, was a dapper little Swiss, intelligent, polite, and not neglectful of his business. I had a vague impression of his appearance, but I remembered him distinctly as soon as I saw him again, and this experience brought a number of other recollections in its train.

I remembered that his wife was a medium-sized, pleasant-faced young woman, but beyond that I could get no precise indication. When I saw her the recollection did not appear so clear as in the case of her husband, but I remembered why I thought her pleasant-looking, as she was healthy, well-balanced, and inclined to smile easily.

In this whole experience it will be seen that the recollection was perfectly good in what I had regarded as the essentials; that is to say, as to whether it was advisable to live in the house; and that the details became dimmer unless some special circumstance associated them with something more or less striking that had particularly demanded attention. Thus I remembered the horses falling as an unpleasant incident, while the work-room of neat-handed and intelligent-looking work-girls struck an interesting note.

I noted similar experiences with regard to other houses, one especially in rue Chaptal which I had inhabited for years, and to which I returned after the lapse of some years. Here, though the whole scope of recollections was larger, yet the persistence or the disappearance of the impressions conformed to the same general principles.

I remembered clearly the impressions that may be summed up as the "atmosphere" of the place. I also remembered a vast multitude of details which were associated with some impression of pleasure, pain, surprise, gaiety, sorrow.

But I had great difficulty in recollecting, for example, the place of the piano. I remembered at length that a young concert singer, since famous in the world of music, Miss Amy Castles, had sung at the piano, and that the situation of the piano too close to the wall had tended to deprive the voice of some of its fine effects. Then I remembered a number of circumstances relating to the piano.

I have other notes regarding houses where a picture has formed the most dominant feature in the impressions of recollection. In these cases there have been special circumstances which have served to weave a great number of strong associations with the picture. It may be admiration or the reverse, or the curiosity of puzzling out the meaning of an obscure symbolist picture, or it may be some strong impulsion towards action or inspiration; or it may be simply continual repetition due to the picture being placed in such a position that it was frequently, and for long periods, in view.

In another case, I had visited a house where I had noticed a picture, a pencil-sketch by Leandre. Years afterwards, owing to a change of residence, I saw the picture again in the hall of another house in entirely unfamiliar surroundings. I had forgotten the picture, but when I saw it again I recognised that I had been acquainted with it. There were two figures in the picture—one of a fine-looking man with a majestic beard, and the other of a thin man. I found that I had entirely forgotten the thin man; on the other hand, I not only recognised the majestic man, but I remembered why the picture had been impressed on my mind, viz. by the resemblance of the majestic man to a lunatic with whom I had some acquaintance.

Immediately afterwards the recollection came to me that the picture was by an artist whose style I knew.

At that moment I observed the name Leandre, and that awaked vague associations which satisfied me, though the picture was not a characteristic example of that artist's work.

Here was a case, but fortunately explained in this particular example, of a mental phenomenon frequently observed. One sees an object, or a composite of objects; an impression is produced in the mind: "I have seen that before, but I cannot recollect where."

This impression may be created in various ways. One may really have seen the thing in question before, as I had seen the picture by Leandre. My recollection of it was so faint that I should never have remembered where I had first seen it but for the fact that I met with it the second time in the house of a friend, and I knew that he had changed his domicile. Then there was the vague recollection of the artist's name, which, even before it was recovered, was (see p. 244) stirring a circle of associations formed round it.

The accident of seeing the name made my vague impressions definite.

But if I had not had these helps to a recollection complete enough to satisfy me it appears, nevertheless, that I would have known that I had at least seen the picture before. There were associations, not only with ideas independent of the composition of the picture such as I afterwards discovered, but with the parts of the picture in itself. I expected, for example, to find with the expression of the eyes, and the form of the nose, the flowing beard of the majestic personage. But there is a recognition in the mind of the fulfilling of the expectancy of associations. It is this that gives us the feeling of certitude in our recollections; but in the ordinary cases the recollection produces associations which, step by step, we can link on with some striking objective experience,

and so again, step by step, to the actual external world which at the given moment surrounds us. It is precisely this recognition which distinguishes realities from dreams.

Now when the recollection is not complete we form associations then and there with the object and the aspect of the external world of the moment, and so we recognise that we are not dreaming; but we cannot link the impression of fulfilment of a certain vague expectancy of associations also with the external world.

Similar impressions of the repetition of a former experience may be produced in a case where no former experience really corresponds. This is when the actual experience has certain elements of resemblance with some past experience; so that the impression of fulfilment of expected associations may arise confusedly, while at the same time there is no chain of associations that would link the experience to the actual external world.

We have already seen (cf. pp. 237 *et seq.*) that a series of associations not yet made evident may nevertheless make some impressions of their existence. Now the fact that they are not evident indicates a want of definition in certain of the associations correlated to converging paths (cf. pp. 95-99). Thus it may happen that experiences, not really related to other associations so as eventually to form a consecutive chain, may nevertheless produce an impression of stirring vague formerly-known associations.

A theory has been put forward, from observation, that such a condition as the vague impression of a formerly-known experience really implies a state premonitory to epilepsy.

It is possible that such a condition might occasionally precede epilepsy; but from the explanation given it will be evident that it is a condition that might happen with

any one. It is simply a condition brought about by faulty recollection of a complex, a portion of which is remembered particularly with regard to associations between the component parts.

Now this faulty recollection may be simply due to the original object not having been observed in such a way as to make much impression—at least in all its parts; or to lapse of time (cf. p. 261); or to absorption of mental energy, or attention by other matters; or to inhibition due to associations arising along wrong tracks; or to fatigue which may be temporary. None of these conditions has any special connection with epilepsy.

On the other hand, if such a condition happened without any of the causes suggested it would indicate some inefficiency in the correlated part of the central nervous system. This might be due to a variety of causes, as for instance to defective nutriment following upon failing heart-action. Or it might be due to causes which, if progressive or cumulative, might lead to epilepsy.[1]

[1] In applying the principles that we have learnt we have been led to a natural explanation of the problem of imperfect reminiscence. To those interested in psychological questions, but who are not experts, it would probably be surprising to find how much literature has grown up around this problem of the *déjà-vu*, as the French call it. Pierre Janet has devoted his powers of analysis, as well as his great physiological learning, to the elucidation of the matter; and, amongst those who have in some way or other contributed to our knowledge in this regard, may be mentioned Leroy, Emile Laurent, Méré, Kindberg, as well as many famous writers—Shelley, Lemaître, and Paul Bourget.

Lemaître calls attention to the profound feeling of surprise and of strange alarm on recognising as *déjà-vu* a place never before entered. Sensational examples of the sort do not disturb the explanations already given. A clue will be found when we study the case of Sir Walter Scott thinking he saw Byron (see p. 313). The observer is not in a normal condition, and the fancy plays a great part. The emotional effect is not produced directly by the *déjà-vu*, but by the associations involved in the things of fancy also. The *déjà-vu* is here only the fuse of the charge of dynamite.

Another aspect of the problem, that of "localisation," has been discussed by Vaschide in *l'Année psychologique*, 1896, and by various others. What

A study of Memory relating mainly to operations having a strong physical base is afforded by the following example:

While waiting for lunch in a restaurant in Bayswater I tried to make a knot called the "Tomfool's knot," which I had learned many years before on shipboard, and which I had since occasionally practised. But at least six years had elapsed since I had made the knot. I found myself unable either to make the knot or even to recollect how it looked when made.

Then there was a recollection that it had a symmetrical appearance and that in the process of forming it the cord should be "thrown" in a certain style. I tried again and again. Then I laid the cord down. Nevertheless, although I had failed I had a strong impression that the obstacles to the recognition of the proper methods were being loosened.

Vaschide here means by localisation is localisation in the whole scheme of the observer's reminiscences. He finds that localisation is not always determined by Association, but is sometimes immediate and direct. The distinction will disappear, however, if we bear in mind that, by Association, it is not intended to imply a series of associations through a chain of ideas already formed.

Now consider what is meant when one asks, Have I seen that house before? We have observed something, either seen before or not; and a series of ideas is invoked which leads step by step to the present time. If the house itself had been known before, then the reminiscences would be associated with the present reality, with the multitude of its impressions, and with the liveliness of its discriminations, all in satisfying concord; but if the house prove to be different to that associated with the first impressions, then we recognise, with a certain shock of surprise, that we are in a new dynamic relation to the series of ideas that has dominated the mind. We wake as from a dream. Dreaming is an extreme case of immersion in ideas where the check of present reality is not available.

We have, then, three distinct conditions, which may be indicated thus: normal memory, where the present reality is in accord with the series of ideas (really reminiscences); illusions of the *déjà-vu*, where the present reality is not in accord; dreams, where the present reality does not appear as a test.

Allied problems are dealt with by various authors, as, for example, Victor Egger, "Le Souvenir dans le Rêve"; Ed. Goblot, "Le Souvenir des Rêves"; Paul Tannery, "Sur la mémoire dans le Rêve"; Bonatelli, "Il fenomeno della ricordanza illusoria"; Kräpelin, "Ueber Erringerungs-falschungen." An article in *Mind* (1887) by F. H. Bradley, "Why do we remember Forwards and not Backwards?" may well be consulted.

Subsequently I took up the cord again; I made an effort which resulted in the cord coming right on one side. I tried twice again; then, in another trial, I found that I had accomplished the knot, but that it had become undone immediately. I did not know how I had made the knot, and I was unable to repeat the performance.

I again laid the cord down. Then, after an interval, I took it up again, and after two or three tentatives I made the knot, recognising also the position of the hands, and remembering, at the same time, that formerly I used to adopt that position as the basis of my manipulation.

Now in all this work I had made no effort whatever to revive the image of the knot, or to trace out logically the consequences of moving my hands in a certain manner and manipulating the cord. I was endeavouring to recover the operations which had become obscured, and with regard to them I made no effort to visualise them or recall them to consciousness in the form of any image.

During my former frequent experiments with the knot I had formed certain paths of associations which had, in relation to the knot, acquired facility of stimulus in a certain order of succession. With lapse of time, and having also learnt other knots, and having been accustomed to use them, I had brought about that the former paths had lost their facility. The tentative efforts finally resulted in finding the right paths, and, when once the right paths were restored, the facility rapidly increased.

Thus, although recollection had become dim, yet a considerable basis for reconstruction remained, for the experiment was not like that which would be observed in tying the knot for the first time, or in inventing it. It will be observed, too, that a considerable part of the

work of reconstruction was subconscious, for I did not even know what I required. I knew that, with each failure, I had not got the knot, for the wrong knots did not awaken any new lines of associations, as I expected, and rightly so, that the proper knot would do.

Part of the process had formerly become automatic; that is to say, referring to physical correlatives, the proper lines of association had become facile, and incongruous lines inhibited. The wrong tentatives helped to inhibit wrong paths, and succeeding efforts made the discovery of the true path easier.

But when one considers the whole mechanism implied, the distribution of the nerves in the hand, the origins of of these nerves in the brachial plexus, the rearrangements of the strands of the nerves in the complex network of the brain; and when one further, on this graphic model, works out the corresponding processes, and when the unconsciousness of much of the process is considered; the conclusion seems legitimate that, as far as physical correlatives are concerned, it is not only in the brain, but *in all parts of the nervous system, that the physical base of* Memory receives its impress from external stimulus, and has, in consequence, its paths facilitated so as to affect repetition and recollection.[1]

Another curious incident which came within my observation may be added. The lunatic acquaintance already mentioned, who had been a sailor in his day, saw a sou'wester hat which he greatly admired. He requested to be allowed to try it on. In doing so he stopped suddenly in surprise, as though something in the recollection puzzled him. After a little reflection he

[1] B. Bourdon has written on Memory for complicated mechanical processes. Henle has taken a more general survey, "Ueber das Gedächtniss in den Sinnen." John A. Bergström (*Am. Jour. of Psych.* 1893) describes "Experiments upon Physiological Memory by means of the Interference of Associations." Vaschide has left behind an interesting book, "Essai sur la Psychologie de la Main" (1907).

remarked, "I remember that at that time I did not wear a beard."

The action of trying on the sou'wester awoke a series, not merely of recollections, but also of operations which did not all make an impression in consciousness. The series was checked owing to the new circumstance of the beard, and this circumstance threw the mind back with a shock to the previous recollection. Then came, distinctly enough evidently to allow the criticism to be made, the recollection that nothing formerly impeded the movements as performed, which, being traced in idea to the point of its arrest, drew attention to the beard as the cause of the check. In such a case it appears certain that there was not at first a clear image of the previous conditions, but simply a series of associations connecting the sight of the sou'wester with the desire of putting it on, which then awoke the series of automatic or partly unconscious processes.

In all our movements we are continually making use of a series of associations to which we have so trained ourselves that they have become automatic. The course of our heredity through countless generations has made certain of these paths so facile at each stage of our development that they serve in part to determine the subsequent development. Many of the paths have become so entirely automatic that they cause no clear and distinct impressions in consciousness, and even when any check happens in their regular course we cannot form a clear image of the condition of the process so obstructed.

We walk without knowing how we walk. Finally, we think without knowing how we think. Even when we desire to learn new combinations of physical processes, such as in bowling a cricket-ball, in playing a violin, or dancing a waltz, a great deal of the actual co-ordinations formed, of nerves and muscles, are beyond the immediate

reach of our consciousness. Our method of learning is not based on an analysis of the factors employed so as afterwards to direct them to desired combinations; it is rather by the persistent expression of the will forcing the physical organism to various tentatives of which those that are suitable are retained, and by incessant practice organically registered. The physical basis is thus formed of a series which perhaps only in incidents makes itself evident in consciousness.

It is generally very difficult to trace out in the organism the successive phases of such an education. One of the most interesting examples with which I am acquainted, where it has been possible to trace out and measure the successive physical modifications due to the effort to conform to a mental impression, is contained in certain studies of Dr. Marage presented to the Académie des Sciences (Jan. 12, 1909) by M. d'Arsonval. Dr. Marage has studied the mechanism of singing by means of photographs representing the vibrations corresponding to the notes emitted, and also by observing the condition of certain of the muscles involved at the moment of emission of these notes.

Singers have generally noted that, in ascending or descending the scale, a curious phenomenon makes itself known by symptoms arising in the larynx. At a certain moment they feel as if they were changing the vocal cords. They say they change from chest-register to head-register, or vice versa. Dr. Marage has shown that if this change is very marked the crico-thyroid muscle, which is the chief agent in making the vocal cords tense, contracts brusquely. With highly trained singers, however, the contraction of the crico-thyroid muscle is gradual, and there is no marked change from chest-voice to head-voice.

A considerable part of the education of a singer

depends, accordingly, on the education of the muscles of the larynx, although the singer may be completely unacquainted with the mechanism of the larynx, or even with the existence of such muscles.[1]

Nevertheless, the correlatives of Memory become established in these muscles and their nerves, and, following impulses derived from certain associations, the whole series is reproduced, while the result only as evidenced by the succession of notes is presented in consciousness.

SELECTED EXPERIMENTS

In most of the preceding observations a certain emotional factor enters, and this would have a different influence with different persons.

I have, however, a series of notes exactly recorded of various experiments which I made at different times with regard to Memory of objects presenting no emotional features whatever. Certain of these experiments extended over years. That is to say, I experimented at one period, and then returned to the same series three or four years afterwards, and pursued a similar course of experiment, and then compared results.

The objects I chose to be memorised were the formulæ of integration as shown in books in the Integral Calculus. I had already been familiar with them, but, after having devoted much time to other studies, I discovered that I had forgotten them almost entirely.

The system I pursued was to commit to Memory

[1] Recently several men of science have turned their attention to the interesting question of the mechanism of the throat as a musical instrument: Dr. W. A. Aikin has published "The Voice, and Introduction to Practical Phonology," in which he joins issue with Helmholtz in regard to the conditions of production of certain vowels; and Drs. E. J. Moure and A. Bouyer have written on "The Abuse of the Singing and Speaking Voice."

a certain number of formulæ—ten on the first day. I repeated these on the following day, and learned ten more. I then repeated the first ten, and marked the errors of recollection. I counted these errors, then learnt ten more; then, on the following day, repeated from the beginning. My notes say that, having learnt the first series of ten so that I could repeat them without mistake, I found that, on learning ten more, and then on the following day repeating the first ten, I failed to remember certain points in some of them: for example, whether a plus or minus sign was found in a certain place, whether x – a or a – x occurred in another place.

I then memorised these points, to which my attention had been thus specially drawn, by associating them with some circumstance of the integral. I then learnt ten more.

I discovered that, on the fourth or fifth day, I sometimes failed to remember what had been perfectly familiar during the other days, and I observed that the faults of Memory generally occurred in those points to which my attention had not been specially called, and where I had not created associations by some direct attention. They were really the points in which the recollection by simple repetition had seemed to be the most facile. I also found that as I learnt still new formulæ the task of remembering the previous ones became more difficult, faults in Memory being found where previously the recollection had been quite sure.

All this is in accord with the observation regarding Memory, that recollection is more tenacious with regard to things associated with the more fundamental, and therefore more frequently exercised, processes of the mind.

The results indicate, too, that a great distinction must be made between two forms of mnemonic effort.

Compare, for example, the memorising of the name of a person well known, but not having been present in the mind for a long time, and the memorising of words or formulæ having no special interest, but which have quite recently been committed to Memory for a special purpose. If at the beginning we are unable to recollect the first name, it will at length by means of the revival of various associations come back to us. On the other hand, the recollection of indifferent things designedly committed to Memory may be quite fresh for a day or two, but it soon fades away, and then cannot usually be restored.

If now we refer to the graphic representation of physical structure correlative to mental operations (cf. pp. 95–99), it would be found in accordance with results if we regarded the actual elementary work of Memory, of adhesiveness of impressions, as a constitutional faculty of each person, and that its value depends on a series of constitutional conditions (cf. pp. 236, 244 *et seq.*).

Repetition, especially when the constitution is young and the organism developing, is a form of physical exercise of the brain which is finally represented in brain development and structure. Hence the Memory of the things which have been familiar to us, or the operations which have been habitual with us, in childhood produce a condition which is veritably organic.

But the Memory of things of recent date is correlated to excitation of nerve filaments which have lost the power of development, and to some extent of responsiveness to excitation. When the temporary excitation disappears, the Memory of the thing diminishes.

An application of these remarks may be made to

what is called "cramming" for examinations. The Memory for things learnt in this way, without deep associations, is merely temporary, and in fact—still speaking in terms of the physical correlatives—it may be found that the over-excitation of a part of the brain may blur the connected paths between a series of cells affected, and so diminish the total Memory. This is a form of degeneration which is the inverse of that form of development which results in the organic associations of early memories.

An extreme case of this would be the obliteration of brain tracts, and their psychical correlatives, by some form of brain disease or brain degeneration.

Similar results may happen even from diseases which are not originally brain diseases, such as in a form of thickening of the arteries, which deprives the parts affected of their efficient nourishment by the blood. Again, some forms of heart-weakness attract the attention of the physician by reason of the patient complaining of failing Memory. In this case the direct effect of malnutrition of the nerve-substance of the brain is made evident.

We may now return to consider further observations in the experiments already mentioned. I found by renewed experiments that repetition without distinct attention is less efficacious in fixing a formula than the special act of attention brought about by the correction of an error, and further that a formula repeated twenty times without an error may on the next occasion become vague or uncertain in some particulars.

If the formulæ be always repeated in a certain order they are easier to remember in that order than in any other order. If they are repeated about the same hour every day they are easier to remember then than at any

other hour. Formulæ learnt at night, even when the brain is in good working order, are more difficult to recall next morning than formulæ learnt during the previous morning.

These experiments I resumed three years afterwards, not having in the meantime specially referred to these formulæ, though continuing to be occupied with various studies in mathematics. I found that of the first thirty formulæ I only remembered six distinctly, but also two others with some uncertainty. I had a faint recollection of some of the others, while some seemed entirely blank. I looked over them once.

A week later I tried the same series. I found that I remembered the first eight quite distinctly. As to the others, I retained only impressions of relative complexity of the formulæ.

I then studied the first twenty-five formulæ for twenty-five minutes. I found that though my recollection had been apparently blank with regard to some of the formulæ, yet that in relearning I had the sense of some of the formulæ being familiar. I recollected observations founded on their form, but which had not been strong enough to reproduce the whole formulæ.

I found, on trial, that altering the order of the formulæ constituted a difficulty. I continued the study for ten more minutes, thus making it thirty-five minutes in all; then tested recollection. I found the first error in No. 13. There were errors of some sort in four others: while in four cases I failed to remember the formulæ even badly.

Then after a short interval I studied the formulæ for seventeen minutes, and then tested the recollection. I found that the errors were fewer. The errors were not quite the same as before, nor in the same formulæ. Two of the previous blanks still remained blank.

Again after an interval of a week I resumed the

study. I found that I remembered the first ten perfectly. There were slight errors in all the others, except in the case of two which remained blank. One of them was a formula which had been blank on a previous occasion; but on the trial immediately preceding these now in question, it was perfectly remembered. The other blank had been blank in every trial.

I then studied the formulæ for thirty-five minutes, and again tested recollection. I found errors in one point in each of six formulæ out of twenty-five, while the formula which had always been blank remained so.

A week later I tested my recollection. I found Memory weak in the formulæ in which it had previously been weak, although the errors were not the same. The formula which had hitherto been always blank in recollection was now perfectly remembered.

I then tried, out of its turn, one which had hitherto been perfectly remembered, and found that my recollection was very defective.

This showed that distinct associations had been formed between the numbers representing the order and the result of the integration, this association being stronger than that of the form under the sign of the integration.

At the period during which these last experiments were made, I was engaged in other studies, and had a considerable amount of extraneous work on hand.

Three years previously, that is to say immediately after the study of the series first recorded of formulæ of the Integral Calculus, I had undertaken a much more fatiguing course. I committed to Memory the formulæ for elliptic functions, which are amongst the most complicated, and at first sight most unattractive in the range of mathematics. At that period I was but little acquainted with the theory of the subject, and that made the learning

more difficult as I had fewer points for association. I selected this series in fact on account of its arduous nature and because it was, under the circumstance, an experiment in memorising a number of expressions conveying no meaning.

I had more leisure than during the experiments last mentioned, and as I had been employing myself with mathematical formulæ for some time previously, it would appear from the results that both these circumstances aided me in the experiment.

My plan was to learn seven formulæ, then repeat them; then to learn seven more; then repeat all from the beginning. I found that as I proceeded it was easier for me to add new formulæ, and to retain the recollection for short periods; but that as I repeated from the beginning the new formulæ rendered it more difficult to retain the formulæ already learnt.

When, however, I made an error, and then corrected it by observing some significant fact about the formulæ, the recollection of these corrected formulæ became firmer than before. In three days I could repeat eighty formulæ of elliptic functions without error. At the same time my recollection was made firmer, by equivalent practice, in the case of the formulæ of integration previously learnt.

Adding a few formulæ day by day I had in three weeks learnt one hundred and sixty-seven formulæ which I repeated with but three errors. Immediately afterwards I repeated sixty-five formulæ of integration without error of any sort.

I found confirmed that even after many repetitions the weakest points were those which, even if at first learnt with great facility, were not associated in the mind with some facts of symmetry, or forms of remarkable sequence, or with some special observation.

I observed that the repetition was easier if it took place at a certain hour every day than if attempted at another time of day.

I also found that there becomes established a certain natural pace at which the repetition is the easiest, and that if that pace be forced blunders are apt to ensue.

The fatigue of recollection and repetition diminished with successive repetitions. It will be noticed also that though the absorption of energy and of emotional interest in one field of study will generally diminish the recollection of the facts of previous study; yet when the mind is greatly occupied with one field of study the learning of an extensive series of facts seems up to a certain point to be favourable to the recollection of another series of a somewhat similar character.

About three years later, having occupied myself with other severe studies during the interval, I returned to the examination of the formulæ of elliptic functions. I had acquired some knowledge of the theory, so that my mind was more receptive with regard to the formulæ. I found, however, that I had no recollection of the series that I had under other circumstances perfectly memorised. I recollected only the impressions of the complexity and of the length of the formulæ.

After studying the formulæ for some time recollection began to appear, and now the recognition of symmetries, and sequences, seemed to be keener than previously.

I have notes of another series of experiments where the task was to learn and remember the doses of drugs as given in a Pharmacopœia.

In this case there is found a slight stimulus in the fact that the series has a more evident meaning than in the case of the formulæ of elliptic functions. Moreover, the actual separate facts to be committed to memory in regard to each drug were simply the lower and upper

limit, generally in minims or grains. There were also aids in the fact that on each page the same dose would be found to apply to several drugs. Moreover, certain drugs, such as arsenious acid, would be expected to have less doses than others such as acetic acid.

The task, therefore, was much less difficult than that of memorising and repeating without fault the intricate formulæ of elliptic functions, even though the total number of drugs was about four hundred.

I had learnt several pages of the drugs in succession until I could repeat them without error. I then tested my recollection in this way. I selected three pages, which for reference we may call the series; a, containing 16 items; b, containing 11 items; c, containing 20 items. Of these I had revised the list in a three hours previously, that in b one day previously, that in c two days previously. I have a note that the test took place at night when I was tired after a hard day's work. The result gave no error in 16 of series a; 1 error in 11 of series b; 11 errors in 20 of series c.

Next morning, while fresh, I again tested the series which had been looked at, in order to verify the results, on the previous night. I found 1 error in 16 of series a; no error in 11 of series b; 10 errors in 20 of series c.

I have a note, however, that in regard to series c, although I did not seem ostensibly to have made much progress, yet there was a clearer notion based on relative quantities, although I was "hazy" about exact numbers. I noticed also a tendency to associate the dose with the position on the page, and the sequence of the drugs.

This remark indicates how in storing up in Memory associations apparently the most simple, and indifferent to emotional interest, we yet find by appropriate tests that associations are specially formed with regard to

every circumstance of which we take cognisance in regard to the object.

I then tested three other series, which may be called d, containing 7 items; e, containing 21 items; f, containing 23 items; and which I had not referred to for three days, four days, and ten weeks respectively. In these the errors were: 7 in 7 for d, 21 in 21 for e, 22 in 23 for f.

But even here the previous work had not been entirely lost, for there was a recollection of relative quantities of different drugs, and also of the relative magnitudes of the lower and upper limits. That the previous study of the series had not been in vain became shown also by a test which I instituted between this series and a new series of a similar kind, which we may call g, containing 23 items.

The first operation began on the night of the last day mentioned, when I was again tired. I tested series f, and found 6 errors in 23. I then memorised the series for two minutes, and found 1 error in 23.

Then I took series g, and learnt the list for six minutes. I found 3 errors in 23. I studied the series g for another two minutes, and found no error in 23.

The following morning, when fresh, I tested the series g, and found 3½ errors in 23; then, after one minute's revision, 1 error in 23. I then tested the series f and found 3½ errors in 23; then after one minute's revision, no error in 23.

On the following day I again tested the series f and g. I found ½ error in 23 for series f; and 3 errors in 23 for series g. I studied series g for one minute, and found no error in 23.

I then took another series, h, which I had not referred to for ten weeks. I found only vague recollections of relative strength, no precise number.

After studying for five minutes, I found 2 errors in

16; each only in one of the limits. I studied the series for one minute, and found 1 error, in one limit, in 16.

Every day I instituted some test of this nature, and I continued the experiments over three weeks.

My notes say that the influence of fatigue in diminishing recollection may be stated thus:

If we have two series, a and b, and the facility and correctness of repetition seem equal at a given time, but if that be because the series b, though committed to Memory with fewer repetitions, has been more recently learnt than the series a; then fatigue will have a greater effect in causing errors in series b than in series a.

I at length arrived at a point when I could repeat the doses of all the drugs without error. I then ceased to refer to them for eleven days. Then I tested my recollection again, having in the meantime increased the number of items in the series. I found in series a 1 error, at one limit only, in 23; in series b, 6 errors in 20; in series c, 4½ errors in 24.

In all the rate of recollection had diminished remarkably, and there was a sense of insecurity even in some cases where the right numbers were given. I found also that my recollection was most certain in cases where I had especially noted corrections previously on account of errors. Also the recollection was perfect in a few cases where I had invented mnemonic aids.

I resumed these trials three years afterwards, beginning under conditions similar to those of the first test, viz. at night when tired.

I found 23 errors in 23 in series a, and 23 errors in 23 in series b.

I then studied series a for two minutes, and then found 10 errors in 23. I then studied series b for six minutes, and found 2½ errors in 23.

On the following morning, while fresh, I found 18

errors in 23 in series a, and 12 errors in 23 in series b. After two minutes' revision I found 3 errors in 23 in series a. Then after six minutes' revision I found no error in 23 in series b. I observed also in both these cases that there was a clearer appreciation of values; that drugs having like doses were better associated; that there was a more marked attention in regard to those doses which were exceptional; that many circumstances of similitude, which I had not noticed on the previous night, now became manifest.[1]

[1] Various writers have treated of analogous subjects. Cf. F. W. Edridge-Green, "Memory and its Cultivation"; Ed. Pick, "Memory and its Doctors." Guicciardi and Cionini have written on the "Effect of Practice in Memory" in the *Riv. Sperim. di Fren*, and Fouillée on "La Survivance et la Sélection des Idées dans la Mémoire" in the *Revue des Deux Mondes*. At an earlier period H. Höffding has studied the effect of repetition, "Die psychologische Bedeutung der Wiederholung," *Viertelj. f. w. Philosophie*, 1883.

SUMMARY OF CONCLUSIONS WITH REGARD TO MEMORY

It may be well to exhibit in summary statements all the observations which we have now gained respecting Memory:

Memory is not a special and separated faculty. Every idea presented in consciousness is reproduced by the Memory that appertains to itself.

Every Unit, or every element of a complex, has its own Memory, and its own associations, some of which may be stronger than the associations formed by the element with the other elements of the complex.

Thus there is no such thing absolutely as good Memory or bad Memory; just as there is no such thing absolutely as muscular strength, considered apart from the strength of each muscle in turn. A man may have almost all his muscles well developed, and yet have some particular muscle weak; or a man may have most of his muscles comparatively weak, but have certain muscles strong. Similarly with Memory, one may have good Memory for certain ideas, but not for others.

Memory may be well considered in regard to its physical correlatives, the nervous structure of the brain.

Memory depends on the quality, the general good health, the nutrition, of this physical base. Memory becomes weakened with respect to certain ideas when the blood supply of the corresponding physical base becomes insufficient.

Generally speaking, any sources of degeneration of nerve-substance tend to impair the Memory.

The pure plastic effort of retention can be "cultivated" only by developing the quality of the physical base.

Memory can be "cultivated" by various arts which

form associations between the idea memorised and some series artificially well established or frequently met with in experience. Mnemonic aids are often useful in this respect.

Memory with regard to any series may be stimulated by repeating the recollection of a series whose physical correlatives are adjacent to the first.

Memory for any set of facts, or series of objects, is increased by moderate exercise of Memory with regard to facts, or objects, of a like character.

Mnemonic aids are most useful when they form direct and strong connection with frequently recurring and well-known series.

The best system in general is to classify well the objects to be remembered, to make associations corresponding to frequently repeated operations of the mind, such as in reasoning; to increase the lines of associations in all directions, in the manner expressed by "thoroughly understanding" a subject.

Memory depends on the emotional disturbances at the time of receiving the impression. Pleasant sentiments, surprise, or, though probably with less force, the opposite of these, aid the Memory.

Repetition is a powerful agent for securing remembrance. Repetition is especially valuable where matters of indifferent interest must be remembered for comparatively short periods of time.

The more recent the original impression, or the more recent the repetition of the exercise of recollection, the more facile and sure is the Memory.

If two series are held in Memory so as to be reproduced at a given time with equal facility; and if this be due in one series to many associations of old date, and in the other to constant repetition some little time previously; then after the lapse of a comparatively

short interval the recollection of the second series may have disappeared, while that of the first remains but little altered.

Memory may be greatly diminished by fatigue, even to temporary extinction.

Memory may be entirely obliterated by some physical condition, such as brain disease; or even, at length, by disuse.

Memory may be greatly diminished with regard to a certain series of ideas, if, even without fatigue, the mind be much occupied with objects having no associations with the first.

Memory may be diminished by any condition contrary to certain favourable physical conditions, such as adequate but not excessive stimulation of the area physically correlated, and a general physical equilibrium not productive of incongruous associations.

Memory may be temporarily impeded through fatigue or unfavourable associations, and in that case the effort of forcing the Memory may have an effect contrary to that intended. It is then generally better to seek to restore recollection through other associations and without much effort.

The rapidity of recollection is generally greater in cases where the recollection is more sure.

Memory is aided by producing external conditions, as of time and place, similar to those that prevailed on the occasion of the original impressions.

The physical substratum correlated to Memory is, in part, composed of nervous structures whose activities are not directly represented in consciousness.[1]

[1] One of the most interesting speculations of a profound thinker on Memory is that of Hering, who believes that Memory may reside in every cell of the body in a manner suggested by the analogy of an algebraical curve being contained implicitly in each of its elements. Hering seems to me to push this theory a little too far in the direction of definiteness; and also, while saying true things, to

throw the importance of Memory out of the perspective: "So here we see that it is Memory to which we are beholden for almost all we are and all we know." I would refer the reader to Hering's own work, "Ueber das Gedächtniss als eine allgemeine Funktion der organischen Materie," which will be found in Ostwald's "Klassiker der exakten Wissenschaften." Cf. also Max Verworn, "Die Zelle als physiologische Grundlage des Gedächtniss," *Zeitsch. f. allgem. Embriol.*, 1906.

In the preceding account of Memory I have relied as far as possible on my own observations and experiments, for beyond that lay a perilous sea of literature. It was from Ebbinghaus that I received the suggestions that led to some of the foregoing experiments. His book, "Ueber das Gedächtniss," should be consulted (it has been translated); also that of Max Offner, "Das Gedächtniss"; the works of Ribot, "Les Maladies de la Mémoire"; of Paul Sollier, "Le Problème de la Mémoire," based on the considerations of physiology and of pathological conditions; Bergson, "Matière et Mémoire," which already contains the germs of his "Evolution Créatrice"; Adolf Lasson, "Das Gedächtniss" (1894); Ferri, "Le Malattie della Memoria"; Ch. Richet, "La Mémoire élémentaire"; Biervliet, "La Mémoire"; an article by I. Madison Bentley in the *Am. J. of Psych.*, 1889, which refers extensively to previous treatises; and various works of Binet and Féré, such as Binet's "Les grands calculateurs et joueurs d'échecs," which contains many useful observations; H. Piéron, "L'Evolution de la Mémoire"; and various passages from Taine, Höffding, Meumann, and Maury.

With regard to special studies there should be mentioned: V. Henri, "La Fatigue intellectuelle"; W. Elder, "The Physical Basis of Memory"; various references in the works of Herbert Spencer, Maudsley, Wundt; Bourdon has written on the Influence of Age in Immediate Memory; G. Müller, G. Schumann, Bigham, Kirkpatrick, and Smith on Retentivity; Beaunier, in a wider scope; W. Lewy has studied Memory in regard to tactile impressions, and to lengths of lines; Bernardini and Ferrari have studied Memory in regard to Music; Wolf in regard to Sounds; Lehmann in regard to Odours; and Arréat has written interestingly on Memory and Imagination.

The journals devoted to Psychology have also contained from time to time remarkable articles on special studies of Memory, as for example: "Un cas d'association latente," by E. Goblot, *Revue philosophique*, 1909; "Histoire d'un Souvenir," by F. Paulhan, *Journal de Psychologie*, 1904; T. W. Harris, "On the Associative Power of Odours," *American Journal of Psychology*, 1908; E. B. Titchener, "Affective Memory," *Phil. Rev.* 1895. Mentz and Kiesow have carried out various experiments in recollection somewhat similar in principle to those here offered. Of older writers, Aristotle and Roger Bacon have written interestingly on Memory.

These few indications will serve to point the way to the literature of the subject.

Book II

REASON

PART I

CHAPTER I

§ I. Formal Examination of Operations of Reason

The study of Reason is by no means facile; and the method of exposition here adopted is therefore that of examining simple cases by aid of the Fundamental Processes, and then proceeding to immerse the subject in wider and deeper relations until the whole scope becomes comprehensible. Let us begin with a tentative definition.

Reasoning is the series of processes by which we proceed from one set of facts, established in certain relations, to another set of facts established in certain other relations; or, as we may express it in language less general but more easily understood: Having given a certain true proposition, reasoning is the mode by which we ascertain that a certain other proposition dependent upon the first is also true.

We cannot at this stage insert any qualifying phrases, such as "by a rigorous line of deduction," or "by unexceptionable proof," or "by steps consistent with the laws of thought," for the problem we have set before us is that of ascertaining in what deduction really consists, or what are the laws of thought, and finally,

what in this regard is the veritable meaning of the words "ascertain" or "proof."

The preceding investigation has already greatly cleared the ground, as might have been expected from the fact that the investigation of the problem of reasoning itself led by successive steps of analysis to the establishing of the Fundamental Processes of Mind (cf. Preface).

If therefore these Processes have been correctly exhibited—and the whole of the previous exposition has had in view that fact—then any mode of proceeding from the truth of one supposition to that of another must be compounded of these Fundamental Processes applied to the actual objects that occur in the experience.

All these Fundamental Processes must be intact in a mind that reasons correctly. It may be well to consider the consequence of a failure of any of them.

Failure of Immediate Presentation must necessarily lead to error in so far as that process is concerned. The extreme cases are where there is no mental phenomenon at all evoked by an objective phenomenon. A man blind from birth cannot reason about the visual aspect of things.

He may certainly form ideas even about colours. Thus sensitive blind persons who have considerable culture assure us that they think of the colour red as of the sound of a trumpet (cf. p. 57). A person not blind, but possessing a highly sensitive organism and a faculty of subtle introspection, might easily find this comparison intelligible. The colour red suggests ideas of a forcible stimulus, normally not of an unpleasant character. With that stimulus are connected various kinds of Impulse, and it is at that level that we form corresponding associations with the bright, lively stimulus of a trumpet. The popular expression of a

person being "loudly" dressed is derived eventually from such associations.

All this, however, does not imply that a blind person can reason about visual phenomena in their own qualities. In so far as such a person can reason there is no failure of Fundamental Presentation, for that is to be found in the recognition of the kinds of Association and Impulse produced.

A person born deaf may learn to speak, but that again is from adopting as symbols, in place of our words, various combinations of positions and movements of the lips and other forms of expression.

It may happen that a brain capable of high development may be encased in a head provided with very inefficient outlook upon the external world. Such is the case of Miss Helen Keller, who though born blind and deaf, has continued by the extraordinary force of her genius to educate herself, and to interpret many phenomena for which one would have been inclined to think the special senses involved were necessary (cf. p. 26). Her mind is deficient in the fund of experience brought by Immediate Presentation through sight or hearing, but there is no lack of such experience in regard to the ideas in which she actually reasons.[1]

It must even be borne in mind that our own normal

[1] Miss Helen Keller has recently published a volume of very remarkable poems, which are not without a "sense" of form and colour. But then she also uses the expression, "I see a young girl," and she writes of birds singing joyously. Evidently she uses these words figuratively, and she forms some representation of the deficient senses by means of others. Besides, it must be remembered that the part of her brain associated with the deficient sense organs is probably intact, and its spontaneous activities seek some kind of expression. Cf. J. Gensel, "Die Wahrheit über Helen Keller" (1909); and W. Stern, "Persönliche Eindrücke" (*Zeits. f. ang. Psychol.* 1910). Stern describes her as appreciating, criticising, and enjoying music. To receive the impressions she placed her hands on the piano while the musician played. Miss Helen Keller is not unique as a blind and deaf person endowed with great intellectual power. Laura Bridgman was celebrated in this respect before her, and books and articles concerning her are numerous. E. C. Sandford has written

senses are very limited in regard to the range of external phenomena which we know to extend beyond the spectrum, or the scales, of our senses (cf. p. 216).

But at a point short of total deprivation there may be failure of Immediate Presentation. If a person looking at a red colour says that it is the same as green we set it down to a defect in his visual apparatus. He differs from the ordinary, therefore we conclude that he is, to that extent, deficient. It would not always be safe to carry out this doctrine quite rigorously, but if we found that the defect arose from a disease which otherwise showed deleterious consequences, we would be more justified.

Attention was called to colour-blindness by the case of the celebrated chemist, Dalton, who, though a Quaker and a man of modest demeanour, appeared at a meeting of the Royal Society in a gay suit of scarlet. He thought he was wearing drab. Dalton does not appear to have been otherwise a victim of disease, but the fact that the overwhelming majority of well-constituted mankind find red different to drab may be accepted as sufficient evidence that Dalton's vision was deficient.[1]

interestingly on her attainments in the *American Journal of Psychology*. Henry H. Donaldson (*Am. J. of Psych.* 1890), G. S. Hall, and M. S. Lamson have also studied her case.

A reference to statistics shows that in all countries the number of deaf and blind is considerable. In 1871 there were 111 in the British census. In the census of 1901 the total number of persons both blind and deaf, whether from birth or otherwise, was given as 470 in England and Wales, and 63 in Scotland.

[1] Some notes have been recently published by a physician, Dr. G. H. Taylor, of New South Wales, Australia, who has had much experience in testing for colour-blindness. He finds that there is in persons so deficient a lack also of animation in the countenance. All this points to a real defect, though possibly of slight character, in the part of the brain corresponding (cf. pp. 53, 56). Mr. E. Nettleship, whose experience is greater than that of most experts, is inclined to believe that there is a connection also between colour-blindness and abnormalities of hearing. Both these conclusions might have been anticipated from anatomical considerations referring to the corresponding nuclei in the brain. Cf. also E. von Tschermak, *Lancet*, 1909.

Now he might reason quite consistently with regard to his own observations, although his range of Discrimination would be limited in one particular; but his reasoning would be faulty in referring to facts of the external world, or to those dealing with this sensation, as expressed by other persons.

Practically, an engine-driver who sees a red danger signal and who, not distinguishing it from green, fails to stop his train, errs not in his process of reasoning; though his reason is defective by virtue of the failure of Immediate Presentation.

Many disputes about questions of art have their origin in divergences of Immediate Presentation. Thus if a painter be short-sighted he will see a landscape in a different manner to that of a normal observer; and if he then paint in an impressionist style, he will see his picture also differently to the ordinary man. He will also range on his side those people whose eyesight is like his own; and so two "schools" may arise, each disparaging the other in various terms of contempt. Cf. Angelucci, "Sur les œuvres des peintres daltoniers,' *Ann. d'ocul.*, 1907.

The defect, or the divergence, of the vision from the normal or the ordinary may not be due to anything so verifiable as short-sightedness. The case of Carrière, the painter, has been investigated. His portraits were noted for good general resemblances and for fine artistic qualities, but the tone of the colours was deficient, consisting mainly of whites and blacks, with a fulgurescent shimmer over all. His eyes were, however, found to be "normal."

Failures of Immediate Presentation are in the majority of cases so serious in regard to their consequences that we find them associated with lunacy. In these circumstances it is difficult to obtain very

secure indications of what the nature of the failure may be. One case has come within my cognisance of temporary aberration of mind where the patient on recovery was able to give clear accounts of certain mental conditions during the period of the malady. On one occasion the patient thought that little seeds were really the hearts of friends. The seeds were of a shape that might suggest that of the heart, but all sense of proportion or scale seems to have been in abeyance, to say nothing of associations of place and function.

In general, hallucinations are not due to failure of Immediate Presentation, for it usually happens that there is no objective reality corresponding to the hallucination. For example, the commonest hallucinations are those of hearing. A person hears a voice frequently urging him to do something, possibly even to commit a crime. He may imagine that enemies are always telephoning to him, and in this way torturing him by insisting on his acting in some way contrary to his own inclinations. In these cases there is no fault of Immediate Presentation in as far as this is considered to be of objective origin, for the external stimulus does not exist.

Referring to the indications we have given of the physical correlatives, and relying upon the results of modern medicine, we should interpret the case thus: There is some source of irritation of the nerve substance, due to some disease which has invaded the brain tissues, and certain nerve-cells become thus stimulated in a manner resembling an original stimulation which represented the physical correlatives of the ideas thus reproduced; that is to say, the nerve-cells corresponding to the words become excited.

It may happen that certain external objects become

associated with hallucinations, but in such a case one must distinguish diverse factors. It may be that the Immediate Presentation has not been faulty except in regard to diminished Discrimination; but there may be, superadded to this, factors not derived from the external object.

This will seem clearer when we consider such a case as that recorded by Sir Walter Scott. He was sitting up late at night, when suddenly he thought he saw Lord Byron before him. The figure was very distinct, and the whole illusion perfect. He subsequently observed that it was the armour of a knight of old, arranged as worn, that had served as a basis for the figure of Byron. On examining the circumstance we see that though the Immediate Presentation was not clear, yet, in as far as that was concerned, there was but an exaggeration of the effects of semi-darkness and distance in diminishing the powers of Discrimination. But to this dim figure were superadded combinations to which no objective reality corresponded. This also is but an exaggeration of what takes place under normal conditions.

Habitually we see only a part of what we believe we see of a person. If we are familiar with the details of the features and dress of an individual, then the sight of some one characteristic part may suffice to call up the image of the whole. This effect is due to the regular manner in which our minds work by Impulse and Association. It is by this means that we appreciate pictures which may only give an indication of the whole image that rises to our minds. A black-and-white sketch by Phil May, for example, if, as usual, struck in with characteristic skill and decision, causes a strong image to rise in the mind. Yet when we examine the actual objective realities we find how small a part these black lines form of a human figure.

This is true of all the senses. It may occasionally be tested for hearing by comparing the notes of the same speech by stenographers. Occasionally a phrase badly heard is rendered quite differently by two stenographers. A story of American journalism takes note of this fact, for when the orator declared: "Amicus Plato, Amicus Socrates, sed major veritas," he was reported as saying: "I may cuss Plato, I may cuss Socrates, said Major Veritas."

A stenographer attentive to his own work will easily recognise that he really does not hear nearly the whole words, still less the whole phrases, which he reports verbatim. This fact becomes very evident when one tries to take a note in a foreign language which has not become quite familiar.

There are certain conditions of the mind, provoked generally by nervous excitability due to want of rest or to over-stimulation or both, when the process of combination by Impulse and Association becomes exaggerated unduly; and if at the same time the process of Discrimination be from any causes greatly diminished; then the conditions are favourable for such illusions as that of Scott (cf. p. 284).

Fortunately, in Sir Walter's case the experience affected a mind of marked common sense as well as of high intellectual power, so that we have been thereby helped to understand the nature of such illusions. When, however, the conditions referred to previously affect minds not well balanced; and when, as for example in elusive moonlight, the power of Discrimination is in great part lost; and where terror, or previously wrought-up anticipations, still further inhibit Discrimination, and increase the excitement leading to Impulse and Association; then we have the familiar phenomena of ghosts.

Pascal, the great French mathematician, who was also

a profound religious enthusiast, met as a young man with an accident by which his carriage was precipitated over the Bridge of Neuilly into the Seine. The whole circumstances made a deep impression on a mind and organism already highly strung and somewhat overwrought. Subsequently Pascal was subject to mental states comparable with that of Scott but showing certain inverse conditions.

He used to be seized with the notion of an abyss lying to his left side, and it required a special effort of his intelligence to make him know that this was but an illusion. Here the effect was to inhibit the proper view of external objects. There was an inhibition of Immediate Presentation, and superadded to that there were positive images associated with the impression of an abyss.

Failure of Immediate Presentation may be due to a state of intoxication, as by alcohol, which is virtually like a temporary condition of lunacy. The eyes "see double," for example. In such a case there may be no great defect of actual vision in each eye, but the images are not overlaid and fused together. Each eye normally forms its own images, and there is indeed a certain energy required to produce the conditions of fusion. This is observed in the eyes of squinting children. If one of the muscles of the eyeball, such as the external rectus be weak, or its innervation be deficient, then the eye is not placed in the position required to secure good superposition of the images. A confusion of the images results. Thus the child acquires the habit of neglecting the image formed by one eye. This eye is left to some extent uncontrolled, and the squint is accentuated. With the intoxicated person there is an inability to adjust the necessary conditions, and the images appear distinct.

Such a state of affairs may be also produced by various forms of brain diseases, such as a tumour in the cerebellum. In this case the mind may be left quite lucid,

and the patient may be able to measure his progress towards convalescence by the degree of proximity to which, under given conditions, he may be able to bring the presentations of the objects.

Hitherto we have discussed Reason by examining possible cases of its failure, and we have confined these cases to that of faultiness of Immediate Presentation. A complete discussion of such cases would take us far afield in the domain of medicine and particularly of Psychiatry, or Mental Aberration. But it has only been necessary for our purpose that we should indicate in what way, as referred to this Fundamental Process, the course of Reasoning may be misleading.

It would be tedious to enter exhaustively into a consideration of the failure of Reason with regard to each Fundamental Process in turn, for this again, as would be expected, must resolve into a discussion of the science of Psychiatry. A few additional indications may therefore suffice, and we will later examine cases of error in Reason where the causes do not seem so fundamental.

Failure to recognise a Unit in mental operations seems inconsistent with any consecutive mental life whatever. There may, however, be a difficulty in dividing any object presented as a Unit into more particular units. The failure, however, in such a case would be due to limitations of Discrimination.

Total failure of Memory or of Association would be also inconsistent with any course of mental life. A similar remark is true with regard to the Feeling of Effort, and Impulse.

The normal course of Impulse may be defeated by some condition, as in lunacy, in which an Impulse arising from morbid processes constantly makes itself evident. Thus a certain patient who had become mentally alienated through the shock of a sudden severe misfortune was

unable to hold any kind of connective conversation; the mind always reverted to certain fixed ideas, thus diverting the ordinary course of associations. A conversation on the weather would be represented thus on her part: "It is a fine day, poor thing. The rain falls, poor thing. And the grass, and he was a lovely man. And the sun too, poor thing."

It may, however, be true that in certain cases the Feeling of Effort may be hidden, or unnoticed; and certainly it is true that Discrimination with regard to this Feeling is capable of being very greatly cultivated.

Without that recognition of sequence that gives the elementary notion of Time, a course of mental life would not be possible. There may, however, be errors of Reason with regard to time by virtue of events which were really anterior to others being supposed to be posterior, and *vice versa*. We shall see that this cause of errors depends on faults of Memory and Association.

A failure to recognise Space as an Immediate Presentation would be inconsistent with a course of mental life. The notion of Space, however, such as we ordinarily possess is built up by degrees, and by diverse factors, consequently there is here considerable scope for error. This question has already been in part considered; it will be dealt with more particularly in the discussion of Externality.[1]

The Hedonic quality of mental impressions has an influence on Reason because it has an influence on Association and on Memory. A pleasurable surprise

[1] I would like to point out here that Time and Space stand in a category, or categories, apart from that of the rest of the series of Fundamental Processes. I have included them simply because in their simplest appearance they mark the unanalysable—as, at least, up to the present, it seems to me (cf. p. 27).

Of recent years the whole question of Time and Space has been re-examined frequently. Meumann and various French writers, Fouillée, Guyau, Bergson, Roisel, Lechalas, derive more or less remotely from Kant. Charles Dunan is influenced greatly by the principles of Experimental Psychology. Czermak finds

accompanying the Immediate Presentation is the surest means of making a strong Association and one that returns in Memory with a lively vivid style.

Repetition, which we have seen to be so powerful a factor in aiding Memory, acts partly by creating a sequence of images of which the facility of Association has itself introduced a peculiar Hedonic quality. This will be observed particularly when the sequence is interrupted unexpectedly, whereupon the attention is called to the failure of the sequence in a little shock of disagreeable character. An exception may be found in that form of play which is immediately followed by a sense of abundant pleasant characteristics which thus gain greater zest. All play, such as for instance we may observe in the mutual teasing of a pair of kittens, is of the nature of mimicry of strife with the disagreeable factors diminished to harmlessness, and the enjoyment of safety as well as of exercise thereby heightened.

The influence of Hedonic quality is often masked when a high development of intellect is considered, for then we have often the sense of following out strategic or tactical lines of conduct in which present conditions must be given importance only in regard to the general scheme and ultimate intention. Thus we may perhaps find a philosopher speaking of the "discipline of an uncongenial study."

Further, the importance of secondary motives, and the pleasurable exercise which their pursuit may involve, obscures at times the primary motives for which alone

a veritable Time-sense. Mach, profound as usual, and issuing here from Herbartian motions, concludes in a manner accordant with the analysis of Motion already given. Heinrich Czolbe finds all sensation extended, and he suggests that Time is the 4th dimension. Here, possibly, he is on the fringe of a great thought. In all this, however, I find nothing to disturb the present exposition. Certainly there is also a question as to all the other Fundamental Processes being on the same plane of co-ordination or of conspicuousness; but these matters are too subtle for a first reading.

they were produced. Herbert Spencer has given so admirable an exposition of this aspect of the subject that it need not be further elaborated. What may be noted as new, however, is that the Hedonic principle which he has well discussed as a great controlling factor in Ethics is here expressed as found in the Fundamental Processes themselves, and as therefore occurring at the base and exercising influence on the course of Reason itself.[1]

Reviewing now the Fundamental Processes, we find that in considering the causes of error in Reason we may direct attention particularly to Discrimination, Generalisation (symbolisation and classification), Memory, and Association. A total and immediate failure with regard to any of these Processes would be inconsistent with a course of mental life.

We shall see that the errors of Reason ordinarily met with may be made clear by discussion with regard to these Processes.

§ II. Forms of Reasoning considered in order to indicate the Amplitude of the Problem

When we speak of Reason, or reasoning, ordinarily we may imply any of a number of operations which have certain features in common, corresponding to the general definition of Reason already given (cf. p. 307), but which may also present considerable differences.

Thus, there may be reasoning along a chain of positions already traced out—as, for example, in verifying the proof of a proposition of Euclid.

Or, again, one is said to use Reason in the endeavour

[1] If this be borne in mind throughout the entire discussion whenever the Hedonic principle is involved, it will be seen that herein is the true sanction of that principle which Epicurus and Herbert Spencer sought in complicated forms, and consequently with certain erroneous implications.

to find the solution of a proposition; that is to say, to find the intermediate positions between the conditions of a proposition and the recognition that a certain other condition, not immediately evident, is thereby necessarily determined as true.

Then, again, there is exercise of Reason when, having ascertained certain conditions in nature, we endeavour to meet with other and as yet unknown conditions which are dependent on the first condition. This is called discovery, or research. Or, again, Reason is employed when we combine known things in a convenient way towards some useful end. This is called invention.

But apart from these formal exercises of Reason, it is equally true that Reason is employed in our ordinary observation of the objects and occurrences that come within our experience. Our minds do not proceed merely by set associations from one point of sequence to another; as, in that case, we should have but an assemblage of facts without meaning. But at every experience we recall past experience; and we compare objects and their associations; we use symbols, and apply them to new objects; and we note differences or resemblances between similar objects; we generalise and we classify.

These exercises of Reason, which are in good part automatic, or nearly so, not only serve their own purpose, but they give us the fund of experience to which we appeal in the more deliberate and definitely enunciated problems of Reason which we may set ourselves.

EXAMINATION OF REASONING IN REGARD TO A SET PROPOSITION

An element of the Process of Reason may be thus expressed; If a is b, then c is d. It may happen, in

some cases, that c is identical with b. But if we examine this simple formula in different fields of application we find that the Fundamental Processes involved are in different combinations.

In order to find an opening for discussion in detail let us consider the fourth proposition of the first book of Euclid. If two triangles have two sides of the one equal to two sides of the other, each to each, and if the included angles be equal, then the bases of the triangles are equal, etc. Now if we had already proved, or if we assumed as true, that three independent "elements," as stated, determined a triangle; and if we further assumed that a triangle did not change its dimensions by changing its situation, then the proposition would be one of those of which the truth is shown by the mere analysis of the terms of enunciation.

Or we might express the fact by saying that any one triangle of the kind is the generalisation of them all in the most exact sense, since, apart from the question of position, it becomes identical with each of the triangles possible.

But if we prove the proposition, as in Euclid, we see that it is still a proposition of which the truth is manifested by the analysis of the terms, if we make certain assumptions, as before, regarding the independence of situation of the magnitudes.

Thus, if we inquire what is the meaning of one side being equal to another we find that, unless we make further assumptions regarding measurements which are not fundamental (cf. p. 125 and p. 216), we can only realise the nature of one side being equal to another by superposing one on the other. This is the first part of Euclid's construction. Now this also applies to the remaining given sides. And this is the third part of Euclid's construction.

Again, if we realise the meaning of one angle being equal to another apart from the notion of measurements, we find the suggestion of superposition with the apices coinciding. This is the second part of Euclid's construction.

Now suppose that we ask this question: What is there in this proposition that would prevent its truth being inevitably apparent from a consideration of the enunciation?

In the first place, we must know the meaning of the terms themselves; that is to say, we must know all that is implied, or connoted, in the symbols. This may appear obvious, but in less simple examples failure in this respect is a fertile cause of error. We are so accustomed to use words fluently that we do not always think of their "connotation" when we employ them.

Even in this case it requires a certain effort of consideration to make us know that, with the proviso mentioned, we can test the equality of the sides only by superposition. If this is fully understood the Impulse towards fulfilling that part of the construction must inevitably arise. With young scholars the question of connotation of the symbols is not merely confined to these few steps of analysis; the difficulty arises with regard to the whole style of the language employed in enunciation. These scholars find the same difficulty as ordinary people experience in reading the "jargon" of a legal document.

The absorption of energy and the confusion resulting prevents due attention being given to the essential part of the enunciation (cf. pp. 140 *et seq.* and p. 236), and thus the Impulse spoken of above, which is the element of all our tentative efforts in Reason, may be lacking. Further, young scholars have a difficulty in forming the notion of a triangle of Euclid. This difficulty, and the processes

involved, have been discussed when considering abstraction (cf. pp. 44 *et seq.*). The difficulty regarding the changing situation of the applied triangle is one of those which escape most students owing to their want of analysis; it occurs only to the philosophic student. A young scholar, however, may have a difficulty in understanding the construction of superposition for quite another reason; for, on the one hand, he has had difficulty with abstraction, and now, in order to form a clear conception of superposition, he must reimpose some material aspect on his triangle.

Another difficulty may be that, even when he has made the constructions separately of the sides, and the included angle, he may not have a notion so clear and unembarrassed that his mind associates their constructions in sequence, and then as a Unit.

There are all sorts of associations formed round the objects of our consciousness, and though to those who are familiar with geometrical constructions the associations required for our proof seem inevitable, yet it must be remembered that, to many minds, the associations lead them away from geometrical ideas altogether. Consequently there must be a check to certain forms of Impulse, and a solution implies guidance of the associations required.

This is true of the next simple step after the construction is formed; for, though it seems evident that if two points be given—the extreme points of the base—only one straight line can lie between, yet this is, in the ordinary demonstration, represented as being a case where two straight lines coincide; and, moreover, the associations formed by the scholar with regard to these two points may not lead him to think at all of the straight line between them.

From the mere description of the steps of the process

it will appear also that the question of Memory must be important; for, by the time the process of demonstration has neared the end, the mind, already tired by the effort of abstraction, and by the unfamiliar terms and associations, may have forgotten what has been already established; and this is more likely to happen by reason of the circumstantial language in which the demonstration is couched.

It will be seen, from this example, that a proposition almost demonstrated by the meaning expressed in its terms may yet appear a stumbling-block to a mind little exercised in such notions.

It is a useful exercise to examine the whole scope of geometry with minute observation so as to determine how much is assumed from experience in the course of the demonstrations. An examination of that character, continued throughout a large field of mathematics, suggested the manner of dealing with geometry of which some indications have been already given (cf. pp. 175 *et seq.*).

We have now, in a very simple case, advanced beyond the failures of Reason due to quite fundamental defects.

Reason may fail from failure of the Impulse producing the required Association, from a misuse of symbolisation, implying a failure of Discrimination, of Generalisation, of classification, and again from inefficiency of Memory.[1]

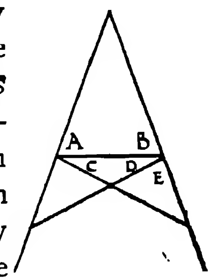
In this case the proposition c is d, if a is b, resolves itself into establishing an equality of two things, where situation is not considered as affecting that equality.

[1] A distinction is sometimes popularly drawn between intellect strong in Reason and intellect strong in Memory.

Differences of intellect do exist which may, in a broad way, be thus referred; but throughout this whole discussion we shall see that Memory is an essential part of Reasoning, and that some of the finest feats of Reason have been dependent on exceptionally good Memory.

If now, to take a further step, we consider a proposition the truth of which is not implicitly contained in the manner of expression, we may select the ordinary demonstration of the fifth proposition of Euclid.

It is not necessary, for our purpose, to enter all the details of the proof; we may say that, the object being to prove that an angle A is equal to an angle B, we first prove that the sum of two angles, A and C, is equal to the sum of two angles B and D; and then we prove that the angle C is equal to the angle D; thus we conclude that the angle A is equal to the angle B (cf. pp. 128-129).



This proposition presents so many difficulties to beginners that, from the olden days, it has been called the *pons asinorum* (the bridge of asses). The difficulties are of the nature mentioned in regard to estimation of numbers when too many objects are presented not clearly classified. (See pp. 140 *et seq.*) There are also difficulties of Memory, for the proof that the sum of the angles A and C is equal to the sum of the angles B and D depends on the fourth proposition, with which, therefore, the scholar should be familiar. Another application of the fourth proposition proves that the angle C is equal to the angle D. The whole demonstration is, moreover, long; and here, again, Memory is taxed.

We know, from what we have already discovered concerning Memory, that these difficulties become less and less by repetition; but, in the meantime, the scholar may become discouraged, especially if he have no apprehension of the importance of these propositions nor notion of whither they lead. Another difficulty arises from the fact that, in this particular application of the fourth proposition, the sides of one triangle lie upon and overlap those of the other. A distinct effort is required

to disassociate the triangles, or to counteract associations which lead the mind away from the conception required.

The subtraction of the angle C from the angle which is the sum of A and C is not a process identical with that of subtraction of numbers (cf. pp. 168, 176, and 183).

For the greater number, known ultimately by counting, must be recognised as made up of any lesser number and the number required to complete the total. Subtraction is the inverse of addition (cf. p. 129). But we prove that the whole angle, here represented by the sum of A and C but really considered at first not with reference to these parts but as a whole, is equal to the corresponding angle represented by the sum of B and D.

To conceive all that is connoted here by equal we must conceive (cf. pp. 124 *et seq.*, p. 148, and p. 217) superposition. Similarly with regard to the equality of the angles C and D.

The process by which we recognise that the residues, after removing the equivalent angles, are equal, is that of Space-measurement (cf. p. 183), which derives from experience involving certain Fundamental Processes in regard to the objects of that experience.

In making use of the fourth proposition it may be observed that we might have again proved the fourth proposition in this particular instance. But the fourth proposition, though relatively simple, as we have seen, requires a series of operations for its full comprehension. It is simpler here, then, to say that the conditions of the fourth proposition are fulfilled, and that the consequences follow. Here is an example of symbolification. The examination as to the fulfilment of the conditions involves Discrimination, so that the same Fundamental Processes, though in different combinations, are employed as in the fourth proposition.

In the case of this proposition also we see that what is implied in the formula of reasoning: if a is b, c is d, is the equality of c and d, with the proviso regarding situation (see p. 183).

Hitherto we have considered propositions in which the sequence of reasoning has been already arranged. Before going further we may inquire what are ultimately the grounds, in such cases as those considered, for our declaring that our Reason is satisfied.

We have recognised that the Fundamental Processes, such as Immediate Presentation, or Time, are essential to any mental life. A narrow examination has convinced us that, in their elementary manifestations, a like statement is true of all.

But in a higher development of thought we speak of generalisation and classification, for example, as processes of value which may not be employed by ordinary minds, which nevertheless enable scientific men to carry on consecutive courses of mental life. We might be tempted to think that there was a distinction here between such processes and the others. For convenience of language and exposition we have seemed to recognise such a difference.

But, looking at the matter still more closely, we find that there are analogous variations, differences, degrees with regard to the other Fundamental Processes, each in its own quality.

Immediate Presentations differ greatly in certain cases (cf. pp. 27, 56, and 94). The process of forming a Unit by new combinations may be thought to be just as truly a scientific faculty as generalisation and classification; if, for example, we are asked to consider as a Unit the amplitude of a Hertzian undulation, or the expansion of the second elliptical function of Jacobi.

We have seen that such Fundamental Processes as

Immediate Presentations, Association, Memory (cf. pp. 114, 288, 401), are independent of our will in their fundamental exercise. We can intentionally produce certain conditions modifying the character of the Immediate Presentation, the Association, or the Memory; but when once the conditions of the external world and of our own constitutions are determined, the process follows. It is only necessary now to consider attentively each of the Fundamental Processes in turn to be convinced that a similar statement is true of all.

We feel, then, our Reason satisfied when all these Fundamental Processes are performed normally. That is to say, if we have to exercise Discrimination we know that, without check, we have exercised our Discrimination to the limit of its necessity in the case, or of its possibility as far as our conception of the case is concerned.

If we exercise Memory we feel assured that our Memory restores the idea of the original in a manner as lively and complete as we demand.

With regard to Association we find, for example, in shifting the position of a triangle, that certain associations are invariable, and inevitable, and it is these that we deal with. And a similar mode of observation applies to all the Fundamental Processes.

With regard to Association certain influences of experience become apparent (cf. p. 28 and p. 47). But this is true also of all the other Fundamental Processes. We continually find, in experience, that our Discrimination is limited, that it may be cultivated, that it is therefore modified by experience. Memory is modified by various conditions (cf. p. 244, p. 248, and p. 251).

The force of these observations will perhaps be better seen if we suppose that the normal exercise of any process is changed. For example, if by a fault, such

as might be due to a defective Memory, we supposed that the fourth proposition proved that the angle formed by the sum of the angles A and C was equal to the angle E, formed by adding another angle to B and D, then the conclusion would be faulty. In such a case we would know it to be faulty, for we had deliberately changed the indications of our Memory; but if the error had arisen unknowingly, then, though it were error, we would have the same satisfaction of our Reason as when the right result was reached, and this satisfaction might remain until we found our results incongruous with some other result obtained also by a course which satisfied our Reason.

Or, again, if we thought the conditions of the fourth proposition applied to a case where they did not really apply, then we would commit an error in symbolisation, an error perhaps finally referable to a fault of Memory or Discrimination; and the result would be wrong.

There is nothing mysterious in all this. Such errors are constantly committed. In attempting to prove a mathematical problem it is not unusual to be "puzzled" through some mistake, or to be convinced that the opposite of the true proposition has been proved. I have referred at this stage particularly to mathematical propositions, because they present no great emotional associations, and the course of demonstration may be easily checked; but in all modes of Reason failure from any of the causes mentioned may arise.

What makes mathematics indeed so sure a science is the fact that, when the appeal is really made to experience, it is experience of a kind that is absolutely fundamental to our mode of existence, experience constantly repeated; so that the road has been traversed, in its elementary parts at least, by a succession of minds, including the keenest of all of humanity; and that, finally, its results

can be checked more or less accurately in calculation. In the higher mathematics the check is less, but the minds themselves that traverse the route are necessarily superior. But in this most certain of all sciences we find Arago, for instance, doubting the validity of some of Abel's propositions in elliptic functions; we find Cauchy correcting and making more rigorous his own proofs in the integration of certain differential equations; and we find the whole characteristic system of Riemann, one of the most genially inspired of all, thrown in doubt.[1]

Now in dealing with the question of Immediate Presentation (cf. p. 216) we saw how limited was the visible spectrum of light compared with the whole extent of what, from other sources, we know of as similar undulations. We know how circumscribed, and often how faulty, are the indications of all the senses.

We have already studied cases of the failure of Memory due to a variety of causes. The microscope and telescope have revealed to us how limited is our power of Discrimination without these aids; and we know, from other senses, of the limitations of these instruments.

The most inevitable kind of Association is that of cause and effect, yet we know that that means, to our present intelligence, no more than invariable sequence.

Thus it will be found, as in the cases we have already examined, that Reason is not only subject to errors, but that when these errors have been, as far as humanly possible, eliminated, then the certitudes of Reason extend no further than those of a concordance between our

[1] In the course of reading I have met with errors, acknowledged or demonstrated, on the part of such brilliant intellects as Legendre, Jacobi, and Abel. Mayer of Leipzig has written an illuminating essay on the history of the Theory of Least Action in which the blunders of Maupertuis, and a lapse of Euler, are discussed.

minds—depending ultimately, as we have seen, on our physical constitutions—and the external universe. In this concordance we assume certain conditions of constancy. For example, if we look at a triangle, and then, after diverting our attention, again look at the triangle, we recognise that it is the same triangle. Underlying this process is the acceptance of a belief, though not expressed overtly or even considered, that our mental condition has remained unchanged, and that the objective reality has remained unchanged.

But, if we looked at a horse in this way, we would say it was the same horse; and similarly if we looked at a stream of water. But as, in these cases, changes at length become obvious, we are led to the conclusion that the basis of our Reason depends also on the limitation of our Discrimination; and hence also, in turn, of all the Fundamental faculties (cf. pp. 35, 36, 93, 207).

If we could "see as God sees," our Reason would be of another character; it would not be Reason, as we generally understand it, at all; it would be of the nature, as far as we can form a conception, of Intuition or the relations of Immediate Presentations. It is from our limitations that we have built up the marvellous processes of Reason which are the glory of our race.

Mathematical examples are useful in the study of Reason because we are less liable to be led astray by accidental associations.

But, keeping in view this danger, we may take other examples. For instance, we read a story concerning the elder Pitt. Certain associations are formed around this symbol. We are acquainted with other stories concerning the Earl of Chatham. Now if we find that the symbols Elder Pitt and Earl of Chatham apply to the same personage, then we may have a proposition which is expressed in its very terms, when we apply all the

associations of the symbol Elder Pitt to the Earl of Chatham.

This example may be compared with that of the fourth proposition, essential features of resemblance being noted and accidental differences disregarded.

In such a case no great confusion is likely to arise, but a reader of French history familiar with the names and with some of the salient points of the career of Marshal Ney, or of Murat, might be puzzled later by reading of the exploits of the Duke of Elchingen, or of the King of Naples. Careless students of Roman history have been known to institute comparisons between the eloquence of Tully and that of Cicero founded on descriptions of their style. On the other hand, taking Disraeli's definition of an archdeacon as one who performed archidiaconal functions, this definition might suffice to inform one who had in some way been made acquainted with the nature of archidiaconal functions.

Referring now to examples more important, we may cite that of certain of Frauenhofer's lines found in the spectrum of the sun being identified with those of the spectrum of sodium. From this identity we proceed to apply the associations that have been gathered around sodium to part of the composition of the sun.

In this case there are really several assumptions. One is that these lines are due to sodium always; another is that, in as far as we apply the same associations, sodium has the same properties in the sun as on the earth.

Such assumptions are constantly being made in reasoning, and, though they are often sources of error, yet if they be true they lead to the establishment of certain laws of nature; and if they be untrue they nevertheless serve a purpose, for the criticism by which they are shown to be untrue reveals new truths.

Thus every generalisation bears an assumption, and the progress of science has been thereby greatly aided.

When Newton enunciated his law of Universal Gravitation he certainly had not considered all the material particles of the universe. Yet the generalisation affords a basis on which we proceed to new discoveries, and the verification of these serves to confirm our belief in the law.[1]

On the other hand, we have seen that a generalisation, not entirely true, may serve to advance science, as in Schwann's germ theory.[2]

But even when the generalisation seems devoid of the chance of error, as in proving a proposition for a given triangle, and so for all of that kind, the certitude is only great because the complex of associations is small; and the certitude is not greater than that of the assumptions we make regarding the invariability of the triangle in a varying situation (pp. 183, 321).

We may now observe how the principles we have investigated may apply to a complex case. The un-

[1] The theories of the Conservation of Energy and the Conservation of Mass are really founded on assumptions which imply the truth of the theories. Of what avail is it to offer the most comprehensive and brilliant display of mathematics, if the whole process be conditioned by these assumptions, and the proof be only to show that the result is in concord with the assumptions? But it is said the problem has been demonstrated. Landolt recently proved that in a reaction between silver nitrate solution and salt solution, producing silver chloride and sodium nitrate, there was no loss of weight. His methods were exquisitely fine, but they leave untouched the question: if a body escaped the law of gravity how could it be known by weighing? More widely, how could it be known at all? Moreover there is an assumption in applying universally a particular result. Finally, however, it does not follow that a principle is untrue because the arguments offered in its support are inadequate.

[2] Schwann's experiments that established the germ theory may be thus briefly indicated: He enclosed some meat in a jar from which access to the external air was excluded. No putrefaction followed. Schwann put forward the theory that all organic changes were occasioned by "germs" which were in the air. Modern research defines the theory more accurately by attributing the changes to enzymes or ferments produced by the germs.

dulatory theory of light depends on the hypothesis of a universal ether which conveys the undulations. We cannot ascertain the presence of ether by any of our senses, we cannot weigh it, we cannot even very well conceive it. The suggestion arises from a study of other undulations—in water, in air; and so, for the phenomena of light, which possess so many subtle associations, we imagine a medium correspondingly subtle.

Physical investigations have shown that, if the ether be real, it must possess qualities which at first we would not be inclined to ascribe to it. Kelvin, for example, thought that it must be solid. Various difficulties that have grown up have suggested to great physicists now living that the hypothesis may be false. Even so, it has served to advance physical science.

Now the phenomena of light have been investigated on the assumption of the undulatory theory, and a highly developed instrument of mathematics has been elaborated to express results and to aid in the search for new truths. Clerk Maxwell, in the course of much investigation, found many points of resemblance between the phenomena of light and of electricity. From the study of these arose the suggestion that the phenomena of electricity were due to undulations of the same ether. And, finally, we arrive at the theory that the phenomena of light, heat, electricity—all forms, in fact, of radiant energy—are due to the undulations of ether.

Here is a case of generalisation based on an hypothesis which may not be true, but which has already produced marvellous results, even in practical domains.

Thus generalisation not only permitted the mathematical apparatus already evolved for light to be applied to electricity, but it suggested to the genial mind of Hertz to attempt to reproduce, or to adapt, for electricity the experiments of reflection, of refraction, and of inter-

ference with which physicists were familiar in the case of light.

Among the consequences of Hertz's experiments may be noted that of wireless telegraphy, which therefore has a remote origin in Clerk Maxwell's brilliant and patient efforts of abstruse thought.

In the examples we have just cited the proposition if a is b, c is d, does not result in a mere identity, or equality. It is true that, by the artifice of language, we might express them as equalities, thus: If these Frauenhofer lines be the lines of sodium then the corresponding material in the sum is identical, as far as constitution is concerned, with the sodium of our planet.

But it is better to follow the natural process which arises in the mind itself, and to say that, if these lines be due to sodium, then we apply the symbol sodium to the material in the sun, and in applying this symbol we associate with the material so symbolised all the facts which we have hitherto ascertained regarding the sodium of our earth.

But even in the case where we say naturally if a is b, c is d, with the implication of equality, or (with certain abstractions) identity, we may express the proposition also as that of associating with d the associations which we already formed about c. Moreover, a close study of the manner of movement of the mind, of the Fundamental Process therefore, will now convince us that that is the meaning ultimately implied in the copula, "is."

We have used the expression "equality, or (with certain abstractions) identity" in order to lead to that conclusion; for we have already seen that there is no equality except that of identity (cf. p. 165); and when we speak of identity, with certain abstractions, we can only mean that there is a process of Disassociation (cf. pp. 44, 160), and that there is a process of Association. The processes

of Disassociation and then Association have already involved the whole series of Fundamental Processes.

The symbol, Belief, is used to indicate a state of mind in which these Processes are formed. No state of mind, it must be remembered, is merely passive (cf. pp. 85, 96). At each experience, or with each proposition accepted, the mind places itself in a certain condition, and this condition is influenced by the physical correlatives; and these, in turn, are the resultant of innumerable forces and conditions which have come to us through countless ages of evolution, through our own individual development, modified by past experience, and now by some actual experience brought to a state which has its expression in consciousness.

All this implies, not only by its factors but also by their resultant, a certain Hedonic quality involved in that state. Moreover, as we find not only the Impulse that produces in sequence some new Association formed in consciousness; but as, from a consideration of the physical correlative we know that there is a magazine of forces there endeavouring to send forth impulses in various directions; we have a state of expectancy created.

Now we have considered Disassociation as the inverse of Association (cf. pp. 44, 47, 160); the same opposition exists in regard to Belief and disbelief.

Belief, accordingly, corresponds to that state of mind in which the natural Process of Association finds uninterrupted exercise, and in which the Hedonic quality is of the same nature as in Association, distinguished from Disassociation.

Belief, then, corresponds on the side of the physical correlatives to a dynamic disposition of forces having a resultant in a determined direction; and in consciousness to an Hedonic condition consonant with the ful-

filment of expectancy in the movement along certain associations.

But as we have seen that Reason depends on the scope of all the Fundamental Processes as applied to definite experiences, and as these Fundamental Processes are all limited and fallible, it follows that Reason is fallible in as far as it may be supposed to offer demonstrations of the relation of things of the external world; but Belief is determined by these processes of which the failure cannot be made apparent, for otherwise they would not be in their regular exercise; consequently Belief gives no real certitude of a correspondence with the external world.

As a matter of fact we constantly meet with people who hold, as fundamental matters of belief, opinions or doctrines which, whether true or false, are inconsistent with the opinions or doctrines which other people hold as fundamental matters of belief.

Such opinions and doctrines cannot all be right. What then is the criterion? Simply this: As we have already seen in geometry, we must seek to rest our Reason on those experiences which are the most fundamental we can find, and to proceed from these by Reason, rigorously examined step by step, towards our new positions and conclusions; to submit these to every kind of test available in correspondence with the external world; and to modify them in accordance with indications so obtained; and, in order to discover the origin of any discrepancy, to submit again the whole process to our Reason (cf. pp. 346, 347).

The discussion of such questions as that of Belief is difficult because it demands on the part of all a close introspection, and moreover it may happen that the words themselves do not convey the same meaning to one person as to another. Hence there has been a

certain insistence, in the foregoing exposition, with the object of making the matter clearer, if possible, by holding the attention to it.

The discussion under a synthetic form (cf. p. 567) will also serve to enable the mind to seize the explanations offered.

§ III. Illustration of Synthetic Form of the Study of Problems of Reason

Throughout the whole course of this exposition we have referred to the similarity of the processes in operation, for example, in different professions. This is easily understood when we consider that the basis to which analysis led us was that of the Fundamental Processes, by the combinations of which the whole of our operations are formed. And the whole of our knowledge is built up by the associations of actual objects of experience with these operations. Thus in quite different activities of life the formal schema of the operations may be similar, but the actual concrete facts by which the schema is illustrated make the difference of knowledge.

Let us consider, then, how by a graphic example we may illustrate that movement of reasoning by which, proceeding from given conditions, we arrive at the demonstration of a certain other condition.

Thus we may express our example: We walk along a passage, not knowing from the beginning what route we may take, but having a sum of previous knowledge and guiding principles so that at each turn we may be able to decide what to do. We meet with a cross chalked on the first door. Associations already learnt indicate to us to take the first turn to the right. Had the mark been a circle, it would have indicated a turn to the left. We proceed according to the directive, and meet a door

on which we see printed: "Open the door and read the instructions left lying on a table in the hall." The instructions tell us to proceed to another place where we see a plan with an arrow pointing in the right direction. Arrived at a certain corner where two roads meet we consult a map in our pocket, and we find ourselves directed to a house. There we see a soldier looking out of a certain window. He advises us to take the road to the left. And so on we proceed to the end of the journey.

Now in a progress of this sort it is not necessary to remember the steps of the journey beyond the step actually dealt with and its path to the next. But step by step we move until finally we associate the end of the journey with the beginning.

Suppose now that, instead of the line of reasoning being traced out in advance, our problem is to find it. For example, taking for convenience of reference an example already considered: what would be the course of an investigator of the fifth proposition of "Euclid" who, believing that the angles at the base of an isosceles triangle are equal, endeavoured to find the best demonstration of the fact?

Demonstrations of this sort are generally sought for analytically; that is to say, the mind proceeds from the conclusion to trace backwards the steps by which it may be reached until in this way the starting-point, viz. that of the given conditions, is arrived at.

Synthetic demonstrations are obtained by reversing this process. The close examination of the method of analysis is especially instructive. Thus, in the fifth proposition of Euclid, if the angles at the base be equal, then the angles formed by adding equal angles to these angles will be equal.

But how does such a suggestion arise? There is

nothing inevitable in that suggestion, for it might as well have been suggested that the angles formed by subtracting equal angles from the angles at the base will be equal, and it is indeed possible to form a demonstration in this way.

It becomes apparent on observation that we allowed freedom to Impulse in forming various associations, and we select those that seem to lie in the way to the goal. There is no certainty in such suggestions, and when the successive steps are very numerous it will be seen that the possible combinations of associations upon association may be enormously large. Hence the difficulty of such problems.

With constant practice certain general principles, such as that of symmetry, the research of simplicity, suggestions from known forms, and previous demonstrations, come to our aid (cf. pp. 140, 179). Moreover, just as we have seen in the case of Memory, so in all these processes the speed of operation is wonderfully increased by continual exercise.

In the fifth proposition the suggestion of angles formed by addition soon leads, since we are dealing with equality of angles, to the application of the fourth proposition, which demonstrates under certain conditions the equality of angles. The next step in this instance is to prepare these conditions. The rest of the demonstration is comparatively easy.

Here, then, we have an example of obtaining a solution by means of tentative efforts; that is to say, by giving free play to associations and testing those that seem to promise some real progress.

These associations are not necessarily confined to those that are habitual or soon apparent. It may happen, in searching for a solution, that the enunciation of the problem must be expressed in some form

of deep generalisation before the appropriate suggestion arises.[1]

§ IV. Analogies between Forms of Reasoning in Diverse Fields

It is interesting to seek corresponding examples in quite different fields of thought; and this exercise will throw light on the developments that may be derived from the positions we have already established. For while the Fundamental Processes remain unchanged the actual objects to which they may be applied differ. Thus the processes of the mind in all combinations may be looked upon as a schema, and the character of the individual experience will determine the concrete details by which the schema becomes illustrated.

Hence we should expect, and we shall see this expectancy more and more clearly satisfied, that mental operations in different professions, or modes of employment having apparently little in common, may be shown to be similar.

The problem of finding a solution for a proposition considered as true might be compared to that of finding one's way from Hyde Park to the Tower of London. Various tentatives are made. Occasionally it is found necessary to abandon one route; occasionally the path

[1] The number of possible combinations to be exhausted may be enormously great—as, for example, in a field which has afforded the highest triumph to mathematical genius, viz. in the search for means of integrating elliptic integrals. Abel found that all possible combinations of ordinary methods seemed to be fruitless, and he also observed the fact, in his examination of the sources of these ordinary methods, that unless the properties of logarithms had been investigated it would have been impossible by those means to have obtained the integral of $\frac{dx}{x}$. Hence he was led to study in what way one might obtain a clear notion of the logarithm by proceeding from the problem of integration of $\frac{dx}{x}$; and the methods so revealed were utilised in the analogous but more difficult problem.

taken seems to be making the journey longer; but, after a series of tentatives, keeping the object of the journey well in view, we succeed in accomplishing our purpose.

Suppose now we had already explored the route from Ludgate Circus; then if, in our tentative efforts, we had arrived at Ludgate Circus and recognised it, the difficulties of the problem would be at an end. But if we had not been perfectly familiar with Ludgate Circus, having observed its aspect only in one direction, then we might reach Ludgate Circus without recognising it.

An analogous case might arise in a mathematical problem where a formula, or a geometrical combination, might be unrecognised because met with in some unfamiliar form.

Such considerations indicate that to the mathematician good Memory is essential, and that especially the recollection of a considerable number of oft-recurring formulæ is very important. The advantage is also made clear of expressing formulæ and propositions in a general form so that they may be recognised even when associated with unfamiliar accidentals.

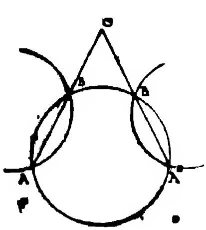
Another example might be drawn from the practice of a surgeon who has diagnosed appendicitis; but who, in his exploration, does not immediately find the appendix. If he recognises the cæcum, and traces it to its junction with the ileum, he is easily led to the discovery of the appendix.

We have often referred by preference to mathematical problems, because illustrations drawn from actual practice in various professions are always confused by complexes of other operations together with their associations; but the exercise is no less useful to find in them also analogies to processes met with elsewhere.

Thus with the warning against confusion we might compare with the problems already considered that of Napoleon Bonaparte in seeking to free Toulon of the British vessels in the harbour. His analysis of the situation led him to consider that a direct attack would be useless, but that if he secured the position called Little Gibraltar he would then be able in comparative safety to cannonade the ships in the harbour. And it so happened that when he seized this position the British ships departed.

At a later period of his career, and at the height of his ambition, he had conceived the project of subjecting the East and drawing therefrom immense resources of troops. In order to render his plans feasible, it was necessary for him to secure the fort of St. Jean d'Acre. Sir Sidney Smith defended it successfully. Napoleon's officers were astonished at his persistency in attempting to seize that *bicoque* (little hole of a place); but in military parlance such positions are the "keys" of the situation. Regarded from the purely intellectual side they are the intermediate steps in the way of the solution of a set problem.

Referring again to a geometrical illustration in order to pose another question: we may require to prove in the course of an investigation that the rectangle O B · O A is equal to the rectangle O B′ · O A′, where A B and A′ B′ are chords of two different circles. The demonstration would be aided if we observed that the points A, B, B′, A′ might be shown to lie on the circumference of a third circle, because then a known proposition would establish the equality, each rectangle being equal to the square of the tangent from O to this circle



But it might require a long series of steps to establish the fact that a certain circle passed through A, B, B′, A′.

The cases where solutions are obtained only by means of a great series of tentative efforts might be compared to the difficulty of finding one's way about a vast city in which at each corner a great number of roadways were found, some of which ended in blind alleys. This will also explain why the persistent exercise of trial and study helps towards the solution of problems. With each tentative some new area is explored and becomes known. We at length form associations not merely between one path, and continuous paths, but also between distant points.

With practice and familiarity also, as we have already seen (cf. pp. 140, 261), the mind is rendered apt for further discrimination, and free to form more associations. Memory is also aided by the greater facility of observations in regions that have hitherto absorbed energy too greatly and have prevented the exercise of associations in other regions (cf. pp. 120, 121, 297).

Thus, for example, in exploring the town the mind may at first at each turn be confused by the multitude of objects presented. At length, in the manner indicated, comes a suggestion that a certain aspect of a building corresponds with what we have learnt of a certain landmark to which a definite symbol is attached. It may happen that, by our previous knowledge, the solution of the problem is known from that point.

Error may occur in the course of such a search for solution in various ways. Let us take an example from the problem of finding our way in a town. Thus, after an absence of some years from Paris, I find myself in a part of the town where I seek the rue de Londres. The rue de Londres begins near the Place de la Trinité, from which it ascends obliquely. To

the right of the rue de Londres another street, which begins at the Place between it and the rue de Londres, also ascends obliquely, but more nearly north and south. This Place is to the east of the Gare St. Lazare. I am in a place which has a similar disposition of streets, one ascending obliquely from a Place, while the second, between it and that Place, ascends more nearly north and south.

The fact is that these streets are to the west of the Gare St. Lazare; but, from the general aspect of the streets, the suggestion comes to me that the street running more westerly is the rue de Londres. Now I have not identified this Place, nor given it a name. The general aspect of the Place, considered as a Unit, has thus become itself the symbol with which I have associated the aspect of the rue de Londres and thence all its known associations.

I proceed to walk along this street, and find it unfamiliar; that is to say, find that the associations presented in idea do not correspond with the Immediate Presentations afforded by the street.

I return to my starting-point, thinking that I may have taken a wrong direction; that is to say, with the intention now of disassociating erroneous associations which led me to the wrong street.

The symbol—that is to say, the general aspect of the Place—has associations so strong that my Discrimination, acting at the disadvantage of a Memory weakened by lapse of time, is not efficient enough to lead to the disassociation of what are really the erroneous associations. I again walk in a street I suppose to be the rue de Londres, and for a time I disregard the unfamiliar aspect; that is to say, I disassociate these associations. Then I see the name rue de Rocher. Here is a new factor which supplies

associations with Impulses strong enough to disassociate the previous associations.

Proceeding along the line of these associations in a contrary direction, I arrive at the examination of the Place itself. Having no other outlet, I am forced to pose the question: Why do I think this the Place corresponding to my former associations of a Place with the rue de Londres?

This condition of the mind, with the attention strongly held to a Place, with the broad general characteristics only known, and with an inhibitive or active disassociation of the previous associations, produces tentatives—that is to say, Impulses bearing new associations. Pursuing these, somewhat at hazard, but with the intention of arriving at a known point, I at length form associations of the other end of the rue de Londres with a situation on the east side of the Gare St. Lazare.

These associations correspond to a good Memory, which brings a chain of recollections; that is to say, associations in ideas, which by their liveliness and their consistency with external objects, and ultimately with the Immediate Presentations, give me a feeling of distinct Hedonic quality, of certitude, or Belief. The Impulses become directives to action. I proceed to find the rue de Londres near the Place de la Trinité.

§ V. Sources of Error in Extended Reasoning: Errors Generally Considered as of False Premises, but Capable of Analysis in Various Ways to Fundamental Processes

Errors of the kind here investigated are formed in every field of Reason. The error might be ex-

pressed by saying that the premises are bad, though the reasoning may be good from that standpoint. But it must be understood that the premises, from which the error flows, may be found at any stage of reasoning, and that the error of the premises may be due to any of numerous causes.

What is called the error of false premises may be due to the failure of any of the Fundamental Processes.

The error of Immediate Presentation of mistaking a red light for a green light may be considered as a case of false premises in regard to the reasoning proceeding from that point.

In fact, as our course of reasoning proceeds in ordinary life, every fresh step accomplished affords the premises for the succeeding steps, so that error of any kind in reasoning may result in false premises.

If we consider the star *a* Centauri of the pointers of the Southern Cross as a single star, and base a course of reasoning on that assumption inconsistent with its being a double star, we have a case of false premises due to wrong comprehension of the Unit. If we see a canvas daubed with paint, and fail to place ourselves so as to recognise the unity of the picture, we may in our reasoning commit errors due to false premises. It will not be necessary to recapitulate every case in turn. The most frequent causes of error due to false premises are probably those which depend on wrong applications of a symbol, or wrong associations with a symbol correctly applied; and these cases in turn may be due to want of Discrimination.

In the example cited of the error regarding the rue de Londres it might have happened that no incident followed which called my attention to the mistake. In that case the associations of the symbol—viz. the general aspect of the Place—would be strengthened. If afterwards,

then, I had learnt and had applied the verbal symbol Place de la Gare St. Lazare to the Place in question, then a strong association would be formed between this symbol and the rue de Londres. If these associations were repeated frequently enough, and especially if they were learnt at an age when the mind is most impressionable and most tenacious, then it might happen afterwards that, even if facts in contradiction with the original error were noticed, the tendency would be to explain them away by some other kind of false reasoning, rather than to revert to and examine the original source of error.

Thus, for instance, I may cite, as bearing on the subject—and it is always useful to trace such relations—a distinguished professor of engineering who used to describe the so-called "practical man" as the man who had perpetrated the same blunders so often that he could not do otherwise.

Taking an example from engineering: A practical man having had occasion frequently to strut up weak structures with extra pieces, proceeded on the assumption that this was always a good plan in case of doubt. He had to deal with a part of a structure consisting of a quadilateral frame in iron with a diagonal piece. Now, as a triangle is fully determined by three sides, so the quadrilateral frame was fully determined by four sides and its diagonal. But the practical man inserted another diagonal crossing the other, and strained the framework almost to breaking before any weight at all was applied. The whole structure subsequently fell.

Another case was where a practical man, having to construct a railway and also a reservoir, hit upon the idea, brilliant in itself, of making the embankment of the railway serve for the bank of the reservoir. He

was under the delusion that the pressure of water on an area of the bank depended, not on the depth of the area below the surface, but on the superficial area of the water in the reservoir. The reservoir was not large in this respect, and accordingly the engineer thought it useless to fortify the bank by a puddle-wall. An appalling disaster duly happened.

Or, again, a medical man, knowing that he must operate at once in certain cases of peritonitis, finds a patient presenting certain of the symptoms, and performs an abdominal operation for what was really a case of acute glaucoma—a disease of the eyes—with intense suffering producing various symptoms, including those observed. Here the false premises have been due to inefficient diagnosis, that is to say, failure of Discrimination.

In many of these cases of false premises a distinct emotional factor plays an important part (see also pp. 356, 359). For example, proceeding from the case just cited, our young surgeon, say, receives an admonition which causes him to take great interest in glaucoma, one of the symptoms of which is a hardness of the eyeball. Soon afterwards he finds a patient with a tension of 3, as he calls it. Now had he expressed himself in language less technical the association would be less strong; but he had learned that tension 3 indicates acute glaucoma. He decides to operate, but he discovers at length that the eye in question is a glass eye.

Recently the Museum in Paris bought a scarabæus from the widow of an eminent Egyptologist. The scarabæus had an inscription which confirmed the recital of Herodotus about the celebrated circumnavigation of Africa organised by the Pharaoh Necho some six hundred years B.C.

In this case also the emotional factor came into play. The inscription was so important and in every way so interesting, that, in short, the desire to give free course to all the associations it evoked prevented a critical examination of the validity of the claim made for the scarabæus. The symbol was erroneously applied through defective Discrimination. The premises were false. The scarabæus had in fact been manufactured in Montmartre.

A case celebrated in literary history is that of the letters of Pascal, which were sold to a distinguished mathematician and member of the Institute of France, M. Chasles, well known for his work on anharmonic co-ordinates and for his researches into the history of the early developments of mathematics. The purport of the letters was to prove that Pascal was really the inventor of the Differential Calculus. The delight of M. Chasles at this discovery blinded him to the lack of evidence of the authenticity of the letters, which were afterwards shown to be false. The argument which Chasles had drawn from them respecting the development of the Differential Calculus was thus erroneous, owing to false premises.[1]

§ VI. Errors of Classification

For errors due to faulty classification the formal statement might be made thus: If a, b, c, d, e be necessary conditions in the widest sense for the production of any phenomenon, and if we have frequent experience of a, b, c, d, e producing x, and if e, for example, be a condition not readily observable in comparison with the others; then it may happen that already when we observe a, b, c, d we anticipate x: and if at

[1] Alphonse Daudet used this story in his delectable novel, "l'Immortel."

any time the series be really a, b, c, d, f we may expect x, and be not aware of our error until the non-appearance of x causes us to examine the conditions more closely. Here we have really had the series a, b, c, d, f wrongly classified as that of a, b, c, d, e; and the results due of a, b, c, d, e have been wrongly associated with a, b, c, d, f.

Popular errors are very frequently due to this cause. Let us consider, for example, those of circumstantial evidence. The house of an artist, a painter, has been robbed. The wife of the artist says that she was bound and gagged by three men—one with a red beard, and another with a black beard—who were in company with a woman. Subsequently the police arrest a poor sculptor and his friend who live in the neighbourhood. The friend has a red beard. His description tallies with that given by the painter's wife. The sculptor has no beard; but it is proved that he used to wear a black beard previous to the robbery. The sculptor and his friend had set out for a long walking tour about the time of the robbery. Previously they had been in want of money. On the walking tour they had plenty of money. On the walking tour they were accompanied by a woman. Their habits were what is known as Bohemian. The public cried, "Down with the assassins!" But, as a matter of fact, the sculptor and his friend were a hundred miles away from the scene on the night of the robbery. The sculptor had removed his beard for reasons of artistic variety. The sudden access of means was due to the sale of his works. And so all the links of the circumstantial evidence fell one after the other. Moreover, the supposed robbery had never taken place.

In a case like this the value of the "chain of evidence" should be estimated in comparison with the value of other possibilities. Thus, in a large city such as Paris there

might be hundreds of men with red beards, and scores of artists who, being pressed for money one day, might be well in funds the next.

Yet at a certain moment the public were for the most part inclined to consider these men the culprits. Such a movement of the mind is popularly called jumping at conclusions, and the fact that it is so designated, the designation acting as a warning, indicates how popular a form of error it is. It is associated often with what is called an impulsive character, and that also indicates its connection with errors of Association; for associations in our typical case between a, b, c, d, x are strong enough to become dominant before they can be overridden by the Disassociation proper to f.

The fact that this error can be demonstrated either as a failure of Classification, or a failure of Association, indicates once more this necessary connection between all the Fundamental Processes (cf. p. 35).

It is convenient, however, to exhibit it as a failure of classification, because in this way attention is formally called to the necessary conditions; and the habit of ascertaining the existence of all the conditions becomes itself an association of influence in such reasoning.

For example, in any chemical experiment we must have certain conditions established before the experiment can be performed. To take one of the simplest instances, we have oxygen and hydrogen mixed together in the proportion of 1 to 2 by bulk in a jar. Here already are essential conditions for the disappearance of these gases, as gases, and the production of water. But another condition is necessary, the passage of an electric spark. Then the whole of the conditions are satisfied, and the expected phenomenon is shown. Again, an amateur chemist reads in a book that if phosphorus be placed in contact with iodine spontaneous combustion will occur.

He exhibits the experiment to a class, and he finds that the expected result does not take place. By experience, however, he learns that the phosphorus, after being taken out of a jar containing water, should be dried carefully. This constitutes a condition which, though essential, is not so prominently brought to the attention as the other conditions.

We see here, too, the effect of training, for each experience of failure calls attention to the condition thus involved. Hence every man in his own trade or profession is relatively efficient in such reasoning, although possibly prone to error in other subjects. And all adopt some form of Classification. Thus a lawyer in reading over a deed of conveyance examines all the necessary provisos. These are the conditions necessary to secure the validity or efficacy of the deed; that is to say, they are the conditions of classification of deeds with which certain associations, such as those of validity, are formed.

A hostess who casts her eye over her table to see that all the necessaries are present uses her faculties in a similar way. A policeman who tries to recognise a man from a description of physical characteristics considers one by one the elements of the classification.

It is the mark of a well-trained mind to examine, in all circumstances, whether the necessary conditions are present before applying a symbol or expecting certain associations. This training gives the character of mind to which popularly the terms judgment, good balance, level-headedness, are applied.

All errors of the application of symbols may be expressed as errors of classification. For example, we call a whale a fish, and thence proceed to associate with the symbol, whale, all the qualities associated with the symbol, fish.

At length we discover certain incongruities, such as

that the whale possesses a breathing apparatus unlike the analogous organs of the fish. We are then led, as a result of our examination, to define more closely the qualities according to which we would apply the name fish or whale. That is to say, we classify. The application of the principles of this classification implies here a good deal of special knowledge. Sometimes the special knowledge may demand a considerable study of a branch of science, and even then the principles of division may be so difficult to express that the opinions of experts may differ. This may occur in the problem of placing low forms of life, such as sponges, in the animal or vegetable kingdoms.[1]

In certain instances the difficulty may be that, even if the classification according to which we apply a symbol be well determined, we may be unable to ascertain whether a particular object possesses the properties indicated in the classification. Thus, if the tuberculosis of cattle were identical with the tuberculosis of human beings, then it would be necessary to take stringent measures in regard to cattle, living or dead, in order to minimise the possibilities of infection. But hitherto there is a great conflict of opinion in regard to this problem. Most of those having some acquaintance with the subject are inclined to believe that the two kinds of tuberculosis are identical. The application of the same name already indicates this, and this symbolisation, again, has a consequence in making observers more liable to adopt associations in accordance with it. But Professor Koch

[1] Sir Ray Lankester has recently (1911) given under the familiar title, "Science from an Easy Chair," two admirable articles on species, and the principles of classification of species, and he there treats of the difficulties of the "systematists," who make a special study of such principles of classification and their application to certain species. In Pleistocene times the muir-fowl, or red grouse of Scotland, and the rype, or willow grouse of Norway, formed one species. Sir Ray Lankester poses the problem as to whether they should now be classified as of distinct species.

and many other bacteriologists of the first rank have held that the two diseases are not identical.[1]

Hence we have a very important practical problem which becomes reduced to the issue of determining the proper application of a symbol. But that determination involves a series of tests, intricate in themselves and demanding a mastery of technique and a power of observation so great that for years after the question had been mooted no solution which convinced scientific authorities had been obtained.[2]

In other cases it may be that the properties possessed by an object are not in doubt, but that the question is: In what manner should those properties be estimated in order to decide the application of a symbol? For example, take the term, Great Man. With this term we associate certain circumstances. After death the body of the Great Man is interred in Westminster Abbey; or in Paris, in the Pantheon.

In Paris, the question was posed as to Zola being

[1] An explanation of these discrepancies may be found in the Report of the Royal Commission on Tuberculosis (Human and Bovine), 1911. (1) The tubercle bacillus shows great variability. The general conclusion, according to Dr. A. Eastwood, is that, with the exception of certain viruses of avian origin, every variety of virus obtained from the lower mammals has also been found in man.

[2] Cases of a somewhat similar kind may arise even in mathematics, but here the difficulty, though demanding a high degree of skill in the demonstration of properties necessary for classification, may become resolved with certainty. For instance, in the investigations in the field of integration we arrive at the tabulation of certain kinds of elliptic integrals. Conditions may be set forth under which expressions differing from those of elliptic integrals may yet, by certain mathematical devices, be reduced to similar expressions. Thus, we may seek to find the curve named Delaunay's Curve, which is traced out by the focus of an ellipse which rolls without sliding on a straight line. In the terms of this problem there is no statement of elliptic integrals; but if, with the knowledge of the properties of the ellipse, we express the conditions of the problem in mathematical symbols; and if we transform according to mathematical principles the expression so obtained, then we arrive at length at a form which we see corresponds to that of an elliptic integral. In this case a high degree of skill is required to demonstrate that the conditions exist necessary for the classification in question; but when once that demonstration is accomplished no doubt remains.

a Great Man worthy of such honour. His admirers urged the greatness of his literary career, and particularly his self-sacrificing endeavour to secure justice in the Dreyfus case. His detractors urged principally the grossness, if not the immorality, of his books. Finally the admirers of Zola carried the day, although not without violent protest on the part of his adversaries.

In England similar questions have been raised, though the consideration of the matter has taken place in a calmer atmosphere, and the less powerful representatives have been in a negligible minority. Thus, after the death of Herbert Spencer his admirers considered that he might be called a Great Man—on the grounds of his having possessed one of the noblest minds of all time, and that this mind was, with marvellous consistency and force, ever devoted to the cause of truth. His adversaries urged his want of conformity to certain opinions accepted by the majority. In this case the title of Great Man, with its necessary association of burial in Westminster Abbey, was denied.

In the case of Sir Henry Irving, the adversaries urged that, even if Irving had been a great actor, no skill in a mimetic art should entitle any one to be called a Great Man. His admirers held that he was the representative of the histrionic profession in England, and in this case the title of Great Man was granted, with its association of burial in Westminster Abbey.

§ VII. Emotional Influences in Reason

Emotional, or generally affective, conditions play a part in influencing what at first might be taken to be cases of pure reasoning (cf. pp. 350, 362).

Let us consider an example used elsewhere for another purpose. A child has been playing with a ruby (cf. p. 567).

Then afterwards it mistakes a piece of live coal for the ruby, and this error leads to unpleasant consequences. Subsequently the child sees the ruby again and proceeds to play with it. The mother of the child, mistaking the ruby for a live coal, calls out suddenly to the child to beware. Instantly the child draws back its hand.

In this case there has been no failure of Discrimination, but a new series of associations of great dynamic Impulse has been set in motion by an external stimulus affecting another part of the brain.

In this case, as far as it is possible for us to judge in such matters, we find in the physical correlatives an actual conflict of physical forces, and the greater part of the whole process is carried out subconsciously.

With more experience and less faith in the maternal advice, or less inclination to obey the maternal command, the child might have relied on its own faculties, and in that case its first desire would have prevailed.

What we have here exhibited as the emotional factor in a simple form becomes developed in accordance with all the faculties which contribute to our reasoning in complex affairs. The emotional factor exists none the less when it has itself taken complex forms, and when it is revealed in the predilections for one form of Belief or another, in politics, in religion, or in the hopes and desires of personal ambitions. Popular observation has indicated these truths in the time-honoured dictum of convincing a man against his will.

Any strong desire to believe the consequences that flow from any fact will create in the mind a tendency to accept that fact. A judge has a prejudice against a prisoner whom he wishes to punish. He is content with scanty evidence of guilt. Or he has a strong predilection in favour of a prisoner, and it is repugnant to him to

inflict a penalty. In that case he requires proof amounting to metaphysical certainty. The story of the man who cried, "I see it, but I don't believe it," may not be apocryphal.

FAULTY ASSOCIATIONS WHERE PREMISES CORRECT

The inverse instances, where premises are not false in themselves but the consequences attached to them are false, may now be considered. The symbol is correctly applied, but the associations have not been found in experience, or have been misapplied through want of proper Discrimination.

The Chinese, for example, were able to predict eclipses. So far, therefore, their application of the symbol corresponding was justified. But they attributed to the eclipse consequences that were not necessarily connected with it.

This view of eclipses pervaded Europe, and Milton refers to it in his "Paradise Lost":

> And with fear of change perplexes monarchs.

Nor were the false associations with regard to eclipses destroyed by any direct arguments, but by the fact that, the Copernican system having been on various grounds well established, all that necessarily derived from it became accepted. Hence a rational view of the causation of eclipses prevailed and no associations derived from the false view were strong enough to counteract the true explanation.

The Chinese, believing that in the eclipse the moon was in danger of being eaten up by a dragon, sought to frighten the dragon away by beating tom-toms. The fact that, after this performance, the moon remained

intact served to convince them that their notion of the dragon was correct.

To express this formally, we would say, suppose that a is associated with b, c, d, by faulty reasoning, and that this reasoning is doubted. Then if, afterwards, there be found veritably associations of the kind indicated between b, c, d, then the tendency is to accept the reasoning, a, b, c, d.

In cases of this kind emotional factors very often exercise influence, as we shall see.

In the meantime it may be noticed in what way errors so arising are related to those arising from consistent reasoning on the basis of false premises.

For example, in searching for the rue de Londres (cf. p. 345), I met with certain experiences that formed a barrier to any further progress of my associations, and at length forced me to call in question my first assumption—that of the "premises," erroneous on account of faulty Discrimination and faulty symbolisation leading to faulty associations.

But if I had in my progress met with a number of experiences consistent with the notion that I had found rue de Londres, although really consistent also with a contrary notion, my Belief would have been strengthened that I had really found rue de Londres.

This observation is of considerable importance practically, for in matters of some complexity, and especially those involving vital interests, or interests greatly exciting the emotions, it will seldom be found that the questions are argued reasonably between opponents, but that the issue to which the argument is conducted is often something quite irrelevant.

In the celebrated dialectical duel between Gladstone and Huxley relative to the morality of the affair of the Gadarene swine, it may be said, without here taking

any side in the contest, that the discussion degenerated into a series of curious arguments on the authenticity of certain records or the weight of certain authorities. The two champions had their warm partisans who followed the wrangle eagerly, and who, as each of these inessential points seemed carried by one or other combatant, felt their whole ethical system confirmed or menaced.

Or, again, a politician argues that the whole principles and aims of the Russian Nihilists are bad. An opponent challenges this view. From point to point the argument is conducted, until it becomes a question whether Father Gapon was really put to death by the Nihilists who had discovered his treason to their cause. In this argument considerable knowledge of facts of Russian life and history is displayed, and the discussion involves a good deal of mental energy. Finally, it is decided that Father Gapon was so put to death, and the anti-Nihilist leaves with the assumption that he has carried his whole position; and, though this may not be granted by his opponent, yet even he feels that a serious blow has been given to his ethical principles.

Or again, Gibbon, in his "Decline and Fall of the Roman Empire," propounds a theory of the downfall being mainly due to the establishment of the Prætorian Guards. This factor seems absurdly inadequate to those who have thought of the immense number and involvement of the forces that make for the development of nations, and consequently of the failures of these forces as accounting for their disintegration. But if the argument turn on the correctness or otherwise of Gibbon's account of the Council of Nice, then the investigation of such a subject requires considerable research, and implies the absorption of a great deal of energy.

If a man, then, be led to agree with Gibbon in this quite extraneous matter, he will, while under the strain of his labours and with the condition of mind brought about by his concurrence, be inclined to accept his positions about the decline and fall of the Roman Empire.

Again, to consider another aspect of this question: If b, c, d be associated with a so that d might be consistent with the series a, b, c, which is well established; but also might be inconsistent without destroying the series a, b, c; then, if the series a, b, c, d has been advanced, and d has been disproved, the tendency is to hold that a, b, c is not established.

For example, it is argued that the whole procedure with regard to the execution of Admiral Byng was correct, and that he was properly executed for his timidity or want of enterprise in not engaging the French battleships near Majorca. The question of procedure is challenged. It is shown that politics affected the decision in the case, that the Cabinet desired a scape-goat. Then the tendency of the mind of the person so persuaded is to absolve Byng in the affair of allowing the French ships to get away.

Again, the whole course of Parnell's regime in Ireland is being investigated by a Special Commission. The most serious charge is that of being the author of letters sold to *The Times* by a certain Pigott. This charge is disproved, and practically the whole case becomes thus disposed of.

Or again Lombroso, the celebrated Italian criminologist, put forward on insufficient grounds a generalisation to the effect that criminals display various physical signs, such as asymmetry; and conversely that persons displaying these signs are criminals. Already Lombroso's arguments have been demolished by various scientists,

who have shown that several of the greatest men of the world, and highest in the ethical scale, have displayed marked facial asymmetry; while others have again observed that asymmetry is normal and almost invariable owing to pre-natal conditions.[1] Prof. Debierre, of Lille, after a minute examination, anthropometrical, anatomical, and histological, of the skulls and the brains of four bandits executed at Bethune in 1909, declared that there were no signs whatever in any of them of any physical degeneracy or anomaly. Thereupon the tendency of those who read these statements was to conclude that no connection existed between the physical condition and moral development.

Again, the theory of Spontaneous Generation of life was held for centuries even by men of ability, from von Helmont to Bastian. Microscopical investigation, showing the great complexity of organisms formerly described as structureless, and the discovery of "germs," served to discredit the theory. It seemed not only untrue but absurd. Then Mr. Burke, of Cambridge, astonished the world, for, in the course of experiments with radium on certain culture media, he obtained some results which he could not explain. Certain appearances were noted which seemed like living organisms, and these he found under conditions that would exclude germs.

Those who satisfied themselves as to the sterilised condition of the media jumped to the conclusion that Mr. Burke had produced life. Those of the general public who were interested reverted largely to the doctrine of Spontaneous Generation. Certain professors of special doctrines went further, and declared that these

[1] Asymmetry is normal not merely with regard to the head but throughout the body. Cf. E. Gaupp, "Die normalen Asymmetrien des Menschlichen Körpers" (1907); and Haemelinck, "L'Asymétrie du sens gustatif" (L.A.P. 1905).

results proved the whole of their teaching. A great creed of religion was declared to have been established by these experiments (cf. p. 356). Then came a reaction. Spontaneous Generation no longer held the field. Thinkers asserted that Mr. Burke had gained his reputation only by his ability to misinterpret phenomena, and that it was now clear that by no means could life be artificially produced.

Prof. Yves Delage announced the result of his researches (cf. p. 487). He had taken the greatest pains to exclude sources of error, and he had with marvellous patience, after incessant failures, arranged conditions for the production of sea-urchins from eggs artificially fecundated. Moreover, he had reared four of these urchins to the full-grown form.

The great reputation of Prof. Delage produced a readier acceptance for these results than could otherwise have been the case. This was such a shattering for preconceived opinions in the case of some, and such a striking confirmation of the hopes of others, that once again far-off and even fantastic theories were considered as established.

In the cases of this kind hitherto noted the emotional element has considerable influence (cf. pp. 350, 356). Its effect is still greater to an individual when the question is one affecting either his interests or some theory deeply rooted in his prejudices or predilections. Darwin recounts that one of his geological friends tried to assure him, against all evidence, that a certain fossil was not found in a given locality, for otherwise the argument of his whole geology of the neighbourhood would be wrong; four of his volumes would go for nothing. This incident made a great impression on Darwin's mind. It showed him that the pursuit of truth was not merely a question of the ordinary exercise of the mind, but that it demanded

qualities of courage and will not less than those of intellect.

We have already noted the tenacity of impressions early created (cf. pp. 95, 291, 292, 348,); and we have noted the force of constant repetition in influencing Memory, and in facilitating associations (cf. p. 291). If, then, these influences be now considered in connexion with the sources of erroneous reasoning we have been discussing, we shall be better able to account for the fact that two persons using the ordinary faculties of ratiocination, but "brought up" under different regimes of religion or politics, may be unable to see any merit in each other's arguments, though each be convinced of the infallibility of his own.

The sole counteractives which it is possible to apply are those which in most cases are inapplicable, viz. a discipline of looking upon truth as the sole great thing in these matters, a disregard of the questions of personal interest in anything but the truth, an absence of ill-founded prejudices or preconceived notions, a habit of searching to the base in the conduct of reason, a freedom of mind towards new or unfamiliar ideas.

If we now revert to the consideration of Belief we may find that our view of the matter has become clearer. The difficulty of a definition of Belief resides in this, that Belief itself, at least in its superficial aspect, is more easily understood than the terms by which we might seek to define it. We have, however, seen that in the operation of Reason the mind proceeds along new experiences, applying the Fundamental Processes, but often erroneously, owing to the defect of any one of them. Errors are caused, for instance, by limited Discrimination, which may be due to imperfect Memory, producing faulty symbolisation, which again brings with it a train of associations inconsistent with the present reality. A

certain experience may cause a closer attention to the sequence of experience. Discrimination may be made more precise. A more correct symbolisation may be applied.

§ VIII. Causes of Error further considered

The study of errors having been introduced we may now more formally consider what has been learned up to this point. In order to define our position by means of the Fundamental Processes, we may revert to the series a, b, c, d, where a, b is a false link in the chain, and where the discussion has turned on c, d, with the result of establishing this link, and producing thereby a tendency of the mind to accept the whole series. With persons little accustomed to connected series of arguments there is often a strong emotional feeling, be it respect, or admiration, or simple wonder, for logical processes in themselves, while the Discrimination is weak as to their correctness or falsity. In such a case the influence of mere authority may produce a Belief, which after some repetition becomes very strong, in regard to such a series as a, b, c, d.

Cases of the kind arise far more often than one might suppose. A favourite author pleases our fancy, our prejudices, or has some special theory, or emotional attraction, that strongly appeals to us. We admit his whole philosophy accordingly. Thus a man may be immensely struck with Carlyle's great passages in "Sartor Resartus" on the Everlasting Nay and the Everlasting Yea, as holding up a vivid and powerfully drawn picture of states of mind within his own experience. His admiration leads him to accept Carlyle's whole philosophy in "Sartor Resartus," even without having rightly comprehended it, or even without examining it.

If subsequently he learns that Carlyle derived his philosophical ideas through Fichte, and ultimately from Kant, he accepts their whole philosophy too, or at least in so far as it coincides with what Carlyle reproduced.

Another aspect of this problem, though complicated with other matters, is that wherein young men of Byron's period, enthusiastic in their admiration, adopted his form of necktie. Or, again, we find it where the influence of Adam Smith's great work, "The Wealth of Nations," was so marked among the more serious young students of the North that they adopted even his tricks of speech. Or, again, a cultured feminine mind may resist George Bernard Shaw's Socialism until it finds in "Candida" a deep insight into the female heart, shown in many subtle nuances; then, under the influence of intense admiration, it accepts not only George Bernard Shaw's Socialism but possibly also his anti-vaccinationism.

With those minds more accustomed to lengthy ratiocination there becomes impressed at length a certain type, or model, of the chain of ratiocination itself. A form of the process of Reason becomes a Unit. In the process of analysis of a proposition, for example, the conditions are examined in which the solution of the problem will be accepted; any of these conditions not accepted may be examined in turn, and the conditions of its acceptance noted; and so on, until we arrive at a condition which is established.

A comparison may be made here with the methods of men of the law in dealing with an intricate case between two parties. The presentation of the case on one side is compared with that on the other; certain facts on which the parties are agreed are accepted; then those in which there is a disagreement are examined. With regard to those which are essential the question resolves itself

into the examination of two versions to discover which is correct, or which is in accordance with the law. Thus the case is brought down to a definite issue, or definite issues.

Now when such an issue is decided the mind reverts to the whole case in a mood of success or disappointment, according as the issue is decided favourably or unfavourably.

Similarly, in the examination by way of analysis of a proposition, say, of geometry. The proof is at length made to depend on a certain condition; and when this is established the mind reverts to the original proposition in a mood of acceptance or Belief.

Now in such a case as that of the series mentioned, a, b, c, d, when the link between a and b is weak, the mind, absorbed by a long and intricate discussion which finally establishes c, d, has forgotten perhaps that the link a, b was not properly established. We have already noted the influences on Memory of mental occupations which absorb attention and demand considerable mental energy. Hence in a failure of Memory of this kind is to be found one source of error.

But it may have been expressed originally that the link a, b was true under certain conditions. The limitations may have escaped from Memory, either because of the subsequent absorption of energy, or because in the first place their importance was not observed. This would be a failure of Discrimination.

But when, after discussion, the link c, d is established, an emotional condition arises which facilitates the acceptance of the link a, b; then we have here first a failure of Discrimination in confusing this problem with that of a known process of correct reasoning, and this failure of Discrimination is confirmed by false symbolisation; and this leads to the false association between the emotional

condition, or intellectual satisfaction, and the condition of acceptance of the link a, b, or Belief, respecting it and its consequences.[1]

We shall have no difficulty with the case when a, b, c, d represents a series where a, b forms a good link, but c, d is determined as a bad link, and where in consequence although c, d is not a necessary consequence of a, b, yet a, b is rejected.

This case bears to the form of demonstration known as *reductio ad absurdum* a relation similar to that which the case previously considered has to the more direct analytic process. The causes of error become easily shown by this comparison.

A common form of error arises from the application of symbols of which the meaning is not fully determined, and where possibly the analysis necessary to express the meaning completely involves other symbols also not fully or clearly defined. Thus we have a symbol A, which covers certain properties, a, b, c, d, etc.; where d is itself a symbol covering z, y, x, w; where w is a symbol which covers m, n, but which has been applied without ascertaining whether it covers p or q.

In such a case it may happen that the labour of a long analysis is itself sufficient to cause error to arise in regard to the proper application of the symbol (cf. pp. 46, 322). Moreover, the analysis may lead to

[1] Even in the severest forms of reasoning, and with those least likely to be influenced by any emotional considerations, errors of this kind may occur. For example, in treating of monodrome functions—that is to say, functions which return to the same value when the variable returns to the same value—we may prove that when the variable has left a point and subsequently returned to it after having moved along a contour enclosing no critical point, then the variable does return to its original value.

We may afterwards erroneously apply this conclusion so obtained, for we may fail to observe that the contour in this instance does enclose a critical point. The inconsistencies which result may cause us to search for the source of error, which may have been faulty Discrimination in the first instance, or simply failure of Memory.

some difficulty which requires exceptional powers of Discrimination.[1]

Now suppose the question to be one of practical import, such as those complex questions of changing the currents of the commerce of nations; the individual makes up his mind the more quickly in proportion as his analysis is limited and thus soon brought to an end. If a sufficient number agree with him, for similar reasons, the laws of the country are determined by what is called collective wisdom. But, as we have observed, there are possibilities of error.

If a proposition be advanced that all fevers except those of the nature of malarial fevers are due to microbes,

[1] Some of the most typical examples are afforded in mathematics, although that is the science in which the terms are all most precisely defined, and the reasoning is rigorous in its exactitude. Let us take an example not really obscure. If we have

$$x = \int_0^y \frac{dy}{\sqrt{(1 - y^2)(1 - k^2 y^2)}} \qquad (a)$$

$$\text{and } K = \int_0^1 \frac{dy}{\sqrt{(1 - y^2)(1 - k^2 y^2)}}$$

And if further we pose $\mathrm{Sn}x = y$, we are asked to conclude, $\mathrm{Sn}K = 1$. In the event of the mind not immediately following the process of inference, it would be necessary to consider what is involved in the sign of integration, and what is the meaning of such an expression as

$$\int_0^y \frac{dy}{\sqrt{(1 - y^2)(1 - k^2 y^2)}}$$

and of the exhibition of its value in such a form as $\psi(y) - \psi(0)$. We should find, as implied in the properties included in this particular form, that $\psi(0) = 0$.

Whence we would take in this case, $x = \psi(y)$,

$$K = \psi(1).$$

Whence if $\mathrm{Sn}x = y$

$$\mathrm{Sn}K = 1.$$

But in each step the meaning of the symbol should be fully comprehended, and care should be taken that in the adaptation of similar forms nothing further is implied in the symbol than is warranted by the general form, as apart from the particular example.

But suppose that a person were insufficiently acquainted with the theory of the integral calculus itself, then in order to obtain a clear explanation of the symbols he would be required to investigate this subject, and to investigate this subject he would be required to analyse until he reached the foundations of mathematics. It is possible that the complexity of the subject would in some instances prevent his forming a clear insight into the proposition from which he started.

we find a similar difficulty. In order to decide the question one way or another, we first seek to define the word "fever." But suppose we content ourselves with some general expression, involving rise of temperature and malaise. Then we have to define what is meant by malarial fever. In this case uncertainty might not arise. But suppose, then, that we included amongst fevers certain chronic diseases such as yaws, because they exhibit at one period rise of temperature and malaise; and if we discovered, in cases of yaws, a microbe of the form of spirochæte, then the problem would be resolved into determining this as the cause of the disease. In that case we might be without adequate means of determination. And so we should be unable to say whether we agreed or disagreed with the original proposition (see p. 375).

We may take another example from subjects not at all technical. A lecturer advances the theory that high imagination is always found associated with low morality.

In this case it is necessary to define what is meant by the word "imagination." Poets are said to be imaginative. Many might consider poetry to mean something expressible in rhythm or rhyme, but without common sense. Or, again, in the poetry of Moore what is called Imagination would seem to be a certain constructive faculty which others would term Fancy. Wordsworth seems to use the word "imagination" in this sense. If Wordworth's "Tintern Abbey" be taken as a poem of Imagination, then the symbol would indicate rather a faculty of reproduction by the force of subtle associations of deep-remembered feelings.

In the case of Keats the term "imagination" might be taken to mean a profound faculty of generalisation, as in scientific imagination, but applied to objects

viewed in a poetic atmosphere, and inspired by a soul exuberant in beautiful forms of association.

If finally the word were applied to the works of Burns, then one aspect of the question would be the consideration also of the term "morality." There are some who would condemn the poet formally and unsparingly on account of breaches of social observances. Others might endeavour to give the word "morality" a wider scope, embracing the whole aim, importance, and effect of a man's work; and they might point out the bold and manly character of Burns, his freedom from meanness, his ardent love of liberty, and the manner in which, giving expression in wonderful lyric form, or in racy pictures, to all that was genial in Scotland, he expressed in immortal tones the best feelings of the people, and quickened the soul of his country to life.

But amid the difficulties of analysis and the uncertainty as to the meanings of the symbols, and since there is conflict of judgment, it is evident that errors must arise.

§ IX. Provisional Résumé in regard to Reason and Belief

But what is the ultimate resort of such an operation of close examination? (cf. p. 337). It is this: That we eventually reach a limit of the exercise of all our processes. We may hesitate with regard to the mind's progress in one direction or another—in Discrimination, for instance. Certain associations, however, are found to be invariable in our experience—such as, for example, simple immediate visible cases of cause and effect; the unchanged character of certain spatial relations in changed situations; the main recurrent phenomena

of the order of the Universe. Now by no determination of physics, by no process of imagination, have we been able to indicate for cause and effect anything more certain than that of sequences invariable in our experiences and what we know of human experience. The constancy of geometrical figures and of certain phenomena of the Universe is a matter of experience, but only in regard to an experience limited in Discrimination and conditioned by limitations also of Time and Space.[1]

Nevertheless, certain sequences and certain constants have so prevailed in the Universe, as it has become known to human beings, that through countless generations their whole physical constitution and their mental processes have, through the actions of various forces and processes, become developed so as to act in accordance with these phenomena. The first necessity for the individual has been the preservation of life; then there have been formed various stimuli to exertion due to living in communities, and there have been corresponding adaptations; then with increasing precision of powers and vast storehouses of Memory, and with a developed symbolisation, and all that it implies, there has been highly evolved that representation in idea which corresponds to experienced realities, and which enables the mind to form ideas also, or notions, of its states and movements; then, in referring these to the present realities, we form the operations which we include under the name Reason.

We have seen with regard to Memory (cf. p. 234) and Association, and it may be noted with regard to all

[1] Recently (1911), at the Philosophical Congress at Bologna, M. Henri Poincaré discussed a question raised by M. Boutroux as to whether the laws of Nature were immutable. M. Poincaré, in accord with Prof. Langevin, celebrated for his work in radio-activity, concluded that what were called laws might change, except, possibly, certain fundamental conditions of the molecules. All that is here written is in accord with these speculations, if it be well noted that they are philosophic and of extraordinarily remote application.

the Fundamental Processes, that once certain conditions are posed the mind moves necessarily in a certain way. This corresponds with all that we have hitherto discussed. But the mind has consciousness of certain of its processes, and, since the processes extend throughout the scope of its consciousness, it has consciousness of some processes as being the removal, or rejection, or the negation, of others. This also applies to those processes which are as fundamental as we know, and which are in the necessary manner of activity of the mind. When the mind, then, in experiences as fundamental as it knows, moves in accordance with the natural exercise of its Fundamental Processes, its condition at each step, corresponding to a certain Hedonic principle involved in the natural exercise of these Processes, is that of Belief. The condition involved in the conscious act of rejection in the natural exercise of these Processes is that of disbelief.

Belief is not a passive state. The mind is in continual progress. Consequently Belief has reference to the activity of the mind in meeting new experiences in accordance with the condition of mind already acquired.

If now we consider that graphic representation which is afforded by the physical substratum itself, the nervous system, we may rightly contemplate it as a complicated dynamic machine, developed in accordance with our world through countless ages; responding to various forces, within a certain scope, of the world; and becoming itself modified by that activity of response. And when we now consider that various of its activities, within certain scope, have correspondences in consciousness, and that its natural activities are accompanied by certain Hedonic conditions, often made more evident by the converse state: then we see that Belief corresponds to what we might in analogy call the resultant of

all the previous forces acting on this dynamic machine; and we note that this resultant implies a certain phase of the dynamic machine in regard to new forces which make themselves manifest.

Now we have seen that the speed with which the mind may entertain ideas may be greatly increased by exercise; but in our progress through the world we meet with a multitude of impressions of all sorts in rapid succession, and, even to those who have formed an analytic habit of mind, time is lacking to reduce our observations to any great degree of certitude. We apply symbols with little Discrimination, for this, though a source of error, is yet the only means by which we can continue our mental activities in the face of realities. Hence arise errors even in cases where the natural exercise of faculties has not been warped by prejudice or early false training.

But when the mind has been early trained wrongly, errors become inevitable which may produce errors on errors in their consequences. But error may be quite consistent with Belief; for example, a symbol may be applied to a certain individual, or to a favourite theory, or to any nation having associations not justified. This symbolisation may be applied in natural exercise, and Belief in its correctness may even cause a refusal to acknowledge the errors which become manifest as a consequence of its application (cf. pp. 345–348).

Belief in errors through a wrongly imposed Belief are, in a high degree, less likely to influence the mind in regard to those sequences of nature which are accepted as invariable. But the whole course of reasoning from such a basis is so long that many causes of errors are by that circumstance introduced. This is shown in the consideration of mathematics, which is yet the best example of such a course of reasoning.

That Belief does not necessarily correspond to the realities of the external world, that it may, in short, depend in great measure on factors peculiar to the individual, is shown by the cases of hesitancy, or doubt, in regard to propositions.

This is not necessarily produced by the failure of any of the Processes on which our reason depends, for such conditions are quite consistent with Belief, even though erroneous Belief. But there are instances where our faculties of Discrimination are near their limit, and we cannot determine definitely whether we observe, or do not observe, either some condition of an external object or some state of relation between objects.

In the dusk all cats are grey, a French proverb says. And to the dusk they also apply the term "between dog and wolf." Here we have a condition of doubt about material things.

Failure of Memory may produce a hesitancy regarding Belief if something be in part remembered. The exact words of a conversation may be a matter of doubt. The guilt of a person accused has been known to turn on the question as to whether he mentioned a crime on a certain day when it had not been made public, or on the following day when it was possible for him to have read it in the newspapers.

The most perplexing form of doubt is that which follows after reading a long argument couched in highly general terms, or after working at a demonstration in which certain assumptions that may be true are made. The mind is unable to decide whether the demonstration is true or false, and this condition bears with it, to some persons, a sense of intellectual humiliation.

The settlement of such a question as that of readjustments of the tariff of a country, for example, demands an acquaintance not only with the principles

of domestic economy, but also with the actual condition of the industries affected, and the condition of the persons whose purchasing power will be influenced by various changes, positive, negative, or of doubtful import, consequent upon the application of a changed tariff. It also demands a fine judgment in regard to the question whether a change of law, positively beneficial in certain definite directions, harmful in others, justifies the alteration of an actual system to which the general policy of the country has been adapted.

The question of tariffs involves also the mutual relations with countries outside the country which may adopt the change of tariff. The consideration of these matters involves not only an acquaintance with foreign politics, but also with the economical conditions of foreign countries. Moreover, the total review of the problem implies a consideration of these factors affecting foreign countries in regard to the advantages or disadvantages they would produce in the home country. There are very few men in any nation capable of forming a good judgment on such questions. There are only a few to whom it has ever occurred even to consider what is the whole scope of the question, so as to appreciate properly in detail and in incidence all the factors involved.

Yet the question of the adjustment of customs duties is made an issue at popular elections, and the ordinary voter decides, even with a certain emphasis of judgment.

This illustrates that sentiment of intellectual humiliation which affects a man when he finds himself unable to say that he believes or disbelieves a proposition that has been presented to his consideration.

Referring to the physical condition corresponding to these mental states, we find that Belief is represented by the manner in which a resultant is formed from our previous physical constitution, particularly of the brain,

and the nerve stimulus produced by the presentation of new objects and their associations.

This Belief has a tendency to be expressed in some form of action, or disposition towards action. A certain Hedonic condition corresponds to it. The condition of hesitancy implies a holding back, or inhibition, of the progress of the nerve stimuli. But this condition is not inactive either. Various tentatives are made which correspond to the flow of nerve-impulses along certain strands. When a correspondence with desired conditions is not found this flow is checked, and in succeeding tentatives it is directed along other strands, or, in hesitancy, again inhibited. With impulsive, or impatient, people this physical condition is unpleasant. Hence the desire to arrive at a solution in general accordance with the temperament of the individual may override the desire to make certain that that solution is correct.

The individuals most impatient of deliberation are those who would not usually examine sufficiently the terms of a difficult classification (cf. pp. 350 *et seq.*), and those again who, on a stimulus from the external world, form associations beyond such as may be warranted by the conditions of the external world which supplied the stimulus (cf. pp. 356–358).

In such a case decisions are not made according to any principle of ratiocination, but mainly on the basis of the individual character, or some dominant feature of an argument which has made an impression, or of direct personal interest, or by influences of environment, or by virtue of the last speech heard, or by following a banner, or by some multiplex prejudice or predilection, or by mere hazard. And in all this there is not the slightest guarantee that the result desired by the person so deciding will ever be realised.

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